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An Optimal Control in Inventory for Deteriorating Items along with Shortages

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ABSTRACT

A model is derived in inventory for deteriorating items with the quadratic constant quadratic type demand rate under three different situations when inventory level reaches to zero. The rate of deterioration is followed by Weibull distributed deterioration. Here, shortages are allowed during the finite planning horizon. In this paper, we determine the total number of deteriorating items, total number of shortage quantity and total number of carrying inventory during the entire planning horizon. The main objective of this model is to develop an optimal policy which minimizes the number of deteriorating items as well as the shortages quantity. Finally, the model is illustrated by a numerical example for each scenario.

Keywords: *Quadratic demand, Inventory level, Deterioration, Weibull distributed deterioration, Inventory, Shortage, Optimal Policy.*

1. INTRODUCTION

Proper management of an inventory is an important responsibility for a manager in an inventory management system. Now a day, business is highly competitive due to the effect of globalization. So, manufacturing firms become very careful to produce a product with the majority of good quality rather than lesser percentage of defective items for the successful survival in the volatile market.

The definition of the deterioration of the products is the spoilage or decay or damage of the product in such a way that the product can no longer be used or they can loss of their usefulness by partially or completely. Deterioration can be classified in various aspects such as its lifetime, its physical damage or decay or its loss of value. In inventory the loss of quality in a perishable product over the remaining time in the inventory, management focuses on reducing and controlling the perishable product by developing several inventory models. There is a difference between the deteriorating items and the perishable items in inventory. The deteriorating items can be remanufactured and can be reused whereas perishable items have a fixed or specified lifetime after which they are considered unsuitable for utilization.

In recent years researchers draw attention on the unit of deteriorated items during the shortage period by developing various types of inventory models. Spoilage of items is continuous in time but proportional to the on-hand inventory. An inventory model of deterioration of goods is developed in [14] at the end of the storage period. In [12] a model is developed in order-level inventory for deteriorating items.

In [1] deterioration for an item is considered at a constant rate and instantaneous replenishment. A generalized economic order quantity model with three parameter Weibull distribution is developed in [11] to determine the time of deterioration. An inventory model with two parameter Weibull distribution for deterioration of item is developed in [9] with finite replenishment.

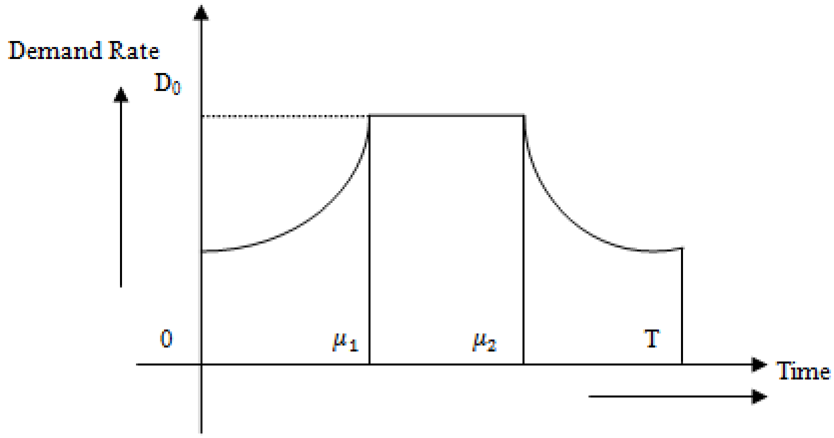
The constant rate of demand is not always applicable to many inventory items such as seasonal fruits, garments, electronic equipments etc. due to the fluctuation of rate of demand. Now a days, many researchers focus on the situation where the rate of demand is dependent on the level of on-hand inventory. An economic order quantity mode is developed in [5] for obtaining the optimal number of replenishments and optimal replenishment times with linear time dependent demand pattern over a finite time horizon. A new replenishment policy is derived in [7] where shortages are allowed in every inventory cycle. In [13] a deterministic inventory model for deteriorating items with price dependent demand rate, finite production rate and time varying deterioration rate is developed over a fixed time horizon. A more general model is developed in [6] where they considered finite production rate is proportional to the demand rate and the deterioration rate is directly proportional to the time. The leakage of dry batteries and life expectancy of ethical drugs could be expressed in terms of Weibull distribution. A model is developed for deteriorating items with variable rate of deterioration is developed in [2]. A strategy of ordering of an item with Weibull deteriorating rate in [15] permitting with discount in payment. Profit maximization with two parameters Weibull distributed deterioration under the assumption of exponentially decreasing demand is developed in [4]. In [3] an inventory model for deteriorating items is developed with instantaneous supply and linearly increasing demand. This is the extension of Philip's model where they considered the time of deterioration as a three parameter Weibull distribution.

An economic production quantity model for deteriorating items is developed in [8] where the optimization method is used for production disruption to reduce the loss occurred due to the disruption of the product. They determined the beginning and the ending of the production when the system gets disrupted. They allowed shortages in their model. A constrained inventory model is developed in [10] where the rate of demand is taken as periodic also the rate of deterioration and shortage are considered as constant. The model is applied using a geometric programming technique.

In our paper, we consider the point of shortage in three different scenarios where we determine the total number of shortage quantity, the total number of deteriorating items and total number of carrying inventory during the specified time interval in each case consequently we determine the total number of shortage quantity and the total number of deteriorating items in the entire inventory management system. Here, we consider the rate of demand is quadratic constant type and the rate of deterioration will be followed by Weibull distributed distribution. Finally, the model is illustrated with a numerical example in each case.

2. NOTATIONS AND ASSUMPTIONS

The fundamental notations and assumptions used in this paper are given below:



(a) The replenishment rate is infinite, thus the replenishment is instantaneous.

(b) The demand rate R(t), which is positive is assumed to be a quadratic function of time, i.e.,

$$R(t)=\begin{cases} a_1+b_1t+c_1t^2; 0\le t\le \mu_1 \\ D_0; \mu_1\le t\le \mu_2 \\ a_2-b_2t-c_2t^2; \mu_2\le t\le T \end{cases}$$

where μ_1 is time point changing from the increasing demand to constant demand and μ_2 is time point changing from constant demand to the decreasing quadratic demand.

(c) $I(t)$ is the level of inventory at any time $t \in [0,T]$

(d) T is the fixed length of each ordering cycle.

(e) Deterioration rate θ follows Weibull Distribution, $\theta = \alpha\beta t^{(\beta-1)}$, where $0 \le \alpha \le 1$, $\beta \ge 1$ and $t \ge 0$

(f) t_1^* is the optimal time point when inventory level reach zero.

(g) S is the maximum inventory level for the ordering cycle, such that $S = I(0)$.

(h) Lead time is zero.

(i) There is no repair or replacement of the deteriorated items.

