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Solution of Assignment Problem Using Vogel's Method

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Abstract : ***Assignment problem is a special case of Transportation problem. It is a minimizing model that assigns numbers of people with equal number of jobs, henceforth minimizing the corresponding costs. In this paper Assignment problem is solved using Vogel's method of transportation problem. The optimality of the solution is checked by diagonal method,with the help of illustrating numerical examples. The results obtained in the examples are compared with the results yielded with Hungarian method. This method is one of the efficient, simple, and accurate method for obtaining an optimal solution of assignment problem.***
Keywords: Assignment Problem, Linear Integer Programming,Cost Minimization; Diagonal sum; Vogel's method

INTRODUCTION:
Assignment problem is a special type of linear programming problem which deals with the allocation of the various resources to the various activities on one to one basis. It does it in such a way that the cost or time involved in the process is minimum and profit or sale is maximum. It is applicable in assigning vehicles to routes, machines to jobs, products to factories, school buses to various routes, aircrafts to particular trips, networking computers etc., In real life, it can also be used to determine marriage partners, friends etc.
Though these problems can be solved by simplex method or by transportation method. but assignment model gives a simpler approach for these problems.
Since its introduction in 1952, many different approaches have been developed for finding best solutions to the assignment problem and various articles have been published on the subject. See [1], [5], [7], [8] and [10] for the history of these methods.
A considerable number of methods has been so far presented for assignment problem in which the Hungarian method is more convenient method among them. This iterative method is based on add or subtract a constant to every element of a row or column of the cost matrix, in a minimization model and create some zeros in the given cost matrix and then try to find a complete assignment in terms of zeros. By a complete assignment for a cost n×n matrix, we mean an assignment plan containing exactly n assigned independent zeros, one in each row and one in each column.
The main concept of assignment problem is to find the optimum allocation of a number of resources to an equal number of demand points. An assignment plan is optimal if optimizes the total cost or effectiveness of doing all the jobs

Mathematical Formulation of Assignment Problem:
Suppose there are n-jobs for a factory and has n-machines to process those jobs. A job i(i =1,2,3,……., n) when processed by machine j (j =1,2,3,….., n) is assumed to incur a cost c_ij . The assignment is to be made in such a way that each job can be associated with one and only one machine. Determine an assignment of jobs to machines so as to minimize the overall cost.

We can define

$$x_{ij} = \begin{cases} 1, \text{if } ith \text{ job is assign to } jth \text{ machine} \\ 0, \text{otherwise} \end{cases}$$

we can assign one job to each machine

$\sum_{i=1}^{n} x_{ij}$=1 and $\sum_{j=1}^{n} x_{ij}$=1

the total assignment cost is given by

$$\sum_{i=1}^{n}\sum_{j=1}^{n} x_{ij}c_{ij}$$

The Algorithm Is as Follows:

1)Locate the two cells that have minimum cost and next to minimum cost. In each row, then write their difference (penalty) along the side of the table against the corresponding row.

2)Locate the two cells that have minimum cost and next to minimum cost in each column, then write their difference (penalty) below the table against the corresponding column.

3)Locate the maximum penalty.

- If it is along the side of the table, make assignment to the cell having minimum cost in that row and delete the corresponding column in which we have made the assignment and go to step 1.
- If it is below the table, make assignment to the cell having minimum cost in that column. delete the corresponding row in which we have made the assignment and go to step 1.

Continue in the same manner until all assignments are made.

4) If the penalties corresponding to two or more rows/columns are equal, find the difference between first and third minimum value. Identify the maximum among them and assign the minimum cost among them. This step gives the initial solution.

Note: if any row or column contains two or more minimum entry then the penalty is 0.

Remarks: For finding the optimal solution, we will follow step 5 and 6

5) Write these assigned costs on the top of the column of original assignment problem. Let a_j be the assigned cost for column j. Subtract a_j from each entry c_{ij} of the corresponding column of assignment matrix.

