A Control in Inventory for Deteriorating Items with Shortages

Sandipa Bhattacharya [1*] and Seema Sarkar (Mondal) [2]

[1] *Research Scholar*
Department of Mathematics
National Institute of Technology, Durgapur
Mahatma Gandhi Avenue, West Bengal-713209, India
[2] *Professor*
Department of Mathematics
National Institute of Technology, Durgapur
Mahatma Gandhi Avenue, West Bengal-713209, India

ABSTRACT

A model is derived in inventory for deteriorating items with the quadratic constant quadratic type demand rate under different situations when inventory level reaches to zero. The rate of deterioration is followed by Weibull distributed deterioration. Here, shortages are allowed during the finite planning horizon. In this paper, we determine the total number of deteriorating items, total number of shortage quantity and total number of carrying inventory during the entire planning horizon. The main objective of this model is to develop an optimal policy which reduces the number of deteriorating items as well as the shortages quantity. Finally, the model is illustrated by a numerical example for each scenario.
Keywords: *Deterioration, Weibull distributed deterioration, Inventory, Shortage.*

I. INTRODUCTION

Deterioration is the spoilage or decay or damage of the product in such a way that the product can no longer be used or they can loss of their usefulness by partially or completely. It can be classified in various aspects such as its lifetime, its physical damage or decay or its loss of value.

A shortage in inventory deals with fewer items in stock and they cannot meet the customer demand. Shortages of stock for an item may result in the cancelation of a customer order or heavy losses in the sale which in turn loss the market value of the product and the goodwill in the market. Increasing the on-hand inventory the number of shortages in inventory will decrease gradually. Keeping all these in view we cannot overstock the inventory so that they become deteriorate or perishable after a certain time interval as well as we cannot keep the inventory under stock so that they fall on the point of shortage. In contrast, we have studied some literature for further processing of our paper which is discussed below.

In recent years researchers draw attention on the unit of deteriorated items during the shortage period by developing various types of inventory models. Spoilage of items is continuous in time but proportional to the on-hand inventory. An inventory model of deterioration of goods is developed in [13] at the end of the storage period. In [10] a model is developed in order-level inventory for deteriorating items. In [1] deterioration for an item is considered at a constant rate and instantaneous replenishment. An inventory model with two parameter Weibull distribution for deterioration of item is developed in [7] with finite replenishment.

The constant rate of demand is not always applicable to many inventory items such as seasonal fruits, garments, electronic equipments etc. due to the fluctuation of rate of demand. Now a days, many researchers focus on the situation where the rate of demand is dependent on the level of on-hand inventory. An economic order quantity model is developed in [5] for obtaining the optimal number of replenishments and optimal replenishment times with linear time dependent demand pattern over a finite time horizon. A new replenishment policy is derived in [6] where shortages are allowed in every inventory cycle. In [2] a heuristic model is developed where demand is proportion with time is considered.

A model is developed for deteriorating items with variable rate of deterioration is developed in [3]. A strategy of ordering of an item with Weibull deteriorating rate in [14] permitting with discount in payment. A generalized economic order quantity model with three parameter Weibull distribution is developed in [8] to determine the time of deterioration. In [4] an inventory model for deteriorating items is developed with instantaneous supply and linearly increasing demand. This is the extension of Philip's model where they considered the time of deterioration as a three parameter Weibull distribution. A model is developed in [9] where the payment is considered in three different cases to obtain an optimal solution for deteriorating items in an entire inventory system. In this paper the holding cost is taken as a linear function and it is directly dependent on time. A deterministic inventory model is developed in [11] with quantity discount, pricing and backordering when the stock of product is deteriorates with times. In [12] a deterministic inventory model for deteriorating items with price dependent demand rate, finite production rate and time varying deterioration rate is developed over a fixed time horizon.