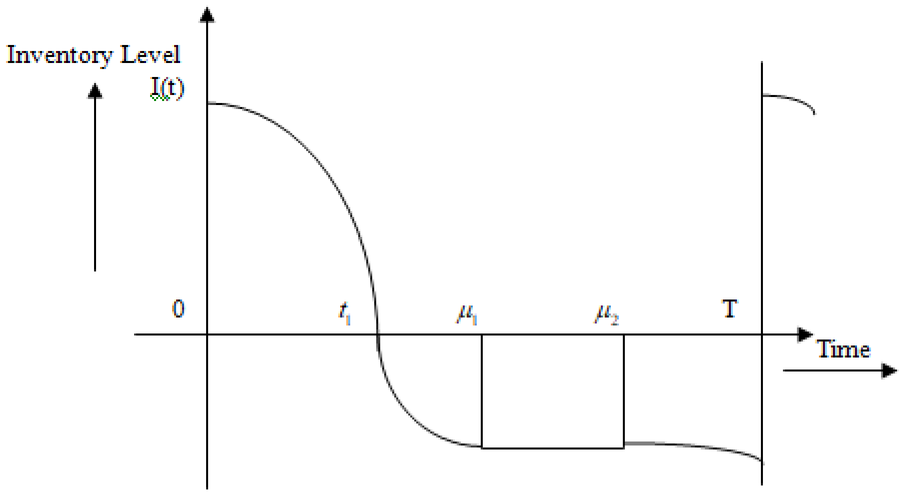
3. MATHEMATICAL FORMULATION

We consider the deteriorating inventory model with quadratic-constant-quadratic type demand rate. Replenishment occurs at time $t = t_1$ when the inventory level attains its maximum. From $t = 0$ to t_1 the inventory level reduces due to demand and deterioration. At t_1, the inventory level becomes zero, then shortage is allowed to occur during the time interval (t_1,T) and all of the documents during the shortage period (t_1,T). is completely backlogged. The total number of backlogged items is replaced by the next replenishment. According to the notations and assumptions mentioned above, the behavior of inventory system at any time can be described by the following differential equations:

$$\frac{d}{dt}I(t) = \begin{cases} -\theta I(t) - R(t); 0 \prec t \prec t_1 \_\_(1) \\ -R(t); t_1 \prec t \prec T \_\_\_\_\_\_\_\_(2) \end{cases}$$

with boundary condition $I(t_1) = 0$. We consider the following possible cases based on the values of t_1, μ_1 and μ_2.

A. *Case1:* $0 \le t \le \mu_1$



Due to reasons of deterioration and quadratic type demand rate, the inventory level gradually diminishes during the period $[0,t_1]$ and ultimately falls to zero at t_1. Then,

$$\frac{d}{dt}I(t)=\begin{cases}-\theta I(t)-\left(a_1+b_1t+c_1t^2\right);0\le t\le t_1 \text{\_\_\_\_\_\_}(3)\\ -\left(a_1+b_1t+c_1t^2\right);t_1\le t\le \mu_1 \text{\_\_\_\_\_\_}(4)\\ -D_0;\mu_1\le t\le \mu_2 \text{\_\_\_\_\_\_}(5)\\ -\left(a_2-b_2t-c_2t^2\right);\mu_2\le t\le T \text{\_\_\_\_\_\_}(6)\end{cases}$$

From equation (3) we obtain;

$$\frac{d}{dt}I(t)=-\alpha\beta t^{(\beta-1)}.I(t)-(a_1+b_1t+c_1t^2);0\le t\le t_1$$

This is a linear differential equation whose, $I.F.=e^{\alpha t^{\beta}}$

Therefore, $I(t).e^{\alpha t^{\beta}}=-\int(a_1+b_1t+c_1t^2).e^{\alpha t^{\beta}}dt=-\int(a_1+b_1t+c_1t^2).(1+\alpha t^{\beta})dt$

(Neglecting the higher terms of α because $\alpha\le 1$)

$I(t).e^{\alpha t^{\beta}}=-\left[a_1t+\frac{b_1}{2}t^2+\frac{c_1}{3}t^3+a_1\alpha\frac{t^{\beta+1}}{\beta+1}+b_1\alpha\frac{t^{\beta+2}}{\beta+2}+c_1\alpha\frac{t^{\beta+3}}{\beta+3}\right]+k$, where k is the constant

Using boundary condition $I(t_1)=0$, we obtain

$$k=a_1t_1+\frac{b_1}{2}t_1^{\,2}+\frac{c_1}{3}t_1^{\,3}+a_1\alpha\frac{t_1^{\,\beta+1}}{\beta+1}+b_1\alpha\frac{t_1^{\,\beta+2}}{\beta+2}+c_1\alpha\frac{t_1^{\,\beta+3}}{\beta+3}$$

Putting the value of k in the above equation, we get,

$$I(t)=\begin{bmatrix}a_1(t_1-t)+\frac{b_1}{2}\left(t_1^{\,2}-t^2\right)+\frac{c_1}{3}\left(t_1^{\,3}-t^3\right)\\ +\frac{a_1\alpha}{\beta+1}\left(t_1^{\,\beta+1}-t^{\beta+1}\right)+\frac{b_1\alpha}{\beta+2}\left(t_1^{\,\beta+2}-t^{\beta+2}\right)+\frac{c_1\alpha}{\beta+3}\left(t_1^{\,\beta+3}-t^{\beta+3}\right)\end{bmatrix}\times e^{\alpha t^{\beta}};0\le t\le t_1 \text{\_\_\_\_\_\_}(7)$$

From equation (4) we obtain,

$I(t)=\left[a_1t+\frac{b_1}{2}t^2+\frac{c_1}{3}t^3\right]+k_1;t_1\le t\le\mu_1$, where k_1 is the constant

Using boundary condition $I(t_1)=0$, we obtain

$$k_1=\left[a_1t_1+\frac{b_1}{2}t_1^{\,2}+\frac{c_1}{3}t_1^{\,3}\right]$$

Putting the value of k_1 in above equation, we get,

$$I(t)=\left[a_1(t_1-t)+\frac{b_1}{2}\left(t_1^{\,2}-t^2\right)+\frac{c_1}{3}\left(t_1^{\,3}-t^3\right)\right];t_1\le t\le\mu_1 \text{\_\_\_\_\_\_}(8)$$

From equation (5) we obtain,

$I(t)=-D_0t+k_2;\mu_1\le t\le\mu_2$, where k_2 is the constant

Using boundary condition $I(t_1)=0$, we obtain

$$I(\mu_1)=\left[a_1(t_1-\mu_1)+\frac{b_1}{2}\left(t_1^{\,2}-\mu_1^{\,2}\right)+\frac{c_1}{3}\left(t_1^{\,3}-\mu_1^{\,3}\right)\right]$$

$$\Rightarrow k_2=\left(a_1t_1+\frac{b_1}{2}t_1^{\,2}+\frac{c_1}{3}t_1^{\,3}\right)+\left(\frac{b_1}{2}\mu_1^{\,2}+\frac{2c_1}{3}\mu_1^{\,3}\right)-\mu_1\left(a_1+b_1\mu_1+c_1\mu_1^2-D_0\right)$$

But $D_0=\begin{cases}a_1+b_1\mu_1+c_1\mu_1^{\,2}\\ a_2-b_2\mu_2-c_2\mu_2^{\,2}\end{cases}$

Therefore, $k_2=a_1t_1+\frac{b_1}{2}\left(t_1^{\,2}+\mu_1^{\,2}\right)+\frac{c_1}{3}\left(t_1^{\,3}+2\mu_1^{\,3}\right)$

Putting the value of k_2 in above equation, we get,

$$I(t) = -D_0 t + a_1 t_1 + \frac{b_1}{2}\left(t_1^{\,2} + \mu_1^{\,2}\right) + \frac{c_1}{3}\left(t_1^{\,3} + 2\mu_1^{\,3}\right); \mu_1 \le t \le \mu_2 \quad \text{\_\_\_\_\_\_(9)}$$

$$I(\mu_2) = -D_0\mu_2 + a_1 t_1 + \frac{b_1}{2}\left(t_1^{\,2} + \mu_1^{\,2}\right) + \frac{c_1}{3}\left(t_1^{\,3} + 2\mu_1^{\,3}\right)$$

