ANALYTICAL SOLUTION OF 3D POISSON EQUATION USING SEPERATION OF VARIABLE METHOD

Prashant Mani¹, Manoj Kumar Pandey²

¹Research Scholar, ²Director
Department of Electronics and Communication Engineering, SRM University NCR Campus Ghaziabad

ABSTRACT

This paper focuses a solution of three dimensional poisson equation using separation of variable method for Fully Depleted Silicon on Insulator (FDSOI) MOSFET. The 3D Approach to solve poisson’s equation using suitable boundary conditions, results high accuracy to calculate the potential of the channel as compare to 1D,2D approach. Simple and accurate analytical expression for surface potential in channel are derived.

The modeling of MOSFET with 3D help to reduce SCE, DIBL, Subthreshold swing etc. Separation of variable method is used in solution of poisson equation, the equation can separated in on side having single variable.

Keywords: SOI MOSFET, Poisson Equation, 3D Solution

I. INTRODUCTION

CMOS CIRCUITS fabricated on silicon-on-insulator (SOI) wafers are gaining prominence in present-day very large-scale integration (VLSI) technology. SOI technology shows better performance over its bulk counterpart because of the following. As the individual devices are perfectly isolated, latchup can be totally eliminated in SOI CMOS circuits. The higher radiation tolerance of SOI MOSFETs, long channel SOI MOSFETs using the solution of one-dimensional (1-D) Poisson’s equation. As the device dimensions continue to scale down to deep submicrometer regime to obtain better performance, analytical modeling of these devices becomes more challenging.

The assumption of constant surface potential used in the charge sharing models is invalid for submicrometer channel lengths. Although these models are simple, they are not as accurate as the models, which solve 2-D Poisson’s equation. The solution of 2-D Poisson’s equation has been obtained using various approaches. The solution of 2-D Poisson’s equation by power series approach has also been obtained [11]. However, the solution of 2-D Poisson’s equation by power series approach is obtained by neglecting the higher order terms and, hence, it is not as accurate as the other approaches.

Analytical solution of 2-D Poisson’s equation by means of Green’s function technique [12] is another method to solve 2-D Poisson’s equation. Another well-known approach to solve 2-D Poisson’s equation is to separate the 2-D Poisson’s equation into a 1-D Poisson’s equation and a 2-D Laplace equation [13]. The ongoing device geometry scaling has pushed MOSFETs to the regime of both short-channel (for higher speed, lower supply voltage) and narrow-width (for higher density and lower power consumption) MOSFETs. Such MOSFETs, called small geometry MOSFETs, are very complex as three-dimensional (3-D) electrostatic effects affect the performance of the transistor. The analytical models for short-channel SOI MOSFETs discussed above take into account the scaling of the channel length and SOI film thickness but fail to take into account the narrow width effects, which become predominant as the width of the transistor is reduced. Narrow-width effects of ultrathin SOI devices are more complicated because of mesa isolation. An analytical model for of mesa-isolated fully depleted (FD) ultrathin SOI MOSFET that takes into account the narrow width effect has been reported [14].

This model solves 3-D Poisson’s equation using the separation of variables method. However, the boundary conditions used for the solution of Poisson’s equation are applicable only for bulk MOSFETs. In fact, to the best of our knowledge, there is no 3-D analytical model for SOI MOSFETs available in the literature that takes into account both short channel and narrow width effects. The 3-D Poisson’s equation is solved analytically using the separation of variables technique in the next section.

II. SOLUTION OF 3D POISSON'S EQUATION

The cross-sectional view of an n-channel SOI MOSFET along the channel length is shown in Fig. 1(a). The source–SOI film and drain–SOI film junctions are located at y=0 and y=Leff, where Leff is the effective channel length. The front and
back interfaces are located at Si-SiO2 at x=0 and x=ts, where ts is SOI film thickness. Toxf and toxb are the thickness of frontgate and backgate oxides, thicknessVgf and Vgb are the applied potential. In order to analyze the structure shown in Fig. 1, we need to solve both Poisson’s equation and current continuity equation. However, in the subthreshold regime, the currents are small and Poisson’s equation alone is sufficient. In this paper, we consider a fully depleted (FD) SOI film. The Poisson’s equation in the FD SOI film region is given by

$$\frac{\partial^2 \psi(x, y, z)}{\partial x^2} + \frac{\partial^2 \psi(x, y, z)}{\partial y^2} + \frac{\partial^2 \psi(x, y, z)}{\partial z^2} = \frac{qN_A}{\varepsilon_{Si}}$$  \hspace{1cm} (1)

where $N_A$ is the doping concentration and is the potential at a particular point ($x,y,z$) in the SOI film. The boundary conditions required to solve the 3-D Poisson’s equation are

$$\psi(x, y, z) = \frac{t_{oxf}}{\varepsilon_{ox}} \left[ \varepsilon_{si} \frac{\partial \psi(x, y, z)}{\partial x} \right]_{x=0} - Q_{st}^f = V_{gfl} - V_{f}^f \hspace{1cm} (2)$$