In our paper, we consider the point of shortage in different scenarios where we determine the total number of shortage quantity, the total number of deteriorating items and total number of carrying inventory during the specified time interval in each case consequently we reduce the total number of shortage quantity and the total number of deteriorating items in inventory for the improvement of the entire inventory management system. Here, we consider the rate of demand is quadratic constant type and the rate of deterioration will be followed by Weibull distributed distribution. Finally, the model is illustrated with a numerical example in each case.

II. NOTATIONS AND ASSUMPTIONS

The fundamental notations and assumptions used in this paper are given below:

(i) The replenishment rate is infinite, thus the replenishment is instantaneous.

(ii) The demand rate $R(t)$ which is positive is assumed to be a quadratic function of time, i.e.,

$$R(t)=\begin{cases} a_1+b_1t+c_1t^2; 0\le t\le \mu_1 \\ D_0; \mu_1\le t\le \mu_2 \\ a_2-b_2t-c_2t^2; \mu_2\le t\le T \end{cases}$$

where μ_1 is time point changing from the increasing demand to constant demand and μ_2 is time point changing from constant demand to the decreasing quadratic demand.

(iii) $I(t)$ is the level of inventory at any time $t\in[0,T]$

(iv) T is the fixed length of each ordering cycle.

(v) Deterioration rate θ follows Weibull Distribution, $\theta=\alpha\beta t^{(\beta-1)}$, where $0\le\alpha\le 1$, $\beta\ge 1$ and $t\ge 0$

(vi) t_1^* is the optimal time point when inventory level reach zero.

(vii) S is the maximum inventory level for the ordering cycle, such that $S=I(0)$.

(viii) Lead time is zero.

(ix) There is no repair or replacement of the deteriorated items.

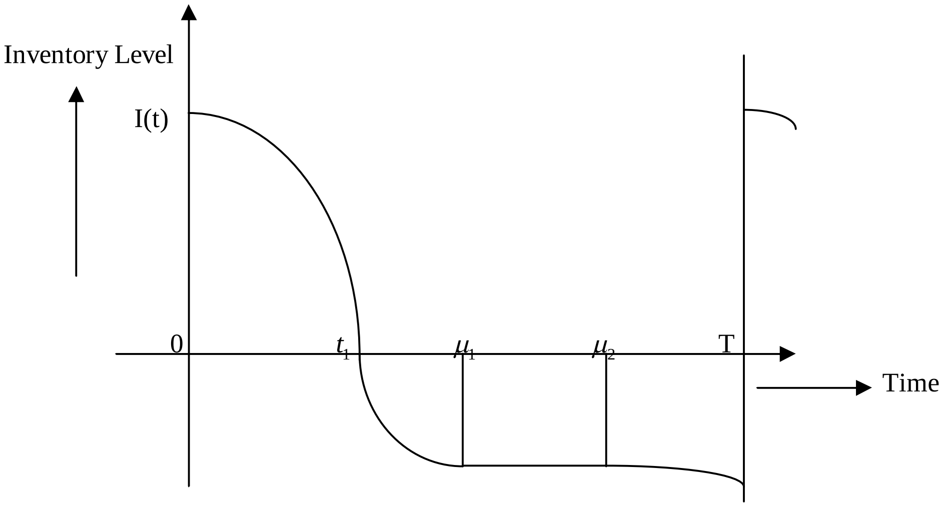
III. MATHEMATICAL FORMULATION

In our paper, we consider an inventory model for deteriorating items where rate of demand is taken as quadratic-constant-quadratic type. Replenishment occurs at time $t=t_1$ when the inventory level attains its maximum. From $t=0$ to t_1 the inventory level gradually reduces due to the demand and deterioration of the items. At t_1, the inventory level becomes zero, then shortage is allowed to occur during the time interval (t_1,T). The items which are completely backlogged are replaced by the next replenishment. In our model the following differential equations are described the total inventory system with the specified time interval:

$$\frac{d}{dt}I(t)=\begin{cases} -\theta I(t)-R(t); 0\prec t\prec t_1 \text{ \_\_(1)} \\ -R(t); t_1\prec t\prec T \text{ \_\_\_\_\_\_\_\_(2)} \end{cases}$$

with boundary condition $I(t_1)=0$. We consider the following possible cases based on the values of t_1, μ_1 and μ_2.