From equation (6) we obtain, $I(t) = -\left[a_2 t - \frac{b_2}{2}t^2 - \frac{c_2}{3}t^3\right] + k_3; \mu_2 \le t \le T$ where k_3 is the constant

Using boundary condition, $I(\mu_2) = 0$, we obtain

$$I(\mu_2) \Rightarrow -D_0\mu_2 + a_1 t_1 + \frac{b_1}{2}\left(t_1^{\,2} + \mu_1^{\,2}\right) + \frac{c_1}{3}\left(t_1^{\,3} + 2\mu_1^{\,3}\right) = -\left[a_2\mu_2 - \frac{b_2}{2}\mu_2^{\,2} - \frac{c_2}{3}\mu_2^{\,3}\right] + k_3$$

Therefore, $k_3 = a_2\mu_2 - b_2\mu_2^{\,2} + \frac{1}{2}b_2\mu_2^{\,2} - c_2\mu_2^{\,3} + \frac{2}{3}c_2\mu_2^{\,3} - D_0\mu_2 + a_1 t_1 + \frac{b_1}{2}\left(t_1^{\,2} + \mu_1^{\,2}\right) + \frac{c_1}{3}\left(t_1^{\,3} + 2\mu_1^{\,3}\right)$

But, $D_0 = \left(a_2 - b_2\mu_2 - c_2\mu_2^{\,2}\right)$

$$\Rightarrow k_3 = \frac{1}{2}b_2\mu_2^{\,2} + \frac{2}{3}c_2\mu_2^{\,3} + a_1 t_1 + \frac{b_1}{2}\left(t_1^{\,2} + \mu_1^{\,2}\right) + \frac{c_1}{3}\left(t_1^{\,3} + 2\mu_1^{\,3}\right)$$

Putting the value of k_3 in above equation, we get,

$$I(t) = a_1 t_1 + \frac{b_1}{2}\left(t_1^{\,2} + \mu_1^{\,2}\right) + \frac{c_1}{3}\left(t_1^{\,3} + 2\mu_1^{\,3}\right) - a_2 t + \frac{b_2}{2}\left(t^2 + \mu_2^{\,2}\right) + \frac{c_2}{3}\left(t^3 + 2\mu_3^{\,3}\right); \mu_2 \le t \le T \quad \text{\_\_\_\_\_\_(10)}$$

The beginning inventory level can be computed as,

$S = I(0)$

Equation (7) gives;

$$S = \left(a_1 t_1 + \frac{b_1}{2}t_1^{\,2} + \frac{c_1}{3}t_1^{\,3} + a_1\alpha\frac{t^{\beta+1}}{\beta+1} + b_1\alpha\frac{t^{\beta+2}}{\beta+2} + c_1\alpha\frac{t^{\beta+3}}{\beta+3}\right) \quad \text{\_\_\_\_\_\_(11)}$$

Total number of items which deteriorate in the inventory $[0, t_1]$ is,

$$D_T = S - \int_0^{t_1} R(t)dt = \left(a_1\alpha\frac{t^{\beta+1}}{\beta+1} + b_1\alpha\frac{t^{\beta+2}}{\beta+2} + c_1\alpha\frac{t^{\beta+3}}{\beta+3}\right) \quad \text{\_\_\_\_\_\_(12)}$$

Total number of inventory carried during $(0, t_1)$ is

Therefore, $H_T = \int_0^{t_1} I(t)dt$

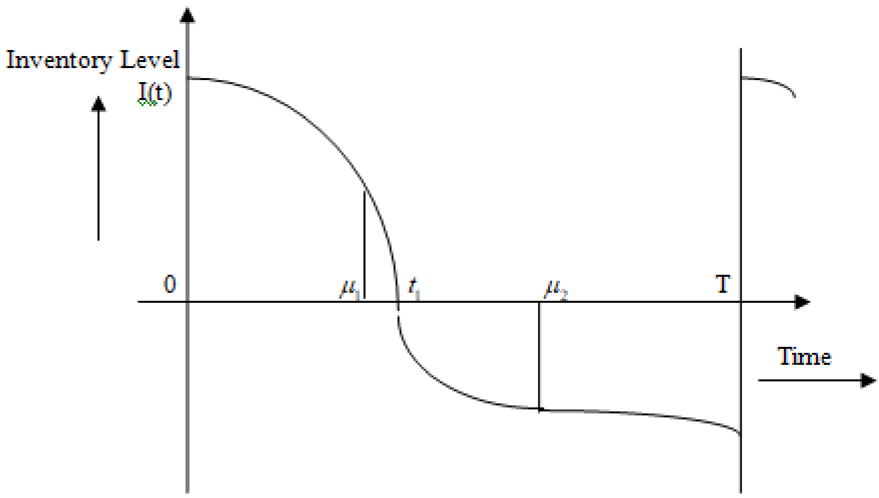
$$= \frac{a_1}{2}t_1^2 + \frac{b_1}{3}t_1^{\,3} + \frac{c_1}{4}t_1^{\,4} + \frac{a_1\alpha\beta}{(\beta+1)\times(\beta+2)}t_1^{\beta+2} + \frac{b_1\alpha\beta}{(\beta+1)\times(\beta+3)}t_1^{\beta+3} + \frac{c_1\alpha\beta}{(\beta+1)\times(\beta+4)}t_1^{\beta+4} \quad \text{\_\_\_\_\_\_(13)}$$

Total shortage quantity during the interval $[t_1, T]$ is

$$B_T = -\int_{t_1}^{T} I(t)dt = -\left[\int_{t_1}^{\mu_1} I(t)dt + \int_{\mu_1}^{\mu_2} I(t)dt + \int_{\mu_2}^{T} I(t)dt\right] \quad \text{\_\_\_\_\_\_(14)}$$

$$\Rightarrow B_T = -\left[\begin{array}{l} \left\{a_1 t_1\mu_1 - \frac{a_1}{2}\left(t_1^2 + \mu_1^2\right) + \frac{b_1}{2}t_1^2\mu_1 - \frac{b_1}{6}\left(\mu_1^3 + 2t_1^3\right) + \frac{c_1}{3}t_1^3\mu_1 - \frac{c_1}{12}\left(\mu_1^4 + 3t_1^4\right)\right\} \\ + \left\{-\frac{D_0}{2}\left(\mu_2^2 - \mu_1^2\right) + a_1 t_1\left(\mu_2 - \mu_1\right) + \frac{b_1}{2}\left(t_1^2 + \mu_1^2\right)\times\left(\mu_2 - \mu_1\right) + \frac{c_1}{3}\left(t_1^3 + 2\mu_1^3\right)\times\left(\mu_2 - \mu_1\right)\right\} \\ + \left\{\begin{array}{l} a_1 t_1\left(T - \mu_2\right) + \frac{b_1}{2}\left(t_1^2 + \mu_1^2\right)\times\left(T - \mu_2\right) + \frac{c_1}{3}\left(t_1^3 + 2\mu_1^3\right)\times\left(T - \mu_2\right) \\ -\frac{a_2}{2}\left(T^2 - \mu_2^2\right) + \frac{b_2}{6}\left(T^3 - \mu_2^3\right) + \frac{b_2}{2}\mu_2^2\left(T - \mu_2\right) + \frac{c_2}{12}\left(T^4 - \mu_2^4\right) + \frac{2c_2}{3}\mu_2^3\left(T - \mu_2\right) \end{array}\right\} \end{array}\right] \quad \text{\_\_\_\_\_\_(15)}$$

