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Optimal Interplanetary Trajectories for Solar Sail

Deepak Gaur[1], M. S. Prasad[2]

[1]M. Tech. (Avionics), Amity Institute of Space Science and Technology, Amity University, Noida, U.P., India

[2]Director, Amity Institute of Space Science and Technology, Amity University, Noida, U.P., India

ABSTRACT

Most interplanetary spacecraft have used finite burns to escape Earth's orbit and be captured by their destination's gravity. Solar sail is an attractive technology to potentially lessen propellent expenditure. Optimal control of the sail coning angle to the Sun can both reduce transit time and the mission's propulsive velocity change. This research sought to determine the savings in hyperbolic excess velocity at departure and arrival for a baseline spacecraft and steering angle.

Keywords: Interplanetary, Solar Sail, Hyperbolic Excess Velocity, Lagrangian Variation of Parameters (VOP)

1. INTRODUCTION

Most interplanetary spacecraft have used finite propulsive burns to escape Earth's orbit and be captured by their destination's gravity. The propellent expended for these maneuvers is a large part of the mass and cost of these missions. If the hyperbolic excess velocity at departure and/or arrival could be reduced, propellant mass could be saved. Solar sail is an attractive technology to potentially optimize propellant expenditure. It converts solar photon momentum to spacecraft acceleration, providing a free, continuous propulsion source. This propulsion system is only limited by sail size, reflective parameters, and orientation to the sun. Methods to optimize transit time and propellant expenditure with solar sails could directly translate to reduced mission cost and/or more scientific exploration of the target.

The methodology of low thrust presented by Vallado details orbit-raising. [1] Raising is accomplished by determining the required control angle between the thrust and velocity vectors. Unfortunately, it requires a multiple-revolution scenario for a closed-form solution, which may be less desirable for a Mars mission. McInnes presents a method using Lagrangian variation of parameters (VOP). [3] Additionally, optimization of the sail angle to provide the maximum force in the desired direction. [3] However, this technique means there will be means that there will usually be a transverse force to that in the desired direction. Stevens et al. were able to find Earth-Mars rendezvous trajectories, but did not keep the coning angle optimal. [4] This research sought to optimally control the sail coning angle to the Sun to reduce transit time and the mission's propulsive velocity change, thus ensuring optimal savings in hyperbolic excess velocity at departure and arrival of the baseline spacecraft and steering angle.

2. METHODOLOGY

2.1 Solar Sail

Solar sail uses solar photon momentum to accelerate the spacecraft. The solar flux has an inverse-square relationship to the distance from the sun, just like gravitation. Therefore, the maximum acceleration provided by the sail and the acceleration due to solar gravity can form the lightness value, the ratio β [4]

$$\beta = \frac{a_{srp}}{a_{grav}} \tag{1}$$

The relationship presented in above equation provides the mission designer the ability to design trajectories without prior knowledge of spacecraft mass, sail area, and sail reflective parameters. It allows the acceleration due to solar radiation pressure be expressed in terms of gravitational acceleration. [3]

The angle between the sun vector and the sail-normal vector is called the coning angle, α. Figure 1 represents two-dimensional representation of the system.

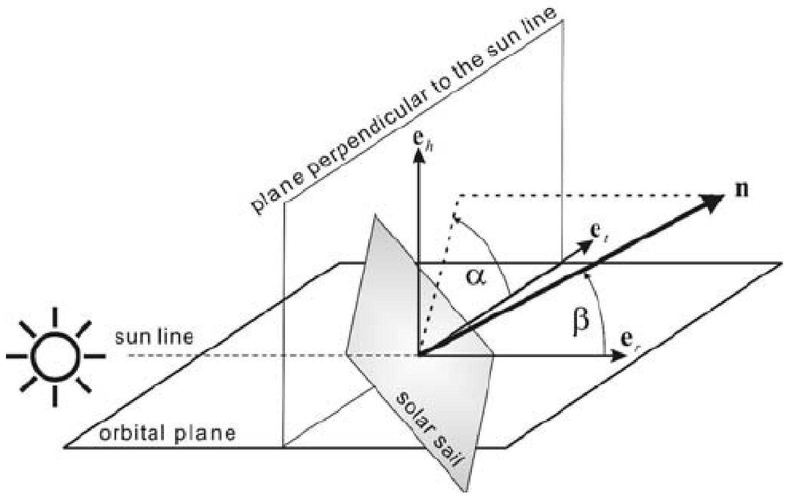

Figure 1 Sail craft coning angle. [5]

