

An Inflationary Trended Inventory Model for Deteriorating and Ameliorating Items under Exponentially Increasing Demand and Partial Backlogging

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Abstract

As the long arm of the grinding, deep financial crisis continues to haunt the global economy, the effects of inflation and time value of money cannot be oblivious to an inventory system. Inflation, defined as a general rise in the prices of goods and services over a period of time, has monetary depreciation as one of its major side effects. And, since inventories correspond to substantial investment in capital for any organization, it would be unethical if the effects of inflation and time value of money are not considered while determining the optimal inventory policy. Moreover, deterioration of items is a phenomenon which cannot be ignored, as it may yield misleading results. But due to lack of considering the influence of demand, the ameliorating items for the amount of inventory is increasing gradually. Amelioration is a natural phenomenon observing in much life stock models. Another important factor is shortages which no retailer would prefer, and in practice are partially backlogged and partially lost. In order to convert the lost sales into sales, the retailer offers such customers an incentive, by charging them the price prevailing at the time of placing an order, instead of the current inflated price. Therefore, bearing in mind these facts, the present paper develops an inventory model for a retailer dealing with deteriorating and ameliorating items with exponential demand under the influence of inflation and time-value of money over a fixed planning horizon. Finally, a numerical example is provided to illustrate the proposed model. Comparative study of the optimal solutions with respect to major parameters under different special cases is carried out graphically and some managerial inferences have been presented.

Key Words: Inventory, deteriorating, ameliorating, inflation, time-value of money, exponential demand, shortages, and partial backlogging.

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1. Introduction:

Several researchers have addressed the importance of the deterioration phenomenon in their field of applications, as a result, many inventory models with deteriorating items have been developed. But due to lack of considering the influence of demand, the ameliorating items for the amount of inventory is increasing gradually. The fast growing animals like broiler, ducks, pigs etc. in the poultry farm, highbred fishes in berry (pond) which are known as ameliorating items. Amelioration is a natural phenomenon observing in much life stock models. Hwang [1] developed an inventory model for ameliorating items only. Again Hwang [2] added to a stock model for both ameliorating and deteriorating things independently. Mallick et al. [3] has considered a creation inventory model for both ameliorating and deteriorating items. Many researchers like Moon et al [4], Law et al [5], L-Q ji [6], Valliathal et al [7], Chen [8], Nodoust [9] are few noteworthy.

In the competitive market, the demand of some product may increase due to the consumer's preference on some eye-catching product. Therefore, the demand of the product at the time of its growth and the phase of declination may be approached by continuous-time-dependent function. These continuous-time-dependent functions may be a function of exponential or linear type. Ritchie [10] discussed the solution of a linear increasing time-dependent demand, which is obtained by Donaldson [11]. Silver and Meal [12] developed a model for deterministic time-varying demand, which also gives an approximate solution procedure termed as Silver-Meal Heuristic. Exponential demand has been developed by Aggarwal and Bahari-Kashani [13], Wee [14] and many other researchers.

Moreover in today's unpredictable market and due to changing consumers' preferences, a stock-out situation may arise in any business. Stock-outs are frustrating for the consumer and costly for the retailer, as they are likely to lose almost

one-half of the intended purchases when a consumer confronts an out-of-stock item. According to the literature of inventory control theory, most of the inventory models were developed under the assumption that “shortages are allowed and completely backlogged”. However nowadays customers are often fickle and increasingly less loyal, which eventually results in only a fraction of the customers waiting for the product till they arrive. In reality, for fashionable commodities and high-tech products with short life cycles, the backorder rate decreases with the length of waiting time. Hence in today’s market structure, partial backlogged shortages are a more practical assumption for better business performance. In this context, we remember the research works of Dye et al.[15], Shah & Shukla [16] , Zhao[17], Biswaranjan Mandal [18] etc.