B. *Case2:* $\mu_1 \le t \le \mu_2$



The differential equations can be expressed as:

$$\frac{d}{dt}I(t)=\begin{cases}-\alpha\beta t^{\beta-1}I(t)-(a_1+b_1t+c_1t^2);0\le t\le \mu_1 \text{ \_\_\_\_\_(16)}\\ -\alpha\beta t^{\beta-1}I(t)-D_0;\mu_1\le t\le t_1 \text{ \_\_\_\_\_(17)}\\ -D_0;t_1\le t\le \mu_2 \text{ \_\_\_\_\_(18)}\\ -(a_2-b_2t-c_2t^2);\mu_2\le t\le T \text{ \_\_\_\_\_(19)}\end{cases}$$

As equation (16) is a linear differential equation whose, $I.F.=e^{\alpha t^{\beta}}$
Therefore, from equation (16), we obtain,

$$I(t).e^{\alpha t^{\beta}}=-\int(a_1+b_1t+c_1t^2).e^{\alpha t^{\beta}}dt=-\int(a_1+b_1t+c_1t^2).(1+\alpha t^{\beta})dt,\ 0\le t\le\mu_1$$

$$I(t).e^{\alpha t^{\beta}}=-\left[a_1t+\frac{b_1}{2}t^2+\frac{c_1}{3}t^3+a_1\alpha\frac{t^{\beta+1}}{\beta+1}+b_1\alpha\frac{t^{\beta+2}}{\beta+2}+c_1\alpha\frac{t^{\beta+3}}{\beta+3}\right]+k$$, where k is the constant _________(20)

From equation (17), we obtain,

$$I(t).e^{\alpha t^{\beta}}=-\int D_0.e^{\alpha t^{\beta}}dt=-\int D_0.\left(1+\alpha t^{\beta}\right)dt=-D_0\left(t+\frac{\alpha}{\beta+1}t^{\beta+1}\right)+k_1$$, where k_1 is a constant

Using boundary condition $I(t_1)=0$, we obtain

$$k_1=D_0\left(t_1-\frac{\alpha}{\beta+1}t_1^{\beta+1}\right)$$

$$I(t)=D_0\left\{\left(t_1-t\right)+\frac{\alpha}{\beta+1}\left(t_1^{\beta+1}-t^{\beta+1}\right)\right\}.e^{-\alpha t^{\beta}}$$

$$I(\mu_1)=D_0\left\{\left(t_1-\mu_1\right)+\frac{\alpha}{\beta+1}\left(t_1^{\beta+1}-\mu_1^{\beta+1}\right)\right\}.e^{-\alpha\mu_1^{\beta}} \text{ \_\_\_\_\_\_\_(21)}$$

Using the above condition for equation (20), we get,

$$D_0\left\{\left(t_1-\mu_1\right)+\frac{\alpha}{\beta+1}\left(t_1^{\beta+1}-\mu_1^{\beta+1}\right)\right\}.e^{-\alpha\mu_1^{\beta}}.e^{\alpha\mu_1^{\beta}}=-\left[a_1\mu_1+\frac{b_1}{2}\mu_1^2+\frac{c_1}{3}\mu_1^3+a_1\alpha\frac{\mu_1^{\beta+1}}{\beta+1}+b_1\alpha\frac{\mu_1^{\beta+2}}{\beta+2}+c_1\alpha\frac{\mu_1^{\beta+3}}{\beta+3}\right]+k$$

$$k = D_0\left\{(t_1-\mu_1)+\frac{\alpha}{\beta+1}\left(t_1^{\beta+1}-\mu_1^{\beta+1}\right)\right\}+\left[a_1\mu_1+\frac{b_1}{2}\mu_1^2+\frac{c_1}{3}\mu_1^3+a_1\alpha\frac{\mu_1^{\beta+1}}{\beta+1}+b_1\alpha\frac{\mu_1^{\beta+2}}{\beta+2}+c_1\alpha\frac{\mu_1^{\beta+3}}{\beta+3}\right]$$

$$I(t)=\left[D_0\left\{(t_1-\mu_1)+\frac{\alpha}{\beta+1}\left(t_1^{\beta+1}-\mu_1^{\beta+1}\right)\right\}\right]$$

$$+\left[\begin{array}{l} a_1(\mu_1-t)+\frac{b_1}{2}\left(\mu_1^2-t^2\right)+\frac{c_1}{3}\left(\mu_1^3-t^3\right) \\ +a_1\alpha\frac{\left(\mu_1^{\beta+1}-t^{\beta+1}\right)}{\beta+1}+b_1\alpha\frac{\left(\mu_1^{\beta+2}-t^{\beta+2}\right)}{\beta+2}+c_1\alpha\frac{\left(\mu_1^{\beta+3}-t^{\beta+3}\right)}{\beta+3}\end{array}\right]\times e^{-\alpha t^\beta};0\le t\le\mu_1 \qquad \text{\_\_\_\_\_\_\_\_\_\_(22)}$$

From equation (18), we obtain,

$I(t)=-D_0t+k_2;t_1\le t\le\mu_2$; where k_2 is a constant

Using boundary condition $I(t_1)=0$, we obtain $k_2=D_0t$

$I(t)=D_0(t_1-t);t_1\le t\le\mu_2$

$I(\mu_2)=D_0(t_1-\mu_2);t_1\le t\le\mu_2$ ________________(23)

From equation (19), we obtain,

$I(t)=-\left(a_2t-\frac{b_2}{2}t^2-\frac{c_2}{3}t^3\right)+k_3;\mu_2\le t\le T$; where k_3 is a constant

Using boundary condition $I(\mu_2)=D_0(t_1-\mu_2)$, we obtain,

$$D_0(t_1-\mu_2)=-\left(a_2\mu_2-\frac{b_2}{2}\mu_2^2-\frac{c_2}{3}\mu_2^3\right)+k_3$$

$$k_3=D_0(t_1-\mu_2)+\left(a_2\mu_2-\frac{b_2}{2}\mu_2^2-\frac{c_2}{3}\mu_2^3\right)$$

$$\Rightarrow I(t)=D_0t_1-a_2t+\frac{b_2}{2}\left(\mu_2^2+t^2\right)+\frac{c_2}{3}\left(2\mu_2^3+t^3\right)+\left\{-D_0+a_2-b_2\mu_2-c_2\mu_2^2\right\}.\mu_2$$

But, $D_0=a_2-b_2\mu_2-c_2\mu_2^2$

$$I(t)=D_0t-a_2t+\frac{b_2}{2}\left(\mu_2^2+t^2\right)+\frac{c_2}{3}\left(2\mu_2^3+t^3\right);\mu_2\le t\le T$$

Equation (22) gives,

$$I(t)=\left[D_0t_1+\frac{D_0\alpha}{\beta+1}\left(t_1^{\beta+1}-\mu_1^{\beta+1}\right)-a_1t-\frac{b_1}{2}\left(\mu_1^2+t^2\right)-\frac{c_1}{3}\left(2\mu_1^3+t^3\right)+\frac{a_1\alpha}{\beta+1}\left(\mu_1^{\beta+1}-t^{\beta+1}\right)+\frac{b_1\alpha}{\beta+2}\left(\mu_1^{\beta+2}-t^{\beta+2}\right)+\frac{c_1\alpha}{\beta+3}\left(\mu_1^{\beta+3}-t^{\beta+3}\right)\right].e^{-\alpha t^\beta};0\le t\le\mu_1 \qquad \text{\_\_\_\_\_\_(22A)}$$