This parameter is steered to produce the sail force in the desired direction. However, it is not optimal to just steer to the coning angle between 0 and $\frac{\pi}{2}$. Considering the sail force direction:

$$n = \cos\alpha\hat{r} + \sin\alpha\,\hat{p}\times\hat{p} \tag{2}$$

And the vector that maximizes sail force to be applied:

$$q = \cos\hat{\alpha}\hat{r} + \sin\hat{\alpha}\,\hat{p}\times\hat{p} \tag{3}$$

The force magnitude f_q is [3]

$$f_q = 2PA\left(n\cdot\hat{r}\right)^2\left(n\cdot q\right) \tag{4}$$

By combining equations (2)-(4) and setting the result’s derivative equal to zero, the optimal coning angle is found by the following expression:

$$\tan\alpha_{opt} = \frac{-3+\sqrt{9+8\tan^2\widehat{\alpha}}}{4\tan\widehat{\alpha}} \tag{5}$$

The coning angle is shown against the desired force angle in Figure 2. Due to the transverse force component being limited by the product of $\sin\alpha$ and $\cos\alpha$, one can see that maximizing the force in the transverse direction (with respect to the Sun) requires a coning angle of 35.25°.
The two-dimensional polar equations of motion for a large gravitational body and solar radiation pressure given as: [3]

$$\ddot{r} - r\dot{\theta}^2 = \frac{\mu}{r^2}\left(\beta\cos^3\alpha - 1\right) \tag{6}$$

$$r\theta + 2\dot{r}\dot{\theta} = \beta\frac{\mu}{r^2}\cos^2\alpha\sin\alpha \tag{7}$$

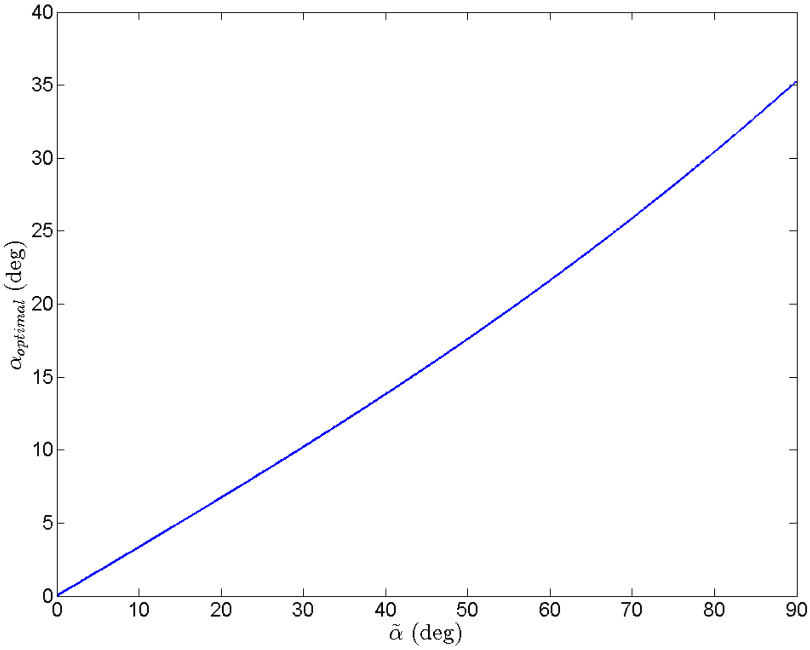

Figure 2 Optimal coning angle.

From equations (2) and (3), it is evident that α provides no sail force at $\frac{\pi}{2}$.