A. *Case1:* $0 \le t \le \mu_1$:



Due to reasons of deterioration and quadratic type demand rate, the inventory level gradually diminishes during the period $[0, t_1]$ and ultimately falls to zero at t_1. Then,

$$\frac{d}{dt}I(t) = \begin{cases} -\theta I(t) - \left(a_1 + b_1 t + c_1 t^2\right); 0 \le t \le t_1 \text{ \_\_\_\_\_\_\_(3)} \\ -\left(a_1 + b_1 t + c_1 t^2\right); t_1 \le t \le \mu_1 \text{ \_\_\_\_\_\_\_\_\_\_\_\_(4)} \\ -D_0; \mu_1 \le t \le \mu_2 \text{ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_(5)} \\ -\left(a_2 - b_2 t - c_2 t^2\right); \mu_2 \le t \le T \text{ \_\_\_\_\_\_\_\_\_\_(6)} \end{cases}$$

From equation (3) we obtain;

$$I(t) = \left[a_1(t_1 - t) + \frac{b_1}{2}\left(t_1^2 - t^2\right) + \frac{c_1}{3}\left(t_1^3 - t^3\right) + \frac{a_1\alpha}{\beta+1}\left(t_1^{\beta+1} - t^{\beta+1}\right) + \frac{b_1\alpha}{\beta+2}\left(t_1^{\beta+2} - t^{\beta+2}\right) + \frac{c_1\alpha}{\beta+3}\left(t_1^{\beta+3} - t^{\beta+3}\right)\right] \times e^{\alpha t^{\beta}}; 0 \le t \le t_1 \text{ \_\_\_\_\_\_\_\_(7)}$$

From equation (4) we obtain,

$$I(t) = \left[a_1(t_1 - t) + \frac{b_1}{2}\left(t_1^2 - t^2\right) + \frac{c_1}{3}\left(t_1^3 - t^3\right)\right]; t_1 \le t \le \mu_1 \text{ \_\_\_\_\_\_\_(8)}$$

From equation (5) we obtain,

$$I(t) = -D_0 t + a_1 t_1 + \frac{b_1}{2}\left(t_1^2 + \mu_1^2\right) + \frac{c_1}{3}\left(t_1^3 + 2\mu_1^3\right); \mu_1 \le t \le \mu_2 \text{ \_\_\_\_\_\_(9)}$$

$$I(\mu_2) = -D_0\mu_2 + a_1 t_1 + \frac{b_1}{2}\left(t_1^2 + \mu_1^2\right) + \frac{c_1}{3}\left(t_1^3 + 2\mu_1^3\right)$$

From equation (6) we obtain,

$$I(t) = a_1 t_1 + \frac{b_1}{2}\left(t_1^2 + \mu_1^2\right) + \frac{c_1}{3}\left(t_1^3 + 2\mu_1^3\right) - a_2 t + \frac{b_2}{2}\left(t^2 + \mu_2^2\right) + \frac{c_2}{3}\left(t^3 + 2\mu_3^3\right); \mu_2 \le t \le T \text{ \_\_\_\_\_\_(10)}$$

From equation (7), **the beginning inventory level** can be computed as,

$$S = \left(a_1 t_1 + \frac{b_1}{2}t_1^2 + \frac{c_1}{3}t_1^3 + a_1\alpha\frac{t^{\beta+1}}{\beta+1} + b_1\alpha\frac{t^{\beta+2}}{\beta+2} + c_1\alpha\frac{t^{\beta+3}}{\beta+3}\right) \text{ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_(11)}$$

Total number of items which deteriorate in the inventory $\left[0, t_1\right]$ **is,**

$$D_T = S - \int_0^{t_1} R(t)dt = \left(a_1\alpha\frac{t^{\beta+1}}{\beta+1} + b_1\alpha\frac{t^{\beta+2}}{\beta+2} + c_1\alpha\frac{t^{\beta+3}}{\beta+3}\right) \text{ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_(12)}$$