Apart from the above mentioned facts, inflation is a crucial attribute of today’s esoteric economy which cannot be shrugged off. The term ‘inflation’ particularly used in an economic context literally means to blow up or get bigger. However the most common economic meaning of inflation is: reduction in the value of money i.e., monetary depreciation. As a result prices of commodities rise which subsequently curbs the purchasing power. Further from the financial perspective, inventories correspond to substantial investment in capital for any organization; hence it would not be ethical if the effects of inflation and time value of money are not considered while determining the optimal inventory policy. Buzacott [19], Bierman & Thomas[20], Moon et al [21], Datta & Pal [22], Mojtaba Kaveh et al [23], Bansal [24], Jaggi et al [25] are mentioned to few.

For these sort of situations, efforts have been made to develop an inventory model in presence of both ameliorating and deteriorating items under the influence of inflation and time-value of money. The demand rate is considered as exponentially increasing demand over a fixed time horizon and shortages which is partially backlogged. Finally the model is illustrated with the help of a numerical example and the comparative study of the optimal solutions with respect to major parameters under different special cases is carried out graphically and some managerial inferences have been presented.

Notations and Assumptions:

The present inventory model is developed under the following notations and assumptions:

Notations:

- I(t) : On hand inventory at time t.
- R(t) : Demand rate.
- Q : On-hand inventory.
- θ : The constant deterioration rate where $0 \leq \theta < 1$
- A : The constant ameliorating rate.
- T : The fixed length of each production cycle.
- i : The inflation rate per unit time.
- r : The discount rate representing the time value of money.
- A_0 : The ordering cost per order during the cycle period.
- p_c : The purchasing cost per unit item.
- h_c : The holding cost per unit item.
- d_c : The deterioration cost per unit item.
- a_c : The cost of amelioration per unit item.
- c_s : The shortage cost per unit item.
- o_c : The opportunity cost per unit item.
- TC : Average total cost per unit time.

Assumptions:

- (i). Lead time is zero.
- (ii). Replenishment rate is infinite but size is finite.
- (iii). The time horizon is finite.
- (iv). There is no repair of deteriorated items occurring during the cycle.

- (v). Amelioration and deterioration occur when the item is effectively in stock.
- (vi). The demand rate is a time dependent exponentially increasing function

$$R(t) = D_0 e^{\lambda t}, D_0 > 0, \lambda \geq 0.$$

(vii). Shortages are allowed and they adopt the notation used in Abad[26], where the unsatisfied demand is backlogged and the fraction of shortages backordered is $e^{-\delta t}$, where δ is a positive constant and t is the waiting time for the next replenishment. We also assume that $t e^{-\delta t}$ is an increasing function used in Skouri et al.[27].

2. Mathematical Formulation and Solution:

In this model, we consider an inventory model starting with no shortage. Replenishment occurs at time $t=0$ and the inventory level attains its maximum. From $t = 0$ to $t = t_1$ the stock will be diminished due to the effect of amelioration, deterioration and demand, and ultimately falls to zero at $t = t_1$. The shortages occur during timeperiod $[t_1, T]$ which are partially backlogged. The behaviour of the model at any time t can be described by the following differential equations:

$$\frac{dI(t)}{dt} + (\theta - A)I(t) = -D_0 e^{\lambda t}, 0 \leq t \leq t_1 \quad (2.1)$$

And
$$\frac{dI(t)}{dt} = -D_0 e^{\lambda t} e^{-\delta(T-t)}, t_1 \leq t \leq T \quad (2.2)$$

The initial condition is $I(0) = Q$ and $I(t_1) = 0 \quad (2.3)$

The solutions of the equations (2.1) and (2.2) using (2.3) are given by the following

$$I(t) = \frac{D_0}{\theta - A + \lambda} e^{\lambda t} \{e^{(\theta - A + \lambda)(t_1 - t)} - 1\}, 0 \leq t \leq t_1 \quad (2.4)$$