2.2 Optimal Trajectories

Optimal trajectories can be found with Lagrangian VOP technique. To optimally increase an orbital parameter, one may take the VOP for that equation and solve for α^2 by means of

$$f_\lambda = 2PA(n\cdot\hat{r})^2(n\cdot\lambda) \tag{8}$$

where λ represents VOP components in the RSW frame. For the coplanar problem, commanded α is

$$\alpha = \arctan\left(\frac{\lambda_R}{\lambda_S}\right) \tag{9}$$

For the orbit semi-major axis to increase, the expression for α is found to be

$$\tan\widehat{\alpha} = \frac{1+e\cos f}{e\sin f} \tag{10}$$

Force along this vector can be minimized by the use of equation (5).

3. SIMULATION AND RESULTS

3.1 Raising Semi-Major Axis

Solar sail trajectories for various values of β are generated to optimally increase the spacecraft semi-major axis to that of Mars and Jupiter. Planetary orbits are assured to be co-planar and circular. Perturbing accelerations are not taken into consideration, just those of solar radiation pressure and gravity of sun. Trajectories are assured to start outside of

Earth's Sphere of Influence (SOI), where $V_{\inf inity}$ is zero. Thus, the only ΔV needed in these trajectories is to inject the spacecraft in the target planet's orbit. Resultant trajectories can be seen for Mars in Figure 3-5.