(Since, $D_0=a_1+b_1\mu_1+c_1\mu_1^2$)

The beginning inventory level is calculated as,

$S=I(0)$

From equation (22), we have

$$S=D_0t_1+\frac{D_0\alpha}{\beta+1}\left(t_1^{\beta+1}-\mu_1^{\beta+1}\right)-\frac{b_1}{2}\mu_1^2-\frac{2c_1}{3}\mu_1^3+\frac{a_1\alpha}{\beta+1}\mu_1^{\beta+1}+\frac{b_1\alpha}{\beta+2}\mu_1^{\beta+2}+\frac{c_1\alpha}{\beta+3}\mu_1^{\beta+3}$$

Total number of items that deteriorates during the interval $[0,t_1]$ is

$$D_T=S-\int_0^{t_1}R(t)dt=S-\int_0^{\mu_1}R(t)dt-\int_{\mu_1}^{t_1}R(t)dt=\frac{D_0\alpha}{\beta+1}\left(t_1^{\beta+1}-\mu_1^{\beta+1}\right)+\frac{a_1\alpha}{\beta+1}\mu_1^{\beta+1}+\frac{b_1\alpha}{\beta+2}\mu_1^{\beta+2}+\frac{c_1\alpha}{\beta+3}\mu_1^{\beta+3};0\le t\le t_1$$

(Since, $D_0=a_1+b_1\mu_1+c_1\mu_1^2$)

The total number of inventory carried during the interval $[0,t_1]$ is

$$H_T=\int_0^{t_1}I(t)dt=-\frac{D_0\alpha}{\beta+1}\mu_1^{\beta+2}-\frac{\mu_1^2}{2}\left(a_1+b_1\mu_1+c_1\mu_1^2-D_0\right)+\frac{b_1}{6}\mu_1^3-\frac{c_1}{2}\mu_1^4+\frac{a_1\alpha}{\beta+2}\mu_1^{\beta+2}+\frac{b_1\alpha}{\beta+3}\mu_1^{\beta+3}+\frac{c_1\alpha}{\beta+4}\mu_1^{\beta+4}+\frac{a_1\alpha}{\beta+2}\mu_1^{\beta+2}+\frac{b_1\alpha}{(\beta+1)}\cdot\frac{(\beta+2)}{(\beta+3)}\mu_1^{\beta+3}+\frac{c_1\alpha}{(\beta+1)}\cdot\frac{(\beta+3)}{(\beta+4)}\mu_1^{\beta+4}+\frac{D_0}{2}t_1^2$$

(Since, $D_0=a_1+b_1\mu_1+c_1\mu_1^2$)

Total inventory carried during the interval $[0,t_1]$ is

$$H_T = \frac{D_0}{2}t_1^2 - \frac{D_0\alpha}{\beta+1}\mu_1^{\beta+2} - \frac{b_1}{6}\mu_1^3 - \frac{c_1}{4}\mu_1^4 + \frac{2a_1\alpha}{\beta+2}\mu_1^{\beta+2} + \frac{b_1\alpha}{\beta+3}\mu_1^{\beta+3} + \frac{c_1\alpha}{\beta+4}\mu_1^{\beta+4} + \left(\frac{b_1\alpha}{\beta+1}\right)\times\left(\frac{\beta+2}{\beta+3}\right)\mu_1^{\beta+3} + \left(\frac{c_1\alpha}{\beta+1}\right)\times\left(\frac{\beta+3}{\beta+4}\right)\mu_1^{\beta+4} \quad \text{\_\_\_\_\_\_(24)}$$

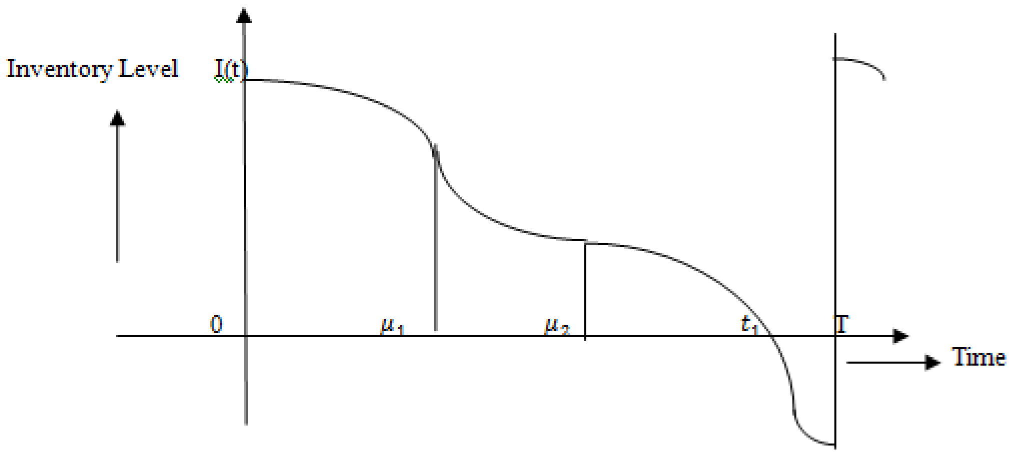
Total shortage quantity during the interval $\left[t_1, T\right]$ **is**

$$B_T = -\int_{t_1}^{T} I(t)dt = -\left[D_0 t_1\mu_2 - \frac{D_0}{2}t_1^2 - D_0\mu_2^{\ 2} + \frac{D_0}{2}T^2 - \frac{a_2}{2}\left(T^2-\mu_2^{\ 2}\right) + \frac{b_2}{6}\mu_1^3 - \frac{2b_2}{3}\mu_2^3 + \frac{b_2}{2}\mu_2^2 T + \frac{c_2}{12}T^4 - \frac{9c_2}{12}\mu_2^4 + \frac{2c_2}{3}\mu_2^3 T\right]$$

$\therefore$ **Shortage in** $t_1 \le t \le T$ **is**

$$B_T = -\int_{t_1}^{T} I(t)dt = \frac{D_0}{2}\left(t_1^2 - 2t_1\mu_2 + 2\mu_2^2 - T^2\right) + \frac{a_2}{2}\left(T^2-\mu_2^2\right) + \frac{b_2}{6}\left(4\mu_2^3 - 3\mu_2^2 T - T^3\right) + \frac{c_2}{12}\left(9\mu_2^4 - 8\mu_2^3 T - T^4\right) \quad \text{\_\_\_\_\_\_(25)}$$

<u>Case 3:</u> $\mu_2 \le t_1 \le T$



The differential equations for this case can be expressed as:

$$\frac{d}{dt}I(t) = \begin{cases} -\alpha\beta t^{\beta-1}I(t) - (a_1 + b_1 t + c_1 t^2); 0 \le t \le \mu_1 \text{\_\_\_\_\_(26)} \\ -\alpha\beta t^{\beta-1}I(t) - D_0; \mu_1 \le t \le \mu_2 \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_(27)} \\ -\alpha\beta t^{\beta-1}I(t) - (a_2 - b_2 t - c_2 t^2); \mu_2 \le t \le t_1 \text{\_\_\_\_\_(28)} \\ -(a_2 - b_2 t - c_2 t^2); t_1 \le t \le T \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_(29)} \end{cases}$$