And
$$I(t) = \frac{D_0}{\lambda + \delta} e^{\lambda t} e^{-\delta(T-t)} \{e^{(\lambda + \delta)(t_1 - t)} - 1\}, t_1 \leq t \leq T \quad (2.5)$$

Since $I(t_1) = 0$, we get the following expression of on-hand inventory from the equation (2.4)

$$Q = \frac{D_0}{\theta - A + \lambda} \{e^{(\theta - A + \lambda)t_1} - 1\} \quad (2.6)$$

The total inventory holding during the time interval $[0, t_1]$ is given by

$$\begin{aligned} I_T &= \int_0^{t_1} I(t) e^{-Rt} dt = \int_0^{t_1} \frac{D_0}{\theta - A + \lambda} e^{\lambda t} \{e^{(\theta - A + \lambda)(t_1 - t)} - 1\} e^{-Rt} dt, R = r - i \\ &= \frac{D_0}{\theta - A + \lambda} \left[\frac{e^{(\lambda - R)t_1}}{\theta - A + R} \{e^{(\theta - A + R)t_1} - 1\} - \frac{1}{\lambda - R} \{e^{(\lambda - R)t_1} - 1\} \right] \quad (2.7) \end{aligned}$$

The total number of deteriorated units during the inventory cycle is given by

$$D_T = \theta \int_0^{t_1} I(t) e^{-Rt} dt, R = r - i$$

$$= \frac{\theta D_0}{\theta - A + \lambda} \left[\frac{e^{(\lambda-R)t_1}}{\theta - A + R} \{e^{(\theta-A+R)t_1} - 1\} - \frac{1}{\lambda - R} \{e^{(\lambda-R)t_1} - 1\} \right] \quad (2.8)$$

The total number of ameliorating units during the inventory cycle is given by

$$A_T = A \int_0^{t_1} I(t) e^{-Rt} dt, \quad R = r - i$$

$$= \frac{AD_0}{\theta - A + \lambda} \left[\frac{e^{(\lambda-R)t_1}}{\theta - A + R} \{e^{(\theta-A+R)t_1} - 1\} - \frac{1}{\lambda - R} \{e^{(\lambda-R)t_1} - 1\} \right] \quad (2.9)$$

The total number of shortages during the period $[t_1, T]$ is given by

$$S_T = \int_{t_1}^T -I(t) e^{-Rt} dt, \quad R = r - i$$

$$= \frac{D_0}{\lambda + \delta} e^{-\delta T} e^{(\lambda+\delta-R)t_1} \left[\frac{1}{R} \{e^{-R(T-t_1)} - 1\} + \frac{1}{\lambda + \delta - R} \{e^{(\lambda+\delta-R)(T-t_1)} - 1\} \right] \quad (2.10)$$

The amount of lost sales during the period $[t_1, T]$ is given by

$$L_T = \int_{t_1}^T R(t) \{1 - e^{-\delta(T-t)}\} e^{-Rt} dt = \int_{t_1}^T D_0 e^{\lambda t} \{1 - e^{-\delta(T-t)}\} e^{-Rt} dt, \quad R = r - i$$

$$= \frac{D_0}{\lambda - R} e^{(\lambda-R)t_1} \{e^{(\lambda-R)(T-t_1)} - 1\} - \frac{D_0}{\lambda + \delta - R} e^{-\delta T} e^{(\lambda+\delta-R)t_1} \{e^{(\lambda+\delta-R)(T-t_1)} - 1\} \quad (2.11)$$

Cost Components:

The total cost over the period $[0, T]$ consists of the following cost components:

(1). Ordering cost (OC) over the period $[0, T] = A_0$ (fixed)

(2). Purchasing cost (PC) over the period $[0, T] = p_c I(0) = p_c Q$

$$= p_c \left[\frac{D_0}{\theta - A + \lambda} \{e^{(\theta-A+\lambda)t_1} - 1\} \right]$$