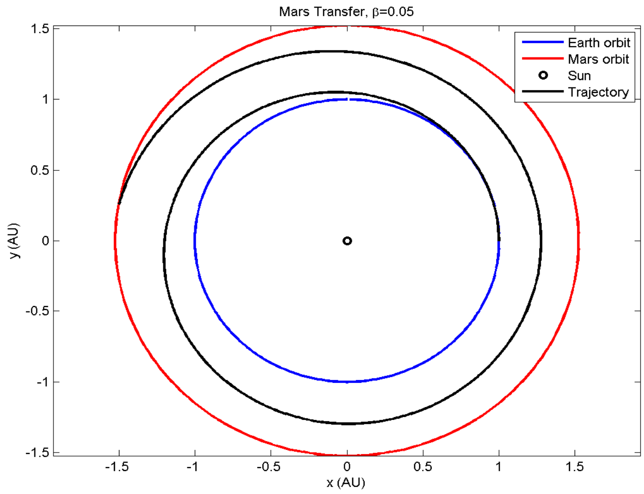

Figure 3 Mars, β= 0.05

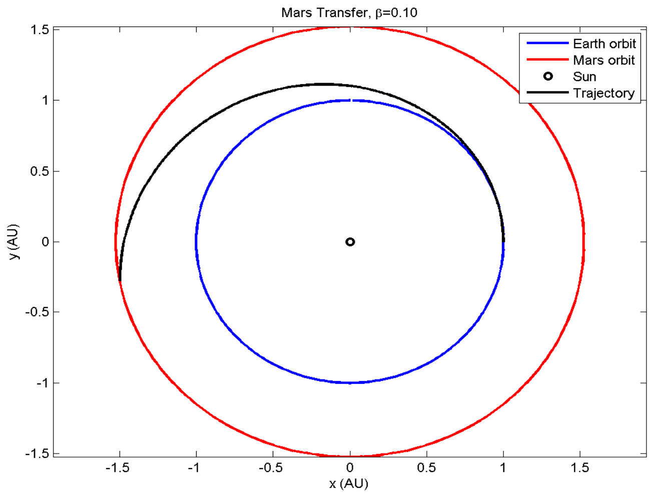

Figure 4 Mars, β= 0.10

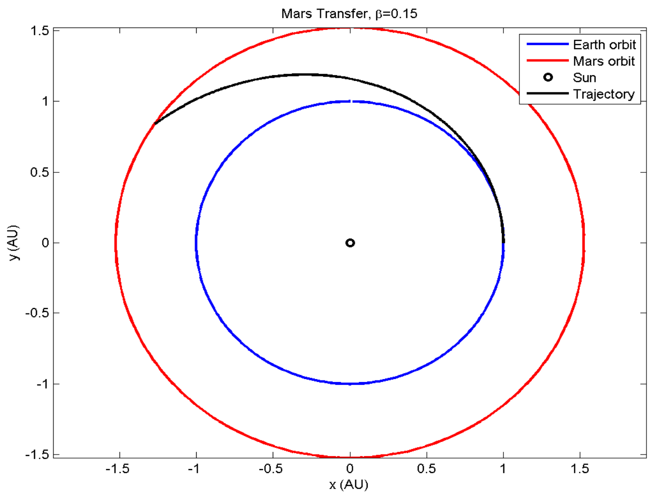

Figure 5 Mars, β= 0.15

Table 1 shows the performance of these trajectories compared to Hohmann transfer.

Table 1: Mars Transfer with Optimal Raise of Semi-Major Axis

β	ΔV_{total} (km/sec)	Transfer Time (days)	Phase Angle (degrees)	Intercept Angle (degrees)
Hohmann	5.594	259	180	0
0.05	2.376	747	530	6
0.10	4.410	251	191	10
0.15	7.552	186	146	18

Trajectories where the semi-major axis is optimally increased provided advantages in ΔV savings over pure Hohmann transfers. For the lowest lightness value simulated, more than 50% of ΔV is saved at the cost of almost three times the transfer time. For β= 0.10, the transfer time is similar, but the continuous sail thrust saves over 1.184 km/sec. This is not as attractive as it seems, however, due to the greater sail area required and the challenges in implementing it. β= 0.15 actually saved more than 2 months of transfer time, but the ΔV exceeded the Hohmann transfer by 1.958 km/sec. The cone angles used in each trajectory is presented in Figure 6.

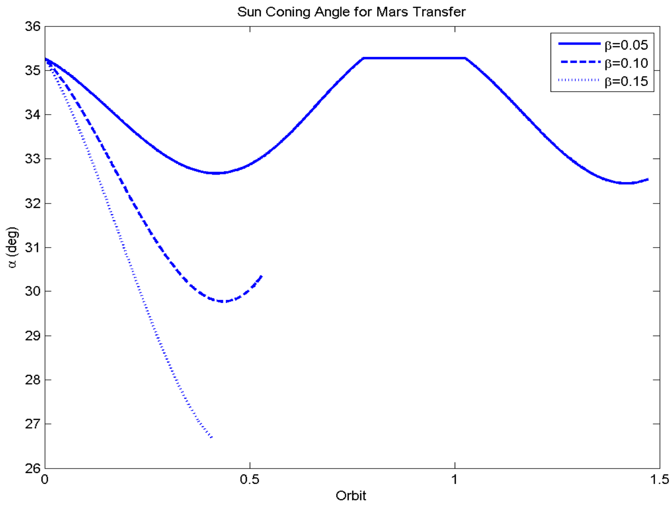

Figure 6 Mars Cone Angles

The cone angles were capped at 35.25°. Higher lightness values allowed for lower cone angles, which generated higher forces.
Resultant trajectories for Jupiter are presented in Figure 7-9.