Equation (26) is a linear differential equation whose, I.F.= $e^{\alpha t^{\beta}}$

Therefore, from equation (26), we obtain,

$$I(t).e^{\alpha t^{\beta}} = -\left[a_1 t + \frac{b_1}{2}t^2 + \frac{c_1}{3}t^3 + a_1\alpha\frac{t^{\beta+1}}{\beta+1} + b_1\alpha\frac{t^{\beta+2}}{\beta+2} + c_1\alpha\frac{t^{\beta+3}}{\beta+3}\right] + k \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_(30)}$$

Solving differential equation (27), we obtain

$$I(t).e^{\alpha t^{\beta}} = -\int D_0 e^{\alpha t^{\beta}}dt = -\int D_0.\left(1+\alpha t^{\beta}\right)dt = -D_0\left(t + \frac{\alpha}{\beta+1}t^{\beta+1}\right) + k_1, \text{ where } k_1 \text{ is a constant} \quad \text{\_\_\_\_\_\_\_\_\_\_\_(31)}$$

Solving differential equation (28), we obtain

$$I(t).e^{\alpha t^{\beta}} = -\left[\left(a_2 - \frac{b_2}{2}t - \frac{c_2}{3}t^2\right) + \left(\frac{a_2\alpha}{\beta+1}t^{\beta+1} - \frac{b_2\alpha}{\beta+2}t^{\beta+2} - \frac{c_2\alpha}{\beta+3}t^{\beta+3}\right)\right] + k_2, \text{ where } k_2 \text{ is a constant} \quad \text{\_\_\_\_\_\_\_\_(32)}$$

Using boundary condition $I(t_1) = 0$, which gives

$$k_2 = \left[\left(a_2 - \frac{b_2}{2}t - \frac{c_2}{3}t^2\right) + \left(\frac{a_2\alpha}{\beta+1}t^{\beta+1} - \frac{b_2\alpha}{\beta+2}t^{\beta+2} - \frac{c_2\alpha}{\beta+3}t^{\beta+3}\right)\right]$$

Putting the value of k_2 in equation (32), we have,

$$\Rightarrow I(t) = \left[a_2(t_1 - t) - \frac{b_2}{2}(t_1^2 - t^2) - \frac{c_2}{3}(t_1^3 - t^3) + \frac{a_2\alpha}{\beta+1}(t_1^{\beta+1} - t^{\beta+1}) - \frac{b_2\alpha}{\beta+2}(t_1^{\beta+2} - t^{\beta+2}) - \frac{c_2\alpha}{\beta+3}(t_1^{\beta+3} - t^{\beta+3})\right].e^{\alpha t^\beta}; \mu_2 \le t \le t_1 \qquad (33)$$

$$\therefore I(\mu_2) = \left[a_2(t_1 - \mu_2) - \frac{b_2}{2}(t_1^2 - {\mu_2}^2) - \frac{c_2}{3}(t_1^3 - {\mu_2}^3) + \frac{a_2\alpha}{\beta+1}(t_1^{\beta+1} - {\mu_2}^{\beta+1}) - \frac{b_2\alpha}{\beta+2}(t_1^{\beta+2} - {\mu_2}^{\beta+2}) - \frac{c_2\alpha}{\beta+3}(t_1^{\beta+3} - {\mu_2}^{\beta+3})\right].e^{\alpha {\mu_2}^\beta}$$

Using boundary condition of $I(\mu_2)$ in equation (31), we get,

$$k_1 = \left[a_2(t_1 - \mu_2) - \frac{b_2}{2}(t_1^2 - {\mu_2}^2) - \frac{c_2}{3}(t_1^3 - {\mu_2}^3) + \frac{a_2\alpha}{\beta+1}(t_1^{\beta+1} - {\mu_2}^{\beta+1}) - \frac{b_2\alpha}{\beta+2}(t_1^{\beta+2} - {\mu_2}^{\beta+2}) - \frac{c_2\alpha}{\beta+3}(t_1^{\beta+3} - {\mu_2}^{\beta+3})\right].e^{\alpha {\mu_2}^\beta}$$
$$+D_0\left[\mu_2 + \frac{\alpha}{\beta+1}{\mu_2}^{\beta+1}\right]$$

Putting the value of k_1 in equation (31), we have,

$$\Rightarrow I(t) = \left[\begin{array}{l}\left\{\begin{array}{l}a_2(t_1 - \mu_2) - \frac{b_2}{2}(t_1^2 - {\mu_2}^2) - \frac{c_2}{3}(t_1^3 - {\mu_2}^3) + \frac{a_2\alpha}{\beta+1}(t_1^{\beta+1} - {\mu_2}^{\beta+1}) \\ -\frac{b_2\alpha}{\beta+2}(t_1^{\beta+2} - {\mu_2}^{\beta+2}) - \frac{c_2\alpha}{\beta+3}(t_1^{\beta+3} - {\mu_2}^{\beta+3})\end{array}\right\} \\ +D_0\left\{(\mu_2 - t) + \frac{\alpha}{\beta+1}({\mu_2}^{\beta+1} - t^{\beta+1})\right\}\end{array}\right].e^{-\alpha t^\beta}; \mu_1 \le t \le \mu_2 \qquad (34)$$

$$\therefore I(\mu_1) = \left[\begin{array}{l}\left\{\begin{array}{l}a_2(t_1 - \mu_2) - \frac{b_2}{2}(t_1^2 - {\mu_2}^2) - \frac{c_2}{3}(t_1^3 - {\mu_2}^3) + \frac{a_2\alpha}{\beta+1}(t_1^{\beta+1} - {\mu_2}^{\beta+1}) \\ -\frac{b_2\alpha}{\beta+2}(t_1^{\beta+2} - {\mu_2}^{\beta+2}) - \frac{c_2\alpha}{\beta+3}(t_1^{\beta+3} - {\mu_2}^{\beta+3})\end{array}\right\} \\ +D_0\left\{(\mu_2 - \mu_1) + \frac{\alpha}{\beta+1}({\mu_2}^{\beta+1} - {\mu_1}^{\beta+1})\right\}\end{array}\right].e^{-\alpha t^\beta}$$

Using boundary condition of $I(\mu_1)$ in equation (30), we get

$$\Rightarrow \left[\begin{array}{l}\left\{\begin{array}{l}a_2(t_1 - \mu_2) - \frac{b_2}{2}(t_1^2 - {\mu_2}^2) - \frac{c_2}{3}(t_1^3 - {\mu_2}^3) \\ +\frac{a_2\alpha}{\beta+1}(t_1^{\beta+1} - {\mu_2}^{\beta+1}) - \frac{b_2\alpha}{\beta+2}(t_1^{\beta+2} - {\mu_2}^{\beta+2}) - \frac{c_2\alpha}{\beta+3}(t_1^{\beta+3} - {\mu_2}^{\beta+3})\end{array}\right\} \\ +D_0\left\{(\mu_2 - \mu_1) + \frac{\alpha}{\beta+1}({\mu_2}^{\beta+1} - {\mu_1}^{\beta+1})\right\}\end{array}\right].e^{-\alpha t^\beta}$$
$$= -\left[a_1\mu_1 + \frac{b_1}{2}\mu_1^2 + \frac{c_1}{3}\mu_1^3 + \frac{a_1\alpha}{\beta+1}\mu_1^{\beta+1} + \frac{b_1\alpha}{\beta+2}\mu_1^{\beta+2} + \frac{c_1\alpha}{\beta+3}\mu_1^{\beta+3}\right] + k$$