(3). Holding cost for carrying inventory (HC) over the period $[0, T] = h_c I_T$

$$= \frac{h_c D_0}{\theta - A + \lambda} \left[\frac{e^{(\lambda-R)t_1}}{\theta - A + R} \{e^{(\theta-A+R)t_1} - 1\} - \frac{1}{\lambda - R} \{e^{(\lambda-R)t_1} - 1\} \right]$$

(4). Cost due to deterioration (CD) over the period $[0, T] = d_c D_T$

$$= \frac{d_c \theta D_0}{\theta - A + \lambda} \left[\frac{e^{(\lambda-R)t_1}}{\theta - A + R} \{e^{(\theta-A+R)t_1} - 1\} - \frac{1}{\lambda - R} \{e^{(\lambda-R)t_1} - 1\} \right]$$

(5).The amelioration cost (AMC)over the period [0,T]= $a_c A_T$

$$= \frac{a_c A D_0}{\theta - A + \lambda} \left[\frac{e^{(\lambda-R)t_1}}{\theta - A + R} \{e^{(\theta-A+R)t_1} - 1\} - \frac{1}{\lambda - R} \{e^{(\lambda-R)t_1} - 1\} \right]$$

(6). Cost due to shortage (CS)over the period [0,T]= $c_s S_T$

$$= \frac{c_s D_0}{\lambda + \delta} e^{-\delta T} e^{(\lambda+\delta-R)t_1} \left[\frac{1}{R} \{e^{-R(T-t_1)} - 1\} + \frac{1}{\lambda + \delta - R} \{e^{(\lambda+\delta-R)(T-t_1)} - 1\} \right]$$

(7). Opportunity Cost due to lost sales (OPC)over the period [0,T]= $o_c L_T$

$$= \frac{o_c D_0}{\lambda - R} e^{(\lambda-R)t_1} \{e^{(\lambda-R)(T-t_1)} - 1\} - \frac{D_0}{\lambda + \delta - R} e^{-\delta T} e^{(\lambda+\delta-R)t_1} \{e^{(\lambda+\delta-R)(T-t_1)} - 1\}$$

The average total cost per unit time of the system during the cycle [0,T] will be

$$\begin{aligned} TC(t_1) &= \frac{1}{T} [\text{OC} + \text{PC} + \text{HC} + \text{CD} + \text{AMC} + \text{CS} + \text{OPC}] \\ &= \frac{1}{T} \left[A_0 + \frac{p_c D_0}{\theta - A + \lambda} \{e^{(\theta-A+\lambda)t_1} - 1\} + \frac{(h_c + d_c \theta + a_c A) D_0}{\theta - A + \lambda} \left[\frac{e^{(\lambda-R)t_1}}{\theta - A + R} \{e^{(\theta-A+R)t_1} - 1\} - \frac{1}{\lambda - R} \{e^{(\lambda-R)t_1} - 1\} \right] \right. \\ &\quad + \frac{c_s D_0}{\lambda + \delta} e^{-\delta T} e^{(\lambda+\delta-R)t_1} \left[\frac{1}{R} \{e^{-R(T-t_1)} - 1\} + \frac{1}{\lambda + \delta - R} \{e^{(\lambda+\delta-R)(T-t_1)} - 1\} \right] \\ &\quad \left. + \frac{o_c D_0}{\lambda - R} e^{(\lambda-R)t_1} \{e^{(\lambda-R)(T-t_1)} - 1\} - \frac{o_c D_0}{\lambda + \delta - R} e^{-\delta T} e^{(\lambda+\delta-R)t_1} \{e^{(\lambda+\delta-R)(T-t_1)} - 1\} \right] \end{aligned} \quad (2.12)$$

For minimum, the necessary condition is $\frac{dTC(t_1)}{dt_1} = 0$

This gives

$$p_c e^{(\theta-A+\lambda)t_1} + \frac{(h_c + d_c \theta + a_c A)}{\theta - A + R} \{e^{(\theta-A+\lambda)t_1} - e^{(\lambda-R)t_1}\} + \frac{c_s e^{-\delta T}}{R} e^{(\lambda+\delta-R)t_1} \{e^{-R(T-t_1)} - 1\} + o_c e^{(\lambda-R)t_1} \{e^{-\delta(T-t_1)} - 1\} = 0 \quad (2.13)$$

For minimum the sufficient condition $\frac{d^2TC(t_1)}{dt_1^2} > 0$ would be satisfied.