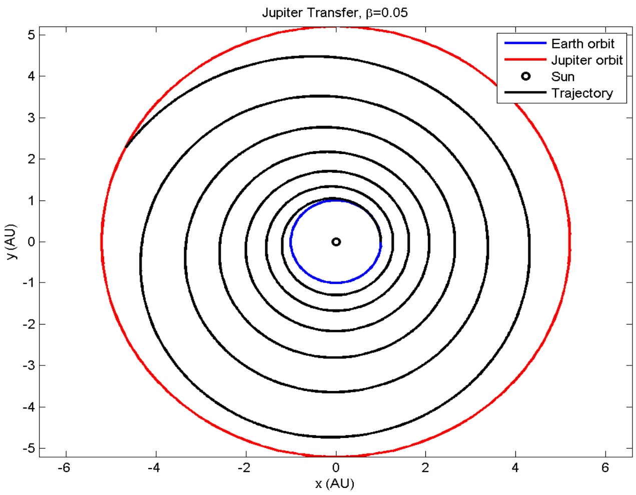

Figure 7 Jupiter, β= 0.05

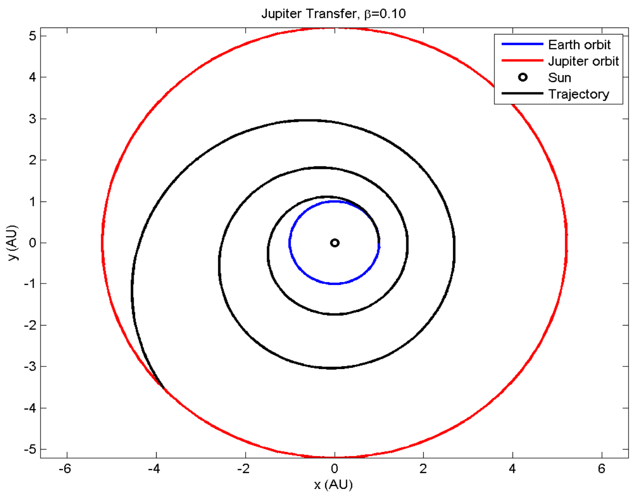

Figure 8 Jupiter, $\beta = 0.10$

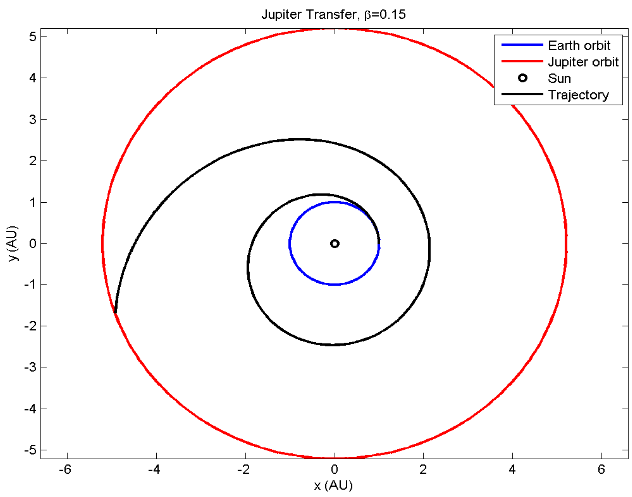

Figure 9 Jupiter, $\beta = 0.15$

Table 2 shows the performance of these trajectories compared to Hohmann transfer.

Table 2: Jupiter Transfer with Optimal Raise of Semi-Major Axis

β	ΔV_{total} (km/sec)	Transfer Time (days)	Phase Angle (degrees)	Intercept Angle (degrees)
Hohmann	7.436	997	180	0
0.05	2.061	10372	2314	9
0.10	2.544	3650	943	9
0.15	5.070	2088	559	23

Trajectories to Jupiter where semi-major axis optimally increased showed fewer desirable qualities than those of Mars. In the case of β = 0.05, the transfer time is over ten times longer, but 5.375 km/sec of ΔV is saved. There is no case where solely optimizing semi-major axis reaches Jupiter in less time than a Hohmann transfer, and the required ΔV for planetary capture increases with lightness value.
The cone angles used in each trajectory for Jupiter transfer is seen in Figure 10.

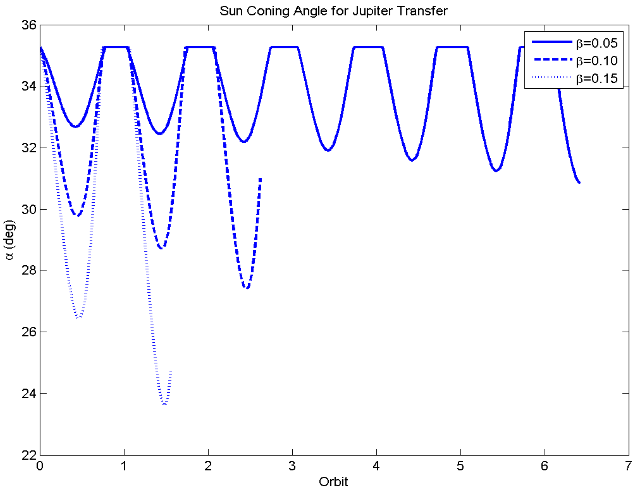

Figure 10 Jupiter Cone Angles

The cone angles are again capped at 35.25°. Minimum cone angles decrease with the higher phasing angles for a given sail lightness β.