$$\psi(x, y, z) = \frac{t_{oxb}}{\varepsilon_{ox}} \left[ \varepsilon_{si} \frac{\partial \psi(x, y, z)}{\partial x} \right]_{x=t_c} + Q_{st}^b = V_{gbl} - V_{fb}^b \hspace{1cm} (3)$$

$$\psi(x, y, 0) = V_{bi} - V_{ds} \hspace{1cm} (4)$$

$$\psi(x, 0, z) = V_{bi} + V_{ds} \hspace{1cm} (5)$$

The boundary conditions given by (2) and (3) indicate that the potential applied at the front (back) gate is the sum of the potential at the front (back) Si–SiO interface and the drop across the front (back) gate oxide. In (2) and (3), and are the flatband voltages and and are the interface trapped charges associated with the front and back gates respectively, and are the permittivities for silicon and silicon dioxide, respectively. Equations (4) and (5) represent the boundary conditions at the source and drain ends of the channel, being the built-in potential of the n-p junctions and is the drain-to-source applied voltage. Equations (6) and (7) indicate that the potential applied at the front gate is the sum of the surface potential at the edge of the transistor and the drop across the sidewall oxide. In order to solve (1), it is separated into 1-D Poisson’s equation, and 2- and 3-D Laplace equation as

$$\frac{d^2 \psi_l(x)}{dx^2} = \frac{qN_A(x)}{\varepsilon_{Si}} \hspace{1cm} (8)$$

$$\frac{\partial^2 \psi_s(x, y)}{\partial x^2} + \frac{\partial^2 \psi_s(x, y)}{\partial y^2} = 0 \hspace{1cm} (9)$$

$$\frac{\partial^2 \psi_u(x, y, z)}{\partial x^2} + \frac{\partial^2 \psi_u(x, y, z)}{\partial y^2} + \frac{\partial^2 \psi_u(x, y, z)}{\partial z^2} = 0 \hspace{1cm} (10)$$

where

$$\psi(x, y, z) = \psi_l(x) + \psi_s(x, y) + \psi_u(x, y, z).$$  \hspace{1cm} (11)

III. Solution of $\Psi_1^{(1)}$
In equation 11, $\psi_1$ is the solution of 1 D POISSON equation, using the boundary condition given below.

$$\psi_1(x) = \frac{t_{ox}}{\varepsilon_{ox}} \left[ \frac{d\psi_1(x)}{dx} \right]_{x=0} - \frac{Q_{it}}{\varepsilon_{ox}} = V_{gs} - V_{ds} \tag{12}$$

$$\psi_1(x) = \frac{t_{ox}}{\varepsilon_{ox}} \left[ \frac{d\psi_1(x)}{dx} \right]_{x=t_{ox}} + \frac{Q_{it}}{\varepsilon_{ox}} = V_{gs} - V_{ds} \tag{13}$$

Now after solving the above boundary condition we find the solution in the form of

$$\psi_1(x) = \psi_{1b} + E_{s1b} (t_s - x) + \frac{q}{2\varepsilon_{s1}} N_A (t_s - x)^2 \tag{14}$$

where $\psi_{1b} = \psi_1(t_s)$ and $E_{s1b} = -((\partial\psi_1/\partial x))_{x=t_s}$.

IV. Solution of $\Psi_s(x,y)$

In equation 11, $\Psi_s$ is the solution of 2 D laplace equation, the boundary conditions required to solve the laplace equation are given below.

$$\psi_s(0, y) = \frac{\varepsilon_{s1}}{\varepsilon_{ox}} t_{ox} \left[ \frac{\partial \psi_s(x, y)}{\partial x} \right]_{x=0} = 0 \tag{15}$$

$$\psi_s(x, 0) = \frac{\varepsilon_{s1}}{\varepsilon_{ox}} t_{ox} \left[ \frac{\partial \psi_s(x, y)}{\partial y} \right]_{x=t_s} = 0 \tag{16}$$

$$\psi_s(x, L_{eff}) = V_{ds} - \psi_s(x) \tag{17}$$

$$\psi_s(x, 0) = V_{ds} - \psi_s(x) \tag{18}$$

The solution comes in the form of summation of sin and sinhyperbolic terms

$$\psi_{s}(x, y) = \sum_{r=1}^{\infty} \frac{1}{[V_r' \sinh(\gamma_r, y) + V_r \sinh(\gamma_r, L_{eff})]} \times \frac{\sinh(\gamma_r x) + \frac{\varepsilon_{s1}}{\varepsilon_{ox}} t_{ox} \gamma_r \cos(\gamma_r x)}{[\sinh(\gamma_r x) + \frac{\varepsilon_{s1}}{\varepsilon_{ox}} t_{ox} \gamma_r \cos(\gamma_r x)]} \tag{19}$$

Here the terms can be explained as below mentions.
\[ V_r = \frac{i_{\text{num1}} + \frac{\varepsilon_{sl}}{\varepsilon_{ox}} t_{\text{cof}} \gamma_r i_{\text{num2}}}{i_{D\text{num}}}, \]

\[ V_r' = V_r + \frac{V_{\text{de}}}{i_{D\text{num}}} \left[ \frac{1}{\gamma_r} \left[ 1 - \cos(\gamma_r t_a) \right] + \frac{\varepsilon_{sl}}{\varepsilon_{ox}} t_{\text{cof}} \sin(\gamma_r t_a) \right] \]

\[ i_{D\text{num}} = \frac{1}{4\gamma_r} \left[ 2\gamma_r t_a - \sin(2\gamma_r t_a) \right] + \left( \frac{\varepsilon_{sk}}{\varepsilon_{ox}} t_{\text{cof}} \gamma_r \right)^2 \times \frac{1}{4\gamma_r} \left[ 2\gamma_r t_a + \sin(2\gamma_r t_a) \right] + \frac{\varepsilon_{sl} t_{\text{cof}}}{2\varepsilon_{ox}} \left[ 1 - \cos(2\gamma_r t_a) \right] \]

\[ i_{\text{num1}} = \frac{q N_A}{\varepsilon_{sl} \gamma_r^2} \left[ 1 - \cos(\gamma_r t_a) \right] + E_{sb} \frac{\sin(\gamma_r t_a)}{\gamma_r^2} \]

\[ + \left[ V_{bi} - \psi_{sb} - E_{sb} t_a - \frac{1}{2\varepsilon_{sl}} N_A t_a^2 \right] \frac{1}{\gamma_r} \]

\[ - \left[ V_{bi} - \psi_{sb} \right] \frac{\cos(\gamma_r t_a)}{\gamma_r} \]

\[ i_{\text{num2}} = \frac{q N_A}{\varepsilon_{sl} \gamma_r^2} \sin(\gamma_r t_a) + E_{sb} \frac{\cos(\gamma_r t_a)}{\gamma_r^2} \]

\[ - \left[ E_{sb} + \frac{q N_A t_a}{\varepsilon_{sl}} \right] \frac{1}{\gamma_r} \left[ V_{bi} - \psi_{sb} \right] \times \frac{\sin(\gamma_r t_a)}{\gamma_r} \]

V. Solution of \( \Psi^{(x,y,z)} \)

In equation \( \Psi_v \) is the solution of 3D laplace equation, the boundary conditions required to solve the laplace equation are given below.

\[ \psi_v(0, y, z) = 0, \psi_v(t_a, y, z) = 0, \psi_v(x, 0, z) = 0, \psi_v(x, t_a, z) = 0 \]

After solving the laplace equation with above given equation we find the solution in the form of

\[ \psi_v(x, y, z) = \left[ A \sin(\alpha y) + B \cos(\alpha y) \right] \left[ C \sin(\beta z) + D \cos(\beta z) \right] \]

\[ \times \left[ E \sinh(\gamma z) + F \cosh(\gamma z) \right] \quad (A.1) \]

Such that

\[ \alpha^2 + \beta^2 = \chi^2 \quad \text{where, } A, B, C, D, E, F, \alpha, \beta, \text{ and } \chi \text{ are arbitrary constants.} \]
\[ \psi_v(x, y, z) = [A \sin(\alpha y) + B \cos(\alpha y)] \times [E \sinh(\chi z) + F \cosh(\chi z)] \times \left[ \sin(\beta x) + \frac{\varepsilon_{si}}{\varepsilon_{ox}} t_{oxf} \beta \cos(\beta x) \right] \]

Where the terms can be expressed as

\[ i_{\text{Dnum}}^{3d} = \left[ \frac{1}{4\beta_r} \left[ 2\beta_r t_s \sin(2\beta_r t_s) \right] + \left( \frac{\varepsilon_{si}}{\varepsilon_{ox}} t_{oxf} \beta_r \right)^2 \frac{1}{4\beta_r} \right] \times \left[ 2\beta_r t_s + \sin(2\beta_r t_s) \right] + \frac{\varepsilon_{si} t_{oxf}}{2\varepsilon_{ox}} \left[ 1 - \cos(2\beta_r t_s) \right] \times \left[ \frac{1}{2 \cos^2(\alpha_s L_{\text{eff}})} \left( L_{\text{eff}} - \frac{\sin(2\alpha_s L_{\text{eff}})}{2\alpha_s} \right) \right] \quad (A.10) \]

\[ i_{\text{num}}^{3d} = \frac{(\cos(\alpha_s L_{\text{eff}}) - 1)}{\alpha_s \cos(\alpha_s L_{\text{eff}})} \left[ i_1 + \frac{\varepsilon_{si}}{\varepsilon_{ox}} t_{oxf} \beta_r i_2 \right] - \frac{1}{\cos(\alpha_s L_{\text{eff}})} \times \sum_{r=1}^{\infty} \frac{i_3}{\sinh(\beta_r L_{\text{eff}})} [V_r i_4 + V_r i_5] \quad (A.11) \]

here the term equation A.10 and A.11 are internal expression of 3D solution of laplace equation.

VI. CONCLUSION

A 3-D model for SOI MOSFET based on an analytical solution of 3-D Poisson’s equation is presented. The model takes into account the narrow-width effects in addition to the SCE. The 3-D Poisson’s equation is solved analytically using the separation of variables technique in the next section. The solution will then extend to obtain the expression of threshold voltage of the small geometry SOI MOSFET.

Journal Papers:


