Investigation of Solar Radiation Pressure on HAMR Objects

Deepak Gaur¹, M. S. Prasad²

¹M. Tech. (Avionics), Amity Institute of Space Science and Technology, Amity University, Noida, U.P., India
²Director, Amity Institute of Space Science and Technology, Amity University, Noida, U.P., India

ABSTRACT
These High area-to-mass ratio (HAMR) objects are a new class of debris objects that have been found in GEO-like orbits with high eccentricities. It is speculated that these objects originated from satellites in GEO and migrated to their current orbits. This migration is likely due to perturbations that have a strong impact on HAMR objects because of their high area and low mass. Solar radiation pressure (SRP), in particular, strongly affects HAMR objects and can cause large variations in eccentricity and inclination over time. Several studies observing and modeling HAMR objects have noted these changes in eccentricity and inclination, and also the periodic behavior of these elements due to the rotation of the Earth about the Sun. Models for HAMR object behavior are currently insufficient, and better knowledge of these orbits is needed due to their close proximity with the highly populated GEO regime. The current research utilizes a simple model for the acceleration due to SRP to model the changes in orbital elements over time for HAMR objects. The results are compared to previous studies and are found to agree well with higher fidelity models as well as observations of HAMR objects. The model is also used to propagate HAMR object orbits with a varying area-to-mass ratio over time. These results confirm that the area-to-mass ratio has a very strong impact on the way the orbital parameters vary with time.

Keywords: High Area-to-Mass Ratio (HAMR), Solar Radiation Pressure (SRP), Perturbations, Geosynchronous Earth Orbit (GEO).

1. INTRODUCTION
High-area-to-mass ratios (HAMR) objects are a fairly new population of debris. They exhibit extremely interesting properties due to their material make-up. They have high surface areas and low mass and are, therefore, more prone to perturbations, particularly those perturbations that are dependent on surface area. Solar radiation pressure (SRP) is a dominant perturbation for HAMR objects and causes unique periodic changes in inclination and eccentricity over time. HAMR objects are also often a flexible material, which results in random, and mostly unpredictable dynamic responses to perturbations. In order to improve space situational awareness and avoid collisions in GEO, a better model for HAMR object orbits and behavior is needed.

2. BACKGROUND
HAMR objects behave in a unique manner due to their high surface area, low mass, and flexible nature. Their unique orbital properties are due to susceptibility to perturbations. Solar radiation pressure is the most significant perturbation that affects HAMR orbits. A better understanding and modeling of HAMR objects under perturbations is needed in order to better predict HAMR orbits.

2.1 HAMR Objects
In 2004, a new class of objects was discovered by the Astronomical Institute of the University of Bern. Standard space objects have an area-to-mass ratio around 0.01 to 0.02 m²/kg. However, this new class of objects has area-to-mass ratios greater 0.1 m²/kg., classifying them as high-area-to-mass ratio (HAMR) objects. These objects are in GEO-like orbits with high eccentricity. There are no potential parent objects in the region where the HAMR objects are found. [6] Therefore, it is theorized that these HAMR objects originated in a different orbit and migrated over time to their current orbits. The mean motion of HAMR objects is concentrated around the nominal value for GEO. [6] It is likely that these objects originated in GEO orbits from explosions, collisions, or deterioration over time, then evolved into highly eccentric orbits. Many spacecraft and upper stages have surfaces that are covered in multi-layer insulation (MLI). MLI can be layers of Dacron nettings or thin sheets of aluminized Mylar, Kapton, or Nomex. [5] MLI is installed to reduce heat loss on board spacecraft. [1] Many HAMR objects are likely pieces of MLI. HAMR objects are subject to all the perturbations that other space objects face. These include, but are not limited to, non-spherical Earth gravity, thermal radiation, solar radiation pressure, central body gravity, as well as the attitude and material properties of the object. [4] Some perturbations are related to the surface area of the object and, therefore, have
a stronger impact on HAMR objects compared to traditional debris and satellites. For example, solar radiation pressure, thermal emission effects, electrostatic surface charging, and interaction with the weak magnetic field all affect HAMR objects more than traditional objects due to their large surface areas and low masses. [3] Solar radiation pressure (SRP), in particular, has a greater impact on HAMR objects than on standard space objects, as solar radiation pressure is directly related to the surface area exposed to the radiation of the sun. HAMR objects are also lightweight so that the force imparted by SRP can significantly change their orbits. Solar radiation pressure is related to the distance the object is from the sun, therefore, its effects are periodical in nature. In fact, the perigees of HAMR objects tend to line up toward the Sun, moving with the sun through the year, as shown in Figure 1. [5]

The strong contribution of solar radiation pressure to HAMR objects creates periodic variations in eccentricity and inclination and explains how the objects evolved from GEO orbits to their current orbits with high eccentricities. However, not all HAMR objects will evolve into the same orbits. In fact, debris with similar area-to-mass ratios can develop very different eccentricities and inclinations, as shown by the plot in Figure 2 where all objects have area-to-mass ratios of 10 to 20 m²/kg, but were released at different epochs. [5]

Another unique feature of HAMR objects compared to traditional space objects is that HAMR objects are often flexible materials. Debris objects are usually considered rigid, but MLI is very flexible and has almost no structural strength. [1]. Flexibility implies that the geometry and orientation of the object can vary with time, therefore, changing the effective surface area of the object. The area of a HAMR object has been observed to vary somewhat randomly with time. [2] In an investigation by Früh and Schildknecht, five HAMR objects were observed. Some objects indicated a constant increase in area-to-mass ratio, while others exhibited a periodic variation, however, all objects showed a
general trend in variations around of 20% of the mean value for the object. [2]. The varying area of HAMR objects affects the evolution of the orbital elements in an unpredictable way. [1] A study by Channumsin, et. al, attempts to model the flexibility of HAMR objects and compare flexible objects with rigid flat plate models that otherwise have identical properties. [1] They found that the eccentricity changed in quite different fashions between the rigid and flexible objects. The rigid body displayed constant growth in eccentricity over the 150-day test period. On the other hand, the flexible object did not grow in eccentricity, but rather oscillated around a mean eccentricity value. It is hypothesized that the periodic nature of the eccentricity of the flexible object was due to the change in cross-sectional area enabled by the flexibility of the object, which in turn affected the magnitude of SRP. It was also found through the use of Euler angles that the attitude of the flexible object rotated far more slowly than the rigid body. This observation can also be related to the deformation of the object allowed by the flexible nature of MLI. The center of mass and moment of inertia are always changing in addition to the fact that some effects of perturbations go into deforming the object rather than spinning the object. This study shall be discussed further in the Previous Studies Section as a baseline for the current investigation.

2.2 Solar Radiation Pressure
Solar radiation pressure is a non-conservative perturbation that all spacecraft experience. SRP becomes much more pronounced at high altitudes where the effects of drag become insignificant. The effect of SRP varies with distance from the sun, and the intensity of solar activity. [7] Solar radiation pressure is caused when photons from the sun strike the surface of a spacecraft and impart momentum to the object. Energy is imparted to the spacecraft from both the photons that are absorbed and the photons that are reflected. The net force of the photons striking the spacecraft acts through the center of pressure of the object and can also produce a torque if the center of pressure is not aligned with the center of mass. On average, $8 \times 10^{17}$ photons/cm² from the Sun reach the Earth, translating to the intensity of SRP (also called the irradiance or solar flux, SF) of 1367 W/m². [7]. The force of solar pressure per unit area can be found using Einstein’s law for energy: [7]

$$E = mc^2 \rightarrow mc = \frac{E}{c} \quad (1)$$

The term $mc$ in Equation (1) equates to momentum, where $m$ is the mass and $c$ is the speed of light. The change in momentum equates to the solar radiation pressure: [7]

$$p_{srp} = \frac{SF}{c} = \frac{1367W}{3 \times 10^8 m^2/s} = \frac{4.57 \times 10^{-6}}{m^2} \quad (2)$$

The acceleration due to SRP is simply the force due to solar radiation pressure divided by the mass of the object according to Newton’s second law. Mathematically, the acceleration due to SRP is: [7]

$$a_{srp} = \frac{F_{srp}}{m} = \frac{p_{srp} C_R A_{\text{sat}}} {m_{object} \left| \vec{r}_{\text{sat}} \right|^2} \quad (3)$$

In Equation (3), $\vec{r}_{\text{sat}}$ represents the Sun, and is a unit vector from the satellite to the sun. The term $A_{\text{sat}}$ is the area of the object that is currently exposed to the sun, rather than the total surface area of an object. This is a very difficult quantity to determine and requires precise knowledge of the object attitude and shape. Finally, the term $C_R$ is the reflectivity of the object. It is a material property that represents how much incoming radiation is reflected and how much is absorbed. This term requires a knowledge of the material properties of the spacecraft, as well as the attitude of the object, particularly if it is non-homogenous. The reflectivity is a value between zero and two. A reflectivity of zero represents a translucent object where SRP imparts no momentum. A reflectivity of one means that all radiation is absorbed (black body) and all momentum from the SRP is transmitted to the object. Lastly, a reflectivity of two indicates that all the radiation is reflected so that twice the momentum from the SRP is transmitted to an object. [7] It should be noted that while values of reflectivity between zero and two are typical and are used for this study, a value between zero and four can also be used when Equation (3) is represented with a $C_R / 2$ term, so that a reflectivity of two actually represents the black body case. This is done in the study by Früh and Schildknecht. [2]

Ultimately, modelling solar radiation pressure is a challenge due in part to the difficulty in modelling and predicting both solar cycles and the cross-sectional area of an object that faces the sun. [7] Much more complex models than Equation (3) would be needed to accurately model SRP. Equation (3) represents the flat plate case, which is actually fairly representative of HAMR objects. In reality, reflections will be somewhat unpredictable since any real surface is not a true flat plate. There are also cannon ball and cylindrical SRP acceleration models. Equation (3) also does not
consider the application point of the net force due to SRP, and instead simply applies an overall acceleration to the object. In reality, some SRP will contribute to a force in the sun-satellite direction, while some will create a torque on the object, unless the centre of pressure is perfectly aligned with the centre of mass. The value of $p_{\text{srp}}$ would also vary with solar cycles and solar storms.

Even if the value of $p_{\text{srp}}$ is assumed to be constant, the effect of SRP is only present when the object is in sunlight. Earth orbiting objects will periodically go in to solar eclipses where the Earth causes the object to be in complete or partial shