Approximation of Variable Cost for the Voltage Support

Sergio B. Barragán G.1, Sarín Montero Corzo2, Nayeli Ramón Lara3

1,2,3 Instituto Politécnico Nacional, México

Abstract
In this article the cost for the voltage support of the generators from a fixed cost and a variable cost is determined. The variable cost is formed by two components: the losses of opportunity cost and the dependent cost of the operating conditions of the generator. The second component of the variable cost sets out to determine from a variable called \( \tau \) which is in function of the active and reactive power generated, considering the capability curve. The simulations are done in the five nodes.

Keywords: Reactive Power, Voltage Support, Ancillary Services, Capability Curve, Variable Cost.

1. INTRODUCTION
The structures of the electric systems have been changing from vertical schemes to horizontal schemes. These changes together with the increment in the demand have provoked that the electric systems operate near their thermal limits. A horizontal scheme is formed by four segments: generation, transmission, distribution and commercialization. Independently from the scheme of the electric system the reactive power is a very important parameter and it has direct link with the voltage magnitude of the system, however in a horizontal structure the reactive power is defined as a part of the set of the auxiliary services and has a commercial aspect in the system. Therefore there exists the necessity to determine a cost for the voltage support of the generators, because this action is considered as an auxiliary service and is reward in an independent way. In this work we determine the cost for generation of reactive power, considering that the total cost for the service of each generator is formed by a fixed cost and a variable cost. The fixed cost is calculated from the annual recovery factor of the invested capital in the generator. The variable cost is formed by the opportunity losses cost and a dependent cost of the operating conditions.

2. REACTIVE POWER DISPATCH
In this paper to solve the reactive power dispatch and to determine the cost of the voltage support in each generator involved in the system. The reactive power dispatch is subject to equality restrictions that represent the balance equations of active and reactive power in each node and inequality restrictions that correspond to the limits in the voltage profiles of all the nodes of the system. The equality restrictions are considered within the problem of optimization with the method of the Lagrange multipliers and the inequality restrictions from quadratic penalty functions [1]. Considering a system of \( n \)-nodes of which the configuration, characteristics of the thermal and the system demand are known, and assuming that the active power generated is maintained constant except in the compensator node. Reduction of the losses in the transmission network is equivalent to minimize the injection of active power of the compensator node, as the programmed active power remains constant [2,3]. Hence, the objective function is:

\[
\min f_0 \quad \iff \quad \min \sum_{k=0}^{m} P_k (V, \delta) \\
\quad \Rightarrow \quad \sum_{k=0}^{m} g_k (v_i^2 + v_j^2 - 2v_i v_j \cos \delta_{ij}) \\
\quad \Rightarrow \quad \sum_{k=0}^{m} P_e (V, \delta) = \sum_{k=0}^{m} P_g - \sum_{k=0}^{m} P_0
\]

where \( f_0 \) are the losses of the active power in the transmission system, \( P_e (V, \delta) \) is the generation of active power of the compensator node, \( P_g \) are the set of nodes of generation, \( P_0 \) are the set of nodes of load.

3. ESTABLISHMENT OF THE COST FOR THE VOLTAGE SUPPORT
The total cost \( (C_t) \) in the short term is the sum of all the costs in which it incise during a productive process [4,5,6,7]. Therefore the total costs in the short term are the sum of the fixed and variable costs. The fixed costs are also named as explicit or direct and are related with the inversion, maintenance and administration costs. The variable costs identified as implicit or indirect are the costs involved around the production process, as well as the set of the necessary energetics.

3.1. Fixed Cost
The principal function of the generators is to produce active power however, they also can absorb or generate reactive power, this function in an electric deregulate system is known as an auxiliary service; hence the fixed cost of the generators to provide support of reactive power is estimated from the retrieval factor of the capital [8]:

\[ C_f = \text{annual recovery factor of the invested capital in the generator} \times \text{cost for the service of each generator} \]
where $\sigma_{g_i}$ is the retrieval factor of the capital of the generator $i$, $I_{g_i}$ is the invested capital in the generator $i$, $i_{g_i}$ is the annual interest in the generator $i$ and $n$ is the useful life of the generator. Therefore, the fixed cost for the support of reactive power is estimated with the annual retrieval of the invested capital in terms of the generated MVA and used in the power triangle to obtain the proportional part of MVar [9,10,11]. The equation is

$$C_{f_{g_i}} = \left[ \frac{\sigma_{g_i}}{8760S_{g_i}} \right] \sin \theta_{g_i} Q_{g_i} \quad i \in ng$$

(3)

where $C_{f_{g_i}}$ is the fixed cost of the generator $i$ in [$/hr]$, $\sigma_{g_i}$ is the annual retrieval factor of the generator $i$, $\theta_{g_i}$ is the power factor angle of the generator $i$, $Q_{g_i}$ is the reactive power of the generator $i$, $S_{g_i}$ is the apparent power in the generator $i$, and $ng$ is the set of generation nodes.

### 3.2. Variable Cost

The variable cost for the voltage support in the generators is formed by two components, the cost for the losses of opportunity and a cost or operating outside its nominal conditions. Normally it is called the capacity load diagram or operating machine map, figure 1[12].

![Figure 1. Capability Curve of the Synchronous Machine.](image)

#### 3.3. Cost for Losses of Opportunity

A synchronous generator is designed to produce active power under its nominal conditions however by the characteristics in the capability curves, there exits a point in which to increase the reactive power generation is necessary to reduce the active power production. This condition is defined as a cost for losses of opportunity [8,13]:

$$C_{v_{g_i}} = \frac{\lambda_c (P_{g_i(a)} - P_{g_i(c)}) - [C_1 (P_{g_i(a)}) - C_1 (P_{g_i(c)})]}{Q_{g_i(c)} - Q_{g_i(a)}} \quad i \in ng$$

(4)

where $C_{v_{g_i}}$ is the variable cost in the generator $i$ for the losses of opportunity in [$/hr], $P_{g_i(a)}$ is the active power generation defined in the primary market, $P_{g_i(c)}$ is the active power generation after the reduction in its generation, $Q_{g_i(a)}$ and $Q_{g_i(c)}$ are the reactive power generations before and after achieving the reduction of active power.
respectively, $\lambda_c$ is the cost for each MW/hr defined in the economic dispatch, $C_v(P_{gi})$ is the cost of active power operation, and $ng$ is the set of generation nodes.

### 3.4 Cost for Operating Outside Nominal Conditions

This one is only applied when the generator does not operate under its nominal conditions most of it is calculated in an equivalent way of the fixed cost however, this component of the variable proposed cost will be in terms of $\tau$. As you can see in figure 2, a $\tau$ is defined as a quadratic function dependant of the power factor angle of the generator, regardless this one operates in an overexcited or underexcited way. The $\tau$ variable is the absolute value of the quotient between the reactive power and the active power generated. The absolute value is considered because when the generator operates in the underexcited conditions the power factor angle and the reactive power generated are negative; hence to obtain the quadratic curve this consideration is realized. In figure 2 it is shown that the $\tau$ variable is increased forth in agreement that the power factor angle increases. This variable tends to the infinity when $\theta \rightarrow 90^\circ$, taking in consideration that a minimum limit of active power generation exists $\tau \neq \infty$ and it will have a maximum value that will be established by the inferior limit of the active power generation. On the other hand, also $\tau \rightarrow 0$ when $\theta \rightarrow 0^\circ$ y $\tau = 0$ only when the reactive power generated is equal to zero. The component of the variable cost for operating outside its nominal conditions is proposed to be determined form the following equation

\[
Cv^i = \frac{\sigmai}{8760Si} \sin \theta_i \tau_i Q_i \quad i \in ng
\]

where $Cv^i$ is the variable cost in the generator $i$ for operating outside its nominal conditions in [$$/hr], $\sigma_i$ is the annual retrieval factor of the generator $i$, $\theta_i$ is the power factor angle of the generator $i$, $Q_i$ is the reactive power in the generator $i$, $Si$ is the apparent nominal power of the generator $i$, $\tau_i$ is the relation of generated powers $\left[\frac{Q_i}{P_i}\right]$ and $ng$ is the set of generation nodes. Therefore this component of the variable cost is $Cv^i \cong 0$ always and when the power factor is approximately equal to the nominal. To maintain the nominal conditions of operation of the generator and $Cv^i = 0$, the increases of reactive power generation and the active power must be equal to $\tau$, figure 3.

**Figure 2.** Relation of the generated powers and the power factor angle.

**Figure 3.** Relation of generated powers and generalized curve.
4. CASES OF STUDY

This system is formed by two generation units and seven lines. For the all cases the operating system conditions are determined firstly solving the active power dispatch, then the reactive power dispatch is applied and the cost for the voltage support of the participating generators is established. In the figure 4 presents the operating conditions of the system. The compensator node is the main contribution of the active power in the system. In figure 5 the magnitudes of the voltage of all the nodes of the system are shown, where the best profiles of the voltage are obtained in the generation nodes.

![Figure 4. The operating conditions of the system.](image)

![Figure 5. Profiles of the voltage of the system.](image)

The cost of the active power operation in the generator \((OC)\) is superior to the cost for the voltage support \((CVS)\), however as it can be observed in figure 6, shows that the CVS is formed by its fixed cost and its variable cost for operating outside its nominal conditions. This is because of the generator works far away from its nominal conditions and its variable \(\tau\) reaches an important value. Even though any generator works under nominal conditions, neither exists the necessity to reduce its active power generation wherefore the \(CV_{V_t} = 0\).

![Figure 6. OC and CVS of the generators.](image)
The total cost of active power operation in each case is superior to the cost for a determined voltage support. The cost for the voltage support is different in each case, although the cost in case three is not the smallest, under these operating conditions of the generators there exists a reduction in the losses of the system and any generator fall into the cost of opportunity losses, see figure 7.

5. CONCLUSIONS

Considering $\tau$ to establish the $Cv$ is determinant, mainly because the variable cost reaches to be a dominant component of the total cost of the service. With the variable $\tau$ also operating conditions of the generators are taken into account since these can operate out of their nominal conditions still being within their limits. The calculation of the $Cv$ as a variable cost can be attractive for the generators since they would still perceive incomes without violating their reactive power generation limits. With the raising to determine the CVS, this depends on the reactive power generation but mainly with the operating conditions of each generator.

REFERENCES

AUTHORS

Sergio Baruch Barragán-Gómez. He received Masters in Electrical Engineering from SEPI-ESIME-IPN, Mexico in 2004. He is currently a electric power systems professor at Department of Electrical Engineering of ESIME-IPN. His research interests: open software, analysis and optimization of electrical power systems.


Nayeli Ramón Lara. Is an associated professor at the ESIME-IPN. M.Sc. in Electrical Engineering at SEPI-ESIME-IPN in 2006. Electronics and Communications engineer graduated from Instituto Tecnológico y de Estudios Superiores de Monterrey en 2002. The interest areas for her are, Electrical Machines Control, Power Electronics and Education.