Total number of inventory carried during $\left(0, t_1\right)$ **is**

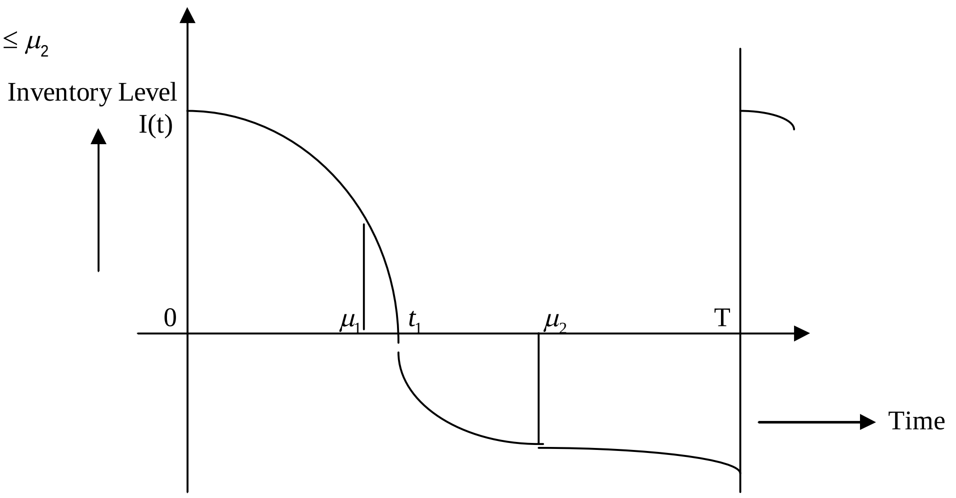
$$H_T = \int_0^{t_1} I(t)dt = \frac{a_1}{2}t_1^2 + \frac{b_1}{3}t_1^3 + \frac{c_1}{4}t_1^4 + \frac{a_1\alpha\beta}{(\beta+1)\times(\beta+2)}t_1^{\beta+2} + \frac{b_1\alpha\beta}{(\beta+1)\times(\beta+3)}t_1^{\beta+3} + \frac{c_1\alpha\beta}{(\beta+1)\times(\beta+4)}t_1^{\beta+4} \quad \text{\_\_\_\_\_(13)}$$

Total shortage quantity during the interval $[t_1, T]$ is

$$B_T = -\int_{t_1}^{T} I(t)dt = -\left[\int_{t_1}^{\mu_1} I(t)dt + \int_{\mu_1}^{\mu_2} I(t)dt + \int_{\mu_2}^{T} I(t)dt\right]$$

$$\Rightarrow B_T = -\left[\begin{array}{l} \left\{a_1t_1\mu_1 - \frac{a_1}{2}\left(t_1^2+\mu_1^2\right) + \frac{b_1}{2}t_1^2\mu_1 - \frac{b_1}{6}\left(\mu_1^3+2t_1^3\right) + \frac{c_1}{3}t_1^3\mu_1 - \frac{c_1}{12}\left(\mu_1^4+3t_1^4\right)\right\} \\ +\left\{-\frac{D_0}{2}\left(\mu_2^2-\mu_1^2\right) + a_1t_1\left(\mu_2-\mu_1\right) + \frac{b_1}{2}\left(t_1^2+\mu_1^2\right)\times\left(\mu_2-\mu_1\right) + \frac{c_1}{3}\left(t_1^3+2\mu_1^3\right)\times\left(\mu_2-\mu_1\right)\right\} \\ +\left\{\begin{array}{l} a_1t_1\left(T-\mu_2\right) + \frac{b_1}{2}\left(t_1^2+\mu_1^2\right)\times\left(T-\mu_2\right) + \frac{c_1}{3}\left(t_1^3+2\mu_1^3\right)\times\left(T-\mu_2\right) \\ -\frac{a_2}{2}\left(T^2-\mu_2^2\right) + \frac{b_2}{6}\left(T^3-\mu_2^3\right) + \frac{b_2}{2}\mu_2^2\left(T-\mu_2\right) + \frac{c_2}{12}\left(T^4-\mu_2^4\right) + \frac{2c_2}{3}\mu_2^3\left(T-\mu_2\right) \end{array}\right\} \end{array}\right] \quad \text{\_\_\_\_\_(14)}$$