Let $t_1 = t_1^*$ be the optimum value of t_1 .

The optimal values Q^* of Q and TC^* of TC are obtained by putting the value $t_1 = t_1^*$ from the expressions (2.6) and (2.12).

3. Some Special Cases:

(a). Absence of deterioration :

If the deterioration of items is switched off i.e. $\theta = 0$, then the expressions (2.6) and (2.12) of on-hand inventory(Q) and average total cost per unit time (TC(t_1)) during the period [0,T] become

$$Q = \frac{D_0}{\lambda - A} \{e^{(\lambda-A)t_1} - 1\} \quad (3.1)$$

$$\begin{aligned} \text{And TC}(t_1) = & \frac{1}{T} [A_0 + \frac{p_c D_0}{\lambda - A} \{e^{(\lambda-A)t_1} - 1\} + \frac{(h_c + a_c A) D_0}{\lambda - A} \left[\frac{e^{(\lambda-R)t_1}}{R-A} \{e^{(R-A)t_1} - 1\} - \frac{1}{\lambda - R} \{e^{(\lambda-R)t_1} - 1\} \right] \\ & + \frac{c_s D_0}{\lambda + \delta} e^{-\delta T} e^{(\lambda+\delta-R)t_1} \left[\frac{1}{R} \{e^{-R(T-t_1)} - 1\} + \frac{1}{\lambda + \delta - R} \{e^{(\lambda+\delta-R)(T-t_1)} - 1\} \right] \\ & + \frac{o_c D_0}{\lambda - R} e^{(\lambda-R)t_1} \{e^{(\lambda-R)(T-t_1)} - 1\} - \frac{o_c D_0}{\lambda + \delta - R} e^{-\delta T} e^{(\lambda+\delta-R)t_1} \{e^{(\lambda+\delta-R)(T-t_1)} - 1\}] \end{aligned} \quad (3.2)$$

The equation (2.13) becomes

$$p_c e^{(\lambda-A)t_1} + \frac{(h_c + a_c A)}{R - A} \{e^{(\lambda-A)t_1} - e^{(\lambda-R)t_1}\} + \frac{c_s e^{-\delta T}}{R} e^{(\lambda+\delta-R)t_1} \{e^{-R(T-t_1)} - 1\} + o_c e^{(\lambda-R)t_1} \{e^{-\delta(T-t_1)} - 1\} = 0 \quad (3.3)$$

This gives the optimum value of t_1 .

(b). Absence of amelioration:

If the amelioration of items is switched off i.e. $A = 0$, then the expressions (2.6) and (2.12) of on-hand inventory(Q) and average total cost per unit time (TC(t_1)) during the period [0,T] become

$$Q = \frac{D_0}{\theta + \lambda} \{e^{(\theta+\lambda)t_1} - 1\} \quad (3.4)$$

$$\begin{aligned} \text{And TC}(t_1) = & \frac{1}{T} [A_0 + \frac{p_c D_0}{\theta + \lambda} \{e^{(\theta+\lambda)t_1} - 1\} + \frac{(h_c + d_c \theta) D_0}{\theta + \lambda} \left[\frac{e^{(\lambda-R)t_1}}{\theta + R} \{e^{(\theta+R)t_1} - 1\} - \frac{1}{\lambda - R} \{e^{(\lambda-R)t_1} - 1\} \right] \\ & + \frac{c_s D_0}{\lambda + \delta} e^{-\delta T} e^{(\lambda+\delta-R)t_1} \left[\frac{1}{R} \{e^{-R(T-t_1)} - 1\} + \frac{1}{\lambda + \delta - R} \{e^{(\lambda+\delta-R)(T-t_1)} - 1\} \right] \\ & + \frac{o_c D_0}{\lambda - R} e^{(\lambda-R)t_1} \{e^{(\lambda-R)(T-t_1)} - 1\} - \frac{o_c D_0}{\lambda + \delta - R} e^{-\delta T} e^{(\lambda+\delta-R)t_1} \{e^{(\lambda+\delta-R)(T-t_1)} - 1\}] \end{aligned} \quad (3.5)$$