$$\Rightarrow k = \left[\left\{\begin{array}{l}a_2(t_1 - \mu_2) - \frac{b_2}{2}(t_1^2 - {\mu_2}^2) - \frac{c_2}{3}(t_1^3 - {\mu_2}^3) \\ +\frac{a_2\alpha}{\beta+1}(t_1^{\beta+1} - {\mu_2}^{\beta+1}) - \frac{b_2\alpha}{\beta+2}(t_1^{\beta+2} - {\mu_2}^{\beta+2}) - \frac{c_2\alpha}{\beta+3}(t_1^{\beta+3} - {\mu_2}^{\beta+3})\end{array}\right\} + D_0\left\{(\mu_2 - \mu_1) + \frac{\alpha}{\beta+1}({\mu_2}^{\beta+1} - {\mu_1}^{\beta+1})\right\}\right].e^{-\alpha t^\beta}$$
$$+\left[a_1\mu_1 + \frac{b_1}{2}\mu_1^2 + \frac{c_1}{3}\mu_1^3 + \frac{a_1\alpha}{\beta+1}\mu_1^{\beta+1} + \frac{b_1\alpha}{\beta+2}\mu_1^{\beta+2} + \frac{c_1\alpha}{\beta+3}\mu_1^{\beta+3}\right]$$

Putting this value of k in equation (30), we obtain

$$I(t)=\left[\begin{array}{l} a_1\left(\mu_1-t\right)-\frac{b_2}{2}\left(\mu_1^2-t^2\right)-\frac{c_2}{3}\left(\mu_1^3-t^3\right) \\ +\frac{a_2\alpha}{\beta+1}\left(\mu_1^{\beta+1}-t^{\beta+1}\right)+\frac{b_2\alpha}{\beta+2}\left(\mu_1^{\beta+2}-t^{\beta+2}\right)+\frac{c_2\alpha}{\beta+3}\left(\mu_1^{\beta+3}-t^{\beta+3}\right) \\ +\left\{\begin{array}{l} a_2\left(t_1-\mu_2\right)-\frac{b_2}{2}\left(t_1^2-\mu_2{}^2\right)-\frac{c_2}{3}\left(t_1^3-\mu_2{}^3\right)+\frac{a_2\alpha}{\beta+1}\left(t_1^{\beta+1}-\mu_2{}^{\beta+1}\right)-\frac{b_2\alpha}{\beta+2}\left(t_1^{\beta+2}-\mu_2{}^{\beta+2}\right) \\ -\frac{c_2\alpha}{\beta+3}\left(t_1^{\beta+3}-\mu_2{}^{\beta+3}\right) \end{array}\right\} \\ +D_0\left\{\left(\mu_2-\mu_1\right)+\frac{\alpha}{\beta+1}\left(\mu_2{}^{\beta+1}-\mu_1{}^{\beta+1}\right)\right\} \end{array}\right].e^{-\alpha t^\beta};0\le t\le \mu_1$$

________________(35)

Solving differential equation (29), we get,

$I(t)=-\left(a_2t-\frac{b_2}{2}t^2-\frac{c_2}{3}t^3\right)+k_3$, where k_3 is a constant

Using boundary condition of $I(t_1)=0$, we obtain

$$\Rightarrow k_3=\left(a_2t_1-\frac{b_2}{2}t_1{}^2-\frac{c_2}{3}t_1{}^3\right)$$

Putting this value of k_3, we get

$$\therefore I(t)=\left[a_2\left(t_1-t\right)-\frac{b_2}{2}\left(t_1{}^2-t^2\right)-\frac{c_2}{3}\left(t_1{}^3-t^3\right)\right];t_1\le t\le T$$ ________________________(36)

The beginning inventory level can be calculated as,

$S=I(0)$

From equation (35), we have

$$S=\left[\begin{array}{l} a_1\mu_1-b_1\mu_1{}^2+\frac{b_2}{2}\mu_1{}^2-c_1\mu_1{}^3+\frac{2c_1}{3}\mu_1{}^3+\frac{a_1\alpha}{\beta+1}\mu_1^{\beta+1}+\frac{b_1\alpha}{\beta+2}\mu_1^{\beta+2}+\frac{c_1\alpha}{\beta+3}\mu_1^{\beta+3} \\ +a_2t_1-a_2\mu_2-\frac{b_2}{2}t_1{}^2-\frac{b_2}{2}\mu_2{}^2+b_2\mu_2{}^2-\frac{c_2}{3}t_1{}^3+c_2\mu_2{}^3-\frac{2c_2}{3}\mu_2{}^3 \\ +\frac{a_2\alpha}{\beta+1}\left(t_1^{\beta+1}-\mu_2{}^{\beta+1}\right)-\frac{b_2\alpha}{\beta+2}\left(t_1^{\beta+2}-\mu_2{}^{\beta+2}\right)-\frac{c_2\alpha}{\beta+3}\left(t_1^{\beta+3}-\mu_2{}^{\beta+3}\right) \\ +D_0\left\{\left(\mu_2-\mu_1\right)+\frac{\alpha}{\beta+1}\left(\mu_2{}^{\beta+1}-\mu_1{}^{\beta+1}\right)\right\} \end{array}\right]$$

But, $a_1+b_1\mu_1+c_1\mu_1{}^2=D_0=a_2-b_2\mu_2-c_2\mu_2{}^2$

$$\therefore S=\left[\begin{array}{l} \frac{a_1\alpha}{\beta+1}\mu_1^{\beta+1}+\frac{b_1\alpha}{\beta+2}\mu_1^{\beta+2}+\frac{c_1\alpha}{\beta+3}\mu_1^{\beta+3}+a_2t_1+\frac{b_2}{2}\mu_1{}^2+\frac{2c_2}{3}\mu_1{}^3-\frac{b_2}{2}\left(t_1^2+\mu_2{}^2\right)-\frac{c_2}{3}\left(t_1^3+2\mu_2{}^3\right) \\ +\frac{a_2\alpha}{\beta+1}\left(t_1^{\beta+1}-\mu_2{}^{\beta+1}\right)-\frac{b_2\alpha}{\beta+2}\left(t_1^{\beta+2}-\mu_2{}^{\beta+2}\right)-\frac{c_2\alpha}{\beta+3}\left(t_1^{\beta+3}-\mu_2{}^{\beta+3}\right)+\frac{D_0\alpha}{\beta+1}\left(\mu_2^{\beta+1}-\mu_2{}^{\beta+1}\right) \end{array}\right]$$