6) Construct a rectangle in such a way that one corner contains negative penalty and remaining two corners are allocated to the assigned cost values in corresponding row and column. Calculate the sum of extreme cells of unassigned diagonal, say d_{ij} . Locate for all d_{ij}< 0. Identify the most negative d_{ij} and exchange the assigned cell of diagonals. Continue the process until all negative penalties are resolved.

Remarks: If any d_{ij}=0, then exchange the cells of diagonals at the end.

Numerical Examples to Illustrate the Method:

This method is illustrated with some numerical examples.

Example 1: Consider an assignment problem with five jobs assign to five workers so that the cost is minimized.

11	17	8	16	20
9	7	12	6	15
13	16	15	12	16
21	24	17	28	26
14	10	12	11	13

Now write the penalty against each row and column and find the maximum penalty and make assignment to the row/column having maximum penalty and delete the corresponding row/column.

	j_1	j_2	j_3	j_4	j_5	Row penalty
i_1	11	17	8	16	20	3
i_2	9	7	12	6	15	1
i_3	13	16	15	12	16	1
i_4	21	24	17	28	26	4
i_5	14	10	12	11	13	1
Columnpenalty	2	3	4	5	2	

The maximum penalty is in $j_4$so we make assignment in j_4 at the minimum element which is 6 and since 6 occurs in i_2 so we delete i_2,now we have

	j_1	j_2	j_3	j_4	j_5	Row penalty
i_1	11	17	8	16	20	3
i_3	13	16	15	12	16	1
i_4	21	24	17	28	26	4
i_5	14	10	12	11	13	1
Columnpenalty	2	6	4		3	

Maximum penalty is in $j_2$so we make assignment in j_2 at the minimum element and since the minimum element is 10 which occurs in i_5 so we delete i_5.
Therefore, we have

	j_1	j_2	j_3	j_4	j_5	Row penalty
i_1	11	17	8	16	20	3
i_3	13	16	15	12	16	1
i_4	21	24	17	28	26	4
Columnpenalty	2		7		4	

Maximum penalty is in $j_3$so we make assignment in$j_3$and since the minimum element is 8 which occurs i_1 so we delete i_1 .Therefore, we have

	j_1	j_2	j_3	j_4	j_5	Row penalty
i_3	13	16	15	12	16	1
i_4	21	24	17	28	26	4
Columnpenalty	8				10	

The maximum penalty is in j_5 so we make assignment in j_5 and since 16 is the minimum entry which occurs in i_3 so we delete i_3,now we have assignments in each row except $i_4$and and each column except j_1 so make assignment at 21.

now using step 6 and 7 we will find if the solution is optimum or not.

All assigned costs are written on the top of the columns of original assignment matrix. Subtract assigned cost from each entry of the corresponding column of the assignment matrix. Identify all the negative penalties.

21 10 8 6 16 →

11	17	8	16	20
9	7	12	6	15
13	16	15	12	16
21	24	17	28	26
14	10	12	11	13

-10	7	0	10	4
-12	-3	4	0	-1
-8	6	7	6	0
0	14	9	22	10
-7	0	4	5	-3

We check the values of $d_{ij}$by using step 7

d_{11}=-1 $d_{22} = 2$

-10	0
0	9

-3	0
0	5

d_{21}=10

-12	0
0	22

$d_{31} = 2$

-8	0
0	10

$d_{25} = 5$ $d_{51} = 7$

0	-1
6	0

0	14
-7	0

$d_{55} = 3$

6	0
0	-3

d_{11}<0, d_{22}>0, d_{21}>0, d_{31}>0, d_{51}>0, d_{25}>0, d_{55}>0

since d_{11}<0 so exchnage the assigned cells of $j_1$and j_3 ,so we get

11 10 17 6 16

11	17	8	16	20
9	7	12	6	15
13	16	15	12	16
21	24	17	28	26
14	10	12	11	13

Now again subtracting the assign entry from the corresponding column we get.