The equation (2.13) becomes

$$p_c e^{(\theta+\lambda)t_1} + \frac{(h_c + d_c \theta)}{\theta + R} \{e^{(\theta+\lambda)t_1} - e^{(\lambda-R)t_1}\} + \frac{c_s e^{-\delta T}}{R} e^{(\lambda+\delta-R)t_1} \{e^{-R(T-t_1)} - 1\} + o_c e^{(\lambda-R)t_1} \{e^{-\delta(T-t_1)} - 1\} = 0 \quad (3.6)$$

This gives the optimum value of t_1 .

(c). Constant demand rate:

If the demand rate is constant in nature i.e. $\lambda = 0$, then the expressions (2.6) and (2.12) of on-hand inventory(Q) and average total cost per unit time (TC(t_1)) during the period [0,T] become

$$Q = \frac{D_0}{\theta - A} \{e^{(\theta-A)t_1} - 1\} \quad (3.7)$$

$$\begin{aligned} \text{And TC}(t_1) = & \frac{1}{T} [A_0 + \frac{p_c D_0}{\theta - A} \{ e^{(\theta-A)t_1} - 1 \} + \frac{(h_c + d_c \theta + a_c A) D_0}{\theta - A} \left[\frac{e^{-Rt_1}}{\theta - A + R} \{ e^{(\theta-A+R)t_1} - 1 \} + \frac{1}{R} \{ e^{-Rt_1} - 1 \} \right] \\ & + \frac{c_s D_0}{\delta} e^{-\delta T} e^{(\delta-R)t_1} \left[\frac{1}{R} \{ e^{-R(T-t_1)} - 1 \} + \frac{1}{\delta - R} \{ e^{(\delta-R)(T-t_1)} - 1 \} \right] \\ & - \frac{o_c D_0}{R} e^{-Rt_1} \{ e^{-R(T-t_1)} - 1 \} - \frac{o_c D_0}{\delta - R} e^{-\delta T} e^{(\delta-R)t_1} \{ e^{(\delta-R)(T-t_1)} - 1 \}] \quad (3.8) \end{aligned}$$

The equation (2.13) becomes

$$p_c e^{(\theta-A)t_1} + \frac{(h_c + d_c \theta + a_c A)}{\theta - A + R} \{ e^{(\theta-A)t_1} - e^{-Rt_1} \} + \frac{c_s e^{-\delta T}}{R} e^{(\delta-R)t_1} \{ e^{-R(T-t_1)} - 1 \} + o_c e^{-Rt_1} \{ e^{-\delta(T-t_1)} - 1 \} = 0 \quad (3.9)$$

This gives the optimum value of t_1 .

(d). Absence of inflationary effect:

If the influence of inflation is ignored i.e. $R = 0$, then the expressions (2.6) and (2.12) of on-hand inventory(Q) and average total cost per unit time (TC(t_1)) during the period [0,T] become

$$Q = \frac{D_0}{\theta - A + \lambda} \{ e^{(\theta-A+\lambda)t_1} - 1 \} \quad (3.9)$$