_________________(37)

Total number of items that deteriorates in the interval $\left[0,t_1\right]$ is

$$D_T=S-\int_0^{t_1}R(t)dt=S-\int_0^{\mu_1}R(t)dt-\int_{\mu_1}^{t_1}R(t)dt$$

But,

$$\int_0^{t_1} I(t)dt = \int_0^{\mu_1} I(t)dt + \int_{\mu_1}^{\mu_2} I(t)dt + \int_{\mu_2}^{t_1} I(t)dt$$

$$\Rightarrow \int_0^{t_1} I(t)dt = \left[\begin{array}{l} -\frac{b_1}{6}\mu_1^{3} - \frac{c_1}{4}\mu_1^{4} - \frac{a_2\alpha}{\beta+1}\mu_2^{\beta+2} + \frac{b_2\alpha}{\beta+2}\mu_2^{\beta+3} + \frac{c_2\alpha}{\beta+3}\mu_2^{\beta+4} \\ +\left(\frac{a_2}{2}\mu_2^{2} - \frac{b_2}{2}\mu_2^{3} - \frac{c_2}{3}\mu_2^{4} - \frac{D_0}{2}\mu_2^{2}\right) + \frac{b_2}{3}\mu_2^{3} - \frac{b_2}{3}t_1^{3} + \frac{5c_2}{12}\mu_2^{4} - \frac{c_2}{4}t_1^{4} \\ +\frac{D_0\alpha\beta}{(\beta+1)(\beta+2)}\mu_2^{\beta+2} + \frac{D_0\alpha}{(\beta+1)}(\mu_2-\mu_1)\mu_2^{\beta+1} + \frac{\alpha}{(\beta+1)}\mu_2^{\beta+1}\left(\frac{b_2}{2}\mu_2^{2} + \frac{2c_2}{3}\mu_2^{3}\right) \\ +\frac{a_2\alpha\beta}{(\beta+1)(\beta+2)}t_1^{\beta+2} - \frac{a_2\alpha\beta}{(\beta+1)(\beta+2)}\mu_2^{\beta+2} - \frac{b_2\alpha\beta}{(\beta+1)(\beta+3)}t_1^{\beta+3} - \frac{b_2\alpha\beta}{2(\beta+2)(\beta+3)}\mu_2^{\beta+3} \\ +\frac{c_2\alpha\beta}{(\beta+1)(\beta+4)}t_1^{\beta+4} - \frac{a_2\alpha\beta}{3(\beta+1)(\beta+4)}\mu_2^{\beta+4} \end{array}\right]$$

Total number of inventory carried during the interval $\left[0, t_1\right]$ is

$$H_T = \int_0^{t_1} I(t)dt = \left[\begin{array}{l} -\frac{b_1}{6}\mu_1^{3} - \frac{c_1}{4}\mu_1^{4} - \frac{2a_2\alpha}{(\beta+2)}\mu_2^{\beta+2} + \frac{3b_2\alpha}{2(\beta+3)}\mu_2^{\beta+3} + \frac{4c_2\alpha}{3(\beta+4)}\mu_2^{\beta+4} + \frac{b_2}{3}\left(\mu_2^{3} - t_1^{3}\right) \\ +\frac{c_2}{12}\left(5\mu_2^{4} - 3t_1^{4}\right) + \frac{D_0\alpha\beta}{(\beta+1)(\beta+2)}\mu_2^{\beta+2} + \frac{D_0\alpha}{(\beta+1)}(\mu_2-\mu_1)\mu_2^{\beta+1} + \frac{\alpha}{(\beta+1)}\mu_2^{\beta+1}\left(\frac{b_2}{2}\mu_2^{2} + \frac{2c_2}{3}\mu_2^{3}\right) \\ +\frac{a_2\alpha\beta}{(\beta+1)(\beta+2)}t_1^{\beta+2} - \frac{b_2\alpha\beta}{(\beta+1)(\beta+3)}t_1^{\beta+3} - \frac{c_2\alpha\beta}{(\beta+1)(\beta+4)}t_1^{\beta+4} + \frac{a_2}{2}t_1^{2} \end{array}\right]$$

$$\therefore \frac{d}{dt}(H_T) = -b_2t_1^{2} - c_2t_1^{3} + \frac{a_2\alpha\beta}{(\beta+1)}t_1^{\beta+1} - \frac{b_2\alpha\beta}{(\beta+1)}t_1^{\beta+2} - \frac{c_2\alpha\beta}{(\beta+1)}t_1^{\beta+3} + a_2t_1$$

$$= \left(a_2 - b_2t_1 - c_2t_1^{2}\right)\left(t_1 + \frac{\alpha\beta}{(\beta+1)}t_1^{\beta+1}\right)$$

Total number of deteriorated items during the interval $\left[0, t_1\right]$ is

$$D_T = S - \int_0^{t_1} R(t)dt = S - \left[\int_0^{\mu_1} R(t)dt + \int_{\mu_1}^{\mu_2} R(t)dt + \int_{\mu_2}^{t_1} R(t)dt\right]$$

$$\Rightarrow D_T = \left[\begin{array}{l} \frac{a_1\alpha}{(\beta+1)}\mu_1^{\beta+1} + \frac{b_1\alpha}{(\beta+2)}\mu_2^{\beta+2} + \frac{c_1\alpha}{(\beta+3)}\mu_2^{\beta+3} \\ +\frac{a_2\alpha}{(\beta+1)}\left(t_1^{\beta+1} - \mu_2^{\beta+1}\right) - \frac{b_2\alpha}{(\beta+2)}\left(t_1^{\beta+2} - \mu_2^{\beta+2}\right) \\ -\frac{c_2\alpha}{(\beta+3)}\left(t_1^{\beta+3} - \mu_2^{\beta+3}\right) - \frac{D_0\alpha}{(\beta+1)}\left(\mu_2^{\beta+1} - \mu_1^{\beta+1}\right) \end{array}\right]$$

$$\therefore \frac{d}{dt}(D_T) = \alpha t_1^{\beta}\left(a_2 - b_2t_1 - c_2t_1^{2}\right)$$

Total number of shortage quantity during the interval $\left[t_1, T\right]$ is

$$B_T = -\int_{t_1}^{T} I(t)dt = \frac{a_2}{2}\left(t_1^{2} + T^{2} - 2t_1T\right) - \frac{b_2}{6}\left(2t_1^{3} + T^{3} - 3t_1^{2}T\right) - \frac{c_2}{12}\left(3t_1^{4} + T^{4} - 4t_1^{3}T\right)$$

$$\therefore \frac{d}{dt}(B_T) = a_2\left(t_1 - T\right) - b_2\left(t_1^{2} - t_1T\right) - c_2\left(t_1^{3} - t_1^{2}T\right)$$

$$= \left(a_2 - b_2t_1 - c_2t_1^{2}\right)\left(t_1 - T\right)$$

4. METHOD OF SOLUTION

In order to solve our model we obtain our following results by using the following parameter values. Here, we apply LINGO software for the solution of our problem where our target is to minimize the deteriorating items in the entire inventory management system.

Values of input parameter:
α=0.03, β=0.9, T=4 weeks, t_2=15 weeks, D_0=5 units, a_1=3, a_2=1, b_1=3.5, b_2=1.1, c_1=3.8, c_2=1.2, μ_1=5, μ_2=8.

Optimal solution:

Case 1:
S=550.41 units, D_T=9.19 units, H_T=77.65 units, B_T=26 units, t_1^*=7 weeks.

Case 2:
S=535.41 units, D_T=5.79 units, H_T=76.22 units, B_T=22 units, t_1^*=5 weeks.

Case 3:
S=570 units, D_T=4.89 units, H_T=72.32 units, B_T=20 units, t_1^*=7.45 weeks.

5. CONCLUSION

In this paper, we consider our model as a minimization problem. Here, we minimize the total number of deteriorating and shortage items in the inventory. Weibull distributed deterioration is applied in our model for deterioration but, some other distribution technique can be applied in future. Here, we have considered our model is in deterministic in nature but we may apply our model also in an uncertain environment. In our problem, we have used LINGO software as a method of solution but, for solution some other advanced soft computing techniques may be applied in future.

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