B. *Case2:* $\mu_1 \le t \le \mu_2$



The differential equations can be expressed as:

$$\frac{d}{dt}I(t) = \begin{cases} -\alpha\beta t^{\beta-1}I(t) - (a_1 + b_1t + c_1t^2); 0 \le t \le \mu_1 & \text{\_\_\_\_\_(15)} \\ -\alpha\beta t^{\beta-1}I(t) - D_0; \mu_1 \le t \le t_1 & \text{\_\_\_\_\_(16)} \\ -D_0; t_1 \le t \le \mu_2 & \text{\_\_\_\_\_(17)} \\ -(a_2 - b_2t - c_2t^2); \mu_2 \le t \le T & \text{\_\_\_\_\_(18)} \end{cases}$$

Therefore, from equation (15), we obtain,

$$I(t) = \left[D_0\left\{\left(t_1-\mu_1\right) + \frac{\alpha}{\beta+1}\left(t_1^{\beta+1}-\mu_1^{\beta+1}\right)\right\}\right] + \left[a_1\left(\mu_1-t\right) + \frac{b_1}{2}\left(\mu_1^2-t^2\right) + \frac{c_1}{3}\left(\mu_1^3-t^3\right) + a_1\alpha\frac{\left(\mu_1^{\beta+1}-t^{\beta+1}\right)}{\beta+1} + b_1\alpha\frac{\left(\mu_1^{\beta+2}-t^{\beta+2}\right)}{\beta+2} + c_1\alpha\frac{\left(\mu_1^{\beta+3}-t^{\beta+3}\right)}{\beta+3}\right]\times e^{-\alpha t^{\beta}}; 0 \le t \le \mu_1 \quad \text{\_\_\_\_\_(19)}$$

From equation (17), we obtain,

$$I(\mu_2) = D_0\left(t_1-\mu_2\right); t_1 \le t \le \mu_2 \quad \text{\_\_\_\_\_(20)}$$

From equation (18), we obtain,

$$I(t) = D_0 t - a_2 t + \frac{b_2}{2}\left(\mu_2^2 + t^2\right) + \frac{c_2}{3}\left(2\mu_2^3 + t^3\right); \mu_2 \le t \le T \_\_\_\_\_\_\_\_ (21)$$

The beginning inventory level is calculated as,

$$S = D_0 t_1 + \frac{D_0\alpha}{\beta+1}\left(t_1^{\beta+1} - \mu_1^{\beta+1}\right) - \frac{b_1}{2}\mu_1^2 - \frac{2c_1}{3}\mu_1^3 + \frac{a_1\alpha}{\beta+1}\mu_1^{\beta+1} + \frac{b_1\alpha}{\beta+2}\mu_1^{\beta+2} + \frac{c_1\alpha}{\beta+3}\mu_1^{\beta+3} \_\_\_\_\_\_\_\_ (22)$$

Total number of items that deteriorates during the interval $\left[0, t_1\right]$ **is**

$$D_T = S - \int_0^{t_1} R(t)dt = \frac{D_0\alpha}{\beta+1}\left(t_1^{\beta+1} - \mu_1^{\beta+1}\right) + \frac{a_1\alpha}{\beta+1}\mu_1^{\beta+1} + \frac{b_1\alpha}{\beta+2}\mu_1^{\beta+2} + \frac{c_1\alpha}{\beta+3}\mu_1^{\beta+3}; 0 \le t \le t_1 \_\_\_\_\_\_\_\_ (23)$$