$$\begin{aligned} \text{And TC}(t_1) = & \frac{1}{T} [A_0 + \frac{p_c D_0}{\theta - A + \lambda} \{ e^{(\theta-A+\lambda)t_1} - 1 \} + \frac{(h_c + d_c \theta + a_c A) D_0}{\theta - A + \lambda} \left[\frac{e^{\lambda t_1}}{\theta - A} \{ e^{(\theta-A)t_1} - 1 \} - \frac{1}{\lambda} \{ e^{\lambda t_1} - 1 \} \right] \\ & - \frac{c_s D_0}{\lambda + \delta} e^{-\delta T} e^{(\lambda+\delta)t_1} \left[T - t_1 - \frac{1}{\lambda + \delta} \{ e^{(\lambda+\delta)(T-t_1)} - 1 \} \right] \\ & + \frac{o_c D_0}{\lambda} e^{\lambda t_1} \{ e^{\lambda(T-t_1)} - 1 \} - \frac{o_c D_0}{\lambda + \delta} e^{-\delta T} e^{(\lambda+\delta)t_1} \{ e^{(\lambda+\delta)(T-t_1)} - 1 \}] \quad (3.10) \end{aligned}$$

The equation (2.13) becomes

$$p_c e^{(\theta-A+\lambda)t_1} + \frac{(h_c + d_c \theta + a_c A)}{\theta - A} \{ e^{(\theta-A+\lambda)t_1} - e^{\lambda t_1} \} - c_s e^{-\delta T} e^{(\lambda+\delta)t_1} (T - t_1) + o_c e^{\lambda t_1} \{ e^{-\delta(T-t_1)} - 1 \} = 0 \quad (22)$$

This gives the optimum value of t_1 .

4. Numerical Example:

To illustrate the developed inventory model, let the values of parameters be as follows:

$D_0 = 50$; $\lambda = 0.1$; $\theta = 0.02$; $A = 0.01$; $\delta = 10$; $A_0 = \$ 500$ per order; $p_c = \$ 5$ per unit, $h_c = \$4$ per unit; $d_c = \$9$ per unit; $a_c = \$6$ per unit; $c_s = \$10$ per unit; $o_c = \$12$ per unit; $i = 0.08$; $r = 2$; $T = 1$ year

Solving the equation (2.13) with the help of computer using the above values of parameters, we find the following optimum outputs

$$t_1^* = 0.35 \text{ year}; Q^* = 17.92 \text{ units and } TC^* = \text{Rs. } 710.04$$

It is checked that this solution satisfies the sufficient condition for optimality.

On the basis of the above parameters, the following solutions are also made.

Special cases	Q^* (units)	TC^* (\$)
Absence of deterioration	17.86	709.24
Absence of amelioration	17.95	710.06
Absence of inflationary effect	46.14	848.66
The constant demand rate	17.61	702.00

Comparative study of the optimal solutions towards different special cases in the inventory model:

The comparative study is also furnished graphically to illustrate the special cases of the inventory model by varying nature of inventory models.