0	7	-9	10	4
-2	-3	-5	0	-1
2	6	-2	6	0
10	14	0	22	10
3	0	-5	5	-3

d_{13} = 1; d_{21}= 8; d_{22}= 2; d_{23} =17

d_{25} = 5; d_{33} = 8; d_{53}= 9; d_{55}= 3

Since, the sum of all unassigned diagonal cells is greater than zero, optimal solution has been successfully achieved. Hence, the minimum cost will be Min Cost = 11+6+16+17+10 = 60.

and minimum cost using Hungarian method is 11+7+17+12+13=60

Example 2:

5 jobs assign to 5 workers such that the cost is minimised

12	8	7	15	4
7	9	1	14	10
9	6	12	6	7
7	6	14	6	10
9	6	12	10	6

Now write the penalty against each row and column and find the maximum penalty and make assignment to the row/column having maximum penalty and delete the corresponding row/column.

	j_1	j_2	j_3	j_4	j_5	Row penalty
i_1	12	8	7	15	4	3
i_2	7	9	1	14	10	6(8)
i_3	9	6	12	6	7	0
i_4	7	6	14	6	10	0
i_5	9	6	12	10	6	3
ColumnPenalty	0	0	6(11)	0	2	

Since the maximum penalty is in $i_2$and $j_3$and using step 5 the maximum penalty is 11 so me make assignment in j_3 at 1 and delete i_2.

Hence, we have,

	j_1	j_2	j_3	j_4	j_5	Row penalty
i_1	12	8	7	15	4	3
i_3	9	6	12	6	7	0
i_4	7	6	14	6	10	0
i_5	9	6	12	10	6	0
Columnpenalty	2	0		0	2	

Now the maximum penalty is in i_1 so me make assignment in $i_1$at 4 and delete j_5 since it contains the minimum element 4 and therefore, we have,

	j_1	j_2	j_3	j_4	Row penalty
i_1	12	8	7	15	
i_3	9	6	12	6	0
i_4	7	6	14	6	0
i_5	9	6	12	10	3
Columnpenalty	2	0		0	

Now the maximum penalty is in i_5 so we make assignment in i_5 at 6 which is the minimum element and delete the column that contains 6 so we deletej_2.hence we have

	j_1	j_3	j_4	Row penalty
i_1	12	7	15	
i_3	9	12	6	3
i_4	7	14	6	1
i_5	9	12	10	
Columnpenalty			0	

Now the maximum penalty is in $i_3$so we make assignment at 7 in $i_3$and delete the column that contains 7 so we delete j_4.Hence, we have,

	j_1	j_3	Row penalty
i_1	12	7	
i_3	9	12	
i_4	7	14	1
i	9	12	
Columnpenalty	2		

Now the only remaining unassigned column isj_1 and only remaining unassigned row is $i_4$so we make assignment at 7.

now using step 6 and 7 we will find if the solution is optimum or not.

All assigned costs are written on the top of the columns of original assignment matrix. Subtract assigned cost from each entry of the corresponding column of the assignment matrix. Identify all the negative penalties.

7 6 1 6 4 →

12	8	7	15	4
7	9	1	14	10
9	6	12	6	7

7	6	14	6	10
9	6	12	10	6

5	2	6	9	0
0	3	0	8	6
2	0	11	0	3
0	0	13	0	6
2	0	11	4	2

Since there are no negative entry so there are no negative d_{ij} therefore, the solution is optimum,and the solution is given by 4+1+6+7+6=24 and using the Hungarian method the solution is 4+1+6+7+6=24.

CONCLUSION:

In this study a Vogel's method to solve assignment problem has been presented to attain an exact optimal solution.This methodcan be used for all kinds of assignment problems, whether maximize or minimize objective. This method is based on Vogel's method of transportation problem and diagonal optimal approach. Ascosnsiderablenumber of methods has been so for presented for assignment problem in which the Hungarian method is more convenient method among them. Also, the comparison between both the methods have beenshown in the paper. Therefore, this paper attempts to propose a method for solving assignment problem which isdifferent from the preceding methods.

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