The total number of inventory carried during the interval $\left[0, t_1\right]$ **is**

$$H_T = \frac{D_0}{2}t_1^2 - \frac{D_0\alpha}{\beta+1}\mu_1^{\beta+2} - \frac{b_1}{6}\mu_1^3 - \frac{c_1}{4}\mu_1^4 + \frac{2a_1\alpha}{\beta+2}\mu_1^{\beta+2} + \frac{b_1\alpha}{\beta+3}\mu_1^{\beta+3} + \frac{c_1\alpha}{\beta+4}\mu_1^{\beta+4} + \left(\frac{b_1\alpha}{\beta+1}\right)\times\left(\frac{\beta+2}{\beta+3}\right)\mu_1^{\beta+3} + \left(\frac{c_1\alpha}{\beta+1}\right)\times\left(\frac{\beta+3}{\beta+4}\right)\mu_1^{\beta+4} \_\_\_\_\_\_\_\_ (24)$$

Total shortage quantity during the interval $\left[t_1, T\right]$ **is**

$$B_T = -\int_{t_1}^{T} I(t)dt = \frac{D_0}{2}\left(t_1^2 - 2t_1\mu_2 + 2\mu_2^2 - T^2\right) + \frac{a_2}{2}\left(T^2 - \mu_2^2\right) + \frac{b_2}{6}\left(4\mu_2^3 - 3\mu_2^2 T - T^3\right) + \frac{c_2}{12}\left(9\mu_2^4 - 8\mu_2^3 T - T^4\right) \_\_\_\_\_\_\_\_ (25)$$

IV. METHOD OF SOLUTION

In order to solve our model we obtain our following results by using the following parameter values. Here, we apply LINGO software for the solution of our problem where our target is to reduce the deteriorating items as well as the shortage quantity in the entire inventory management system.

Values of input parameter:
α=0.03, β =1.1, T=4 weeks, t_2 =15 weeks, D_0 =5 units, a_1 =3, a_2 =1, b_1 =3.5, b_2 =1.1, c_1 =3.8, c_2 =1.2, μ_1 =5, μ_2 =8.

DIFFERENT SITUATIONS	OPTIMAL SOLUTION					
	BEGINNING INVENTORY (S) IN UNITS	DETERIORATING ITEMS (D_T) IN UNITS	CARRYING INVENTORY (H_T) IN UNITS	SHORTAGE QUANTITY (B_T) IN UNITS	OPTIMAL TIME (t_1^*) IN WEEKS	MAXIMUM INVENTORY LEVEL $I(t)$ IN UNITS
A A. CASE 1:	552.67	11.45	79.84	26	7	1045.78
B B. CASE 2:	557.53	10.26	80.12	22.33	7.1	1105.38

In our model, our objective is to reduce the deterioration items and a huge number of shortages in an inventory management system. We consider the different situations for controlling these. Here, we observed that the total number of deteriorating items and the total number of shortage quantity are gradually decreased by considering two different scenarios. In first case, the deteriorating item is 11.45 units and the number of shortage quantity is 26 units whereas in the second case, the deteriorating item is 10.26 units and the number of shortage quantity is 22.33 units. So, we can control the total number of deteriorating items and the number of shortage quantity in the entire inventory management system by considering variable situations.

V. CONCLUSION

In this paper, we consider our model is an inventory problem where we determine the total number of deteriorating items and total number of shortage quantity. Here, we reduce the deteriorating items and the shortage quantity by considering two different situations but, it can also be applied for other situations. Weibull distributed deterioration is applied in our model for deterioration but, some other distribution techniques can be applied in future. In our model, the deteriorating items are not considered to replace or repair or they cannot be reused but in practical situation, it is not always possible, they may be remanufactured or may be reused in future. Here, we have considered our model is in deterministic in nature but we may apply our model also in an uncertain environment. In our problem, we have used LINGO software as a method of solution but, for solution some other advanced soft computing techniques may be applied in future.

REFERENCES

[1] Aggarwal, S. P., "A Note on an Order-Level Inventory Model for a System with Constant Rate of Deterioration", Opsearch, 15, pp. 184-187, 1978.

[2] Bahari-Kashani, H., "Replenishment Schedule for Deteriorating Items with Time-Proportional Demand", Journal of the Operational Research Society, 40, pp. 75-81, 1989.

[3] Berrotoni, J. N., "Practical Applications of Weibull Distribution", ASQC Tech. Conf. Trans., pp. 303-323, 1962.

[4] Chakrabarty, T., Giri B.C. and Chaudhuri K. S., "An EOQ Models for Items with Weibull Distribution Deterioration, Shortages and Trended Demand: An Extension of Philip's Model", Computers Ops. Res., Vol. 25, No. 7/8, pp. 649-657, 1998.

[5] Donaldson, W. A., "Inventory Replenishment Policy for a Linear Trend in Demand-an Analytical Solution", Operational Research Quarterly, 28, pp. 663-670, 1977.

[6] Goyal, S. K., Morin D. and Nebebe, F., "The Finite Horizon Trended Inventory Replenishment Problem with Shortages", Journal of the Operational Research Society, 43, pp. 1173-1178, 1992.

[7] Misra, R. B., "Optimum Production Lot-Size Model for a System with Deteriorating Inventory", International Journal of Production Research, 13, pp. 495-505, 1975.

[8] Philip, G. C., "A Generalized EOQ Model for Items with Weibull Distribution Deterioration", AIIE Transactions, 6, pp. 159-162, 1974.

[9] Sen, N. and Saha, S., "An Inventory Model for Deteriorating Items with Time Dependent Holding Cost and Shortages under Permissible Delay in Payment", International Journal of Procurement Management, Vol. 11, No. 4, pp. 518-531, 2018.

[10] Shah, Y. K. and Jaiswal, M. C., "An Order-Level Inventory Model for a System with Constant Rate of Deterioration", Opsearch, 14, pp. 174-184, 1977.

[11] Wee, H. M., "Deteriorating Inventory Model with Quantity Discount, Pricing and Partial Backordering", International Journal of Production Economics, 59, pp. 511-518, 1999.

[12] Wee, H. M. and Law, S. T., "Replenishment and Pricing Policy for Deteriorating Items Taking Account of the Time Value of Money", International Journal of Production Economics, 71, pp. 213-220, 2001.

[13] Whitin, T. M., "Theory of Inventory Management", Princeton University Press, Princeton, NJ, 1957.

[14] Wu, K. S., "An Ordering Policy for Items with Weibull Distribution Deterioration under Permissible Delay in Payments", Tamsui Oxford Journal of Mathematical Science, Vol. 14, pp. 39-54, 1998.

AUTHORS

Sandipa Bhattacharya is PhD research scholar from Department of Mathematics of National Institute of Technology Durgapur, West Bengal, India. She received her B.Sc. degree in Mathematics from Burdwan University, West Bengal and Masters in Computer Applications from West Bengal University of Technology, (Presently MAKAUT). She obtained her M.Tech degree in Operations Research from Department of Mathematics of National Institute of Technology Durgapur. She has participated in many National and International Workshops and Conferences in India. Her research interests include in Operations Research, Supply Chain Management, Soft Computing and some other related fields in Applied Mathematics.

Dr. Seema Sarkar (Mondal) is Professor of Department of Mathematics of National Institute of Technology Durgapur, West Bengal, India. She received her B.Sc. degree in Mathematics from Presidency College (Presently Presidency University) Kolkata and M.Sc , M.Phil & PhD degree in Applied Mathematics from University of Calcutta, Kolkata.
Her research interests include Geophysics, Operations Research and some other related fields. She has authored/coauthored more than 25 publications and received 'Best Paper Award' in International Conference on ‘Information and Management Science’ in China during August, 2010. She had also received “National Scholarship” during Secondary Examination.
Six students have completed their PhD and awarded doctorate degree under her supervision and 6 more are pursuing their doctoral study.
She is a Life Member of 'Calcutta Mathematical Society' and 'Operational Research Society of India', Kolkata , Chapter.