3.2 Raising Semi-Major Axis

To determine the effect of intercept angle and continued thrusting, a coast phase is introduced immediately after the oscillating aphelion radius is equal to the target planet's radius. The result for Mars is shown in Table 3.

Table 3: Mars Transfer with Optimal Raise to Target Aphelion, followed by Coast

β	ΔV_{total} (km/sec)	Transfer Time (days)	Phase Angle (degrees)	Intercept Angle (degrees)
0.05	1.116	811	563	0
0.10	2.559	302	215	0
0.15	2.793	266	186	0

As one would expect, the transfer time modestly in all cases. However, significant changes in total ΔV are obtained for higher sail lightness values. Table 4 shows the results for Jupiter transfers with the same strategy.

Table 4: Jupiter Transfer with Optimal Raise to Target Aphelion, followed by Coast

β	ΔV_{total} (km/sec)	Transfer Time (days)	Phase Angle (degrees)	Intercept Angle (degrees)
0.05	0.692	11011	2367	0
0.10	2.138	3839	956	0
0.15	3.109	2407	580	0

Jupiter transfers utilizing the coast stage also show high ΔV savings at the expense of increased transfer time.

4. RESULTS

The use of solar sail can provide the energy required to reach the orbits of the outer planets without initial hyperbolic excess velocity. However, it generally comes with the cost of increased transfer times or increased ΔV to insert into planetary orbit.

Increasing the semi-major axis optimally until the sail craft reached the target radius resulted in lower values of β requiring different ΔV at arrival. Higher value of β required a total ΔV at Mars that exceeded two Hohmann burns, but sped up transfer by 73 days. For all β values on a Jupiter transfer, transfer times are increased but there is much excess velocity that is trimmed.

Coasting after the desired aphelion radius is reached, decreased final ΔV in all cases studied. While the transfer time increased dramatically for Jupiter, the highest value of β resulted in a transfer time only 1.5 year longer than the Juno mission cruise phase.

The potential savings in propellant by using solar sails is obvious in all cases studied. Results showed, by using optimal methods, sail craft can open a wide array of mission options.

References

[1] D. A. Vallado, “Fundamentals of Astrodynamics and Applications,” McGraw-Hill, 1997.
[2] D. A. Vallado, “Continuous-Thrust Transfers,” Fundamentals of Astrodynamics and Applications, The Space Technology Library, 4th ed., 2013.
[3] C. McInnes, “Solar Sailing: Technology, Dynamics, and Mission Applications,” Springer, 2004.
[4] R. Stevens, I.M. Ross, S. Matousek, “Earth-Mars Return Trajectories Using Solar-Sails,” 55th International Astronautical Congress, IAC-04-A.2.08, Canada, 2004.
[5] B. Dachwals, W. Seboldt, B. Hausler, “Performance Requirements for Near-Term Interplanetary Solar Sailcrafts Missions,” 6th International AAAF Symposium on Space Propulsion: Propulsion for Space Transportation of the XXIst Century, 2002.
[6] J. L. Wright, “Space Sailing,” Gordon and Beach Science Publishers, Philadelphia, 1992.
[7] C. G. Sauer, “Optimum solar-sail interplanetary trajectories,” AIAA/AAS Astrodynamics Conference, San Diego, 2000.
[8] M. Leipols, “To the sun and pluto with solar sails and micro-sciencecraft,” Acta Astronautica, 45(4-9):549-555. 1999.