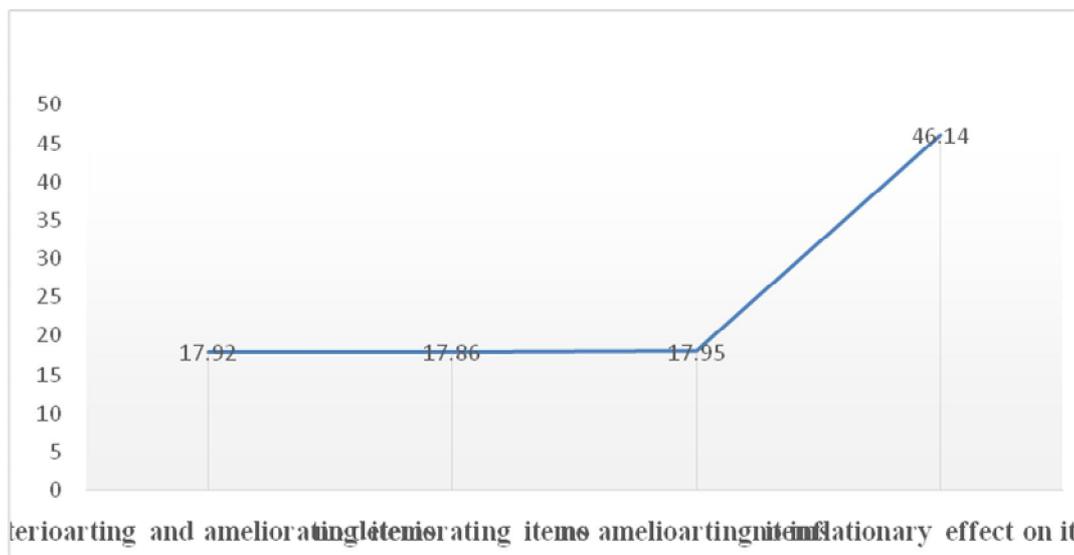


Fig. 1: Inventory items vs Optimal On-hand Inventory

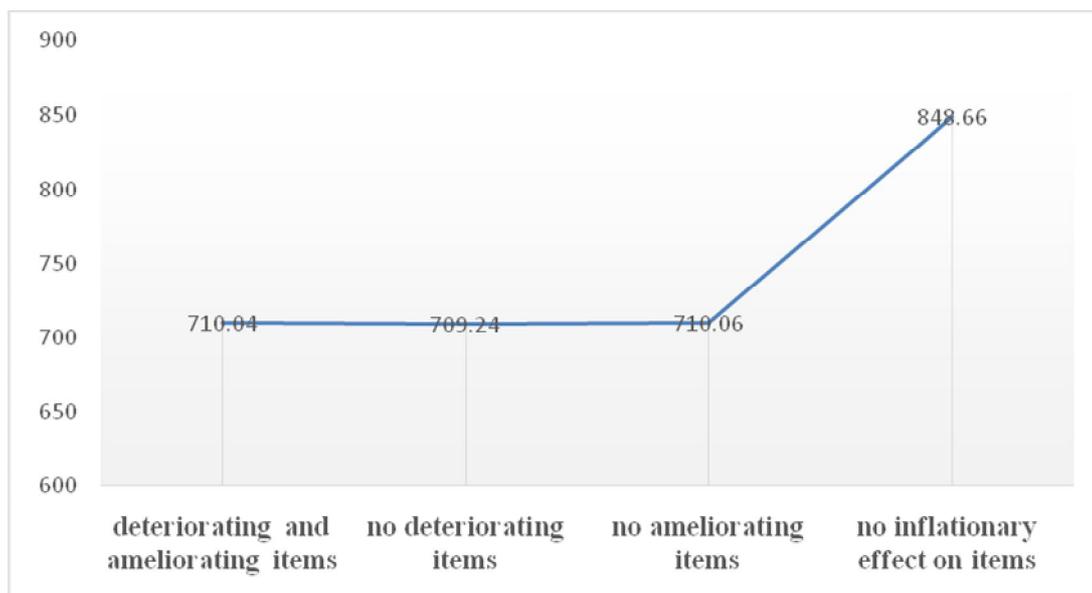


Fig. 2: Inventory items vs Optimal Inventory Total Cost

Concluding Remarks:

In this model, an inventory management policy has been framed in presence of both ameliorating and deteriorating items with exponentially increasing demand under the influence of inflation and time-value of money over a fixed planning horizon. Shortages are allowed which are partially backlogged. The model is developed analytically as well as computationally with graphical representation.

Efforts are given on comparative study graphically between optimal inventory total cost and optimal on-hand inventory considering deteriorating and ameliorating, no deteriorating, no ameliorating and no inflationary effects on inventory items. Analyzing Fig. 1 and Fig. 2, it is observed that optimality of inventory total cost and on-hand inventory level are changing very less sensitively for all kinds of inventory goods except model where influence of inflation is ignored. Moreover it is also observed that the optimal inventory total cost is significantly very high for the inventory model where the effect of inflation is switched off. So inflation plays an important role for estimation of the optimality of inventory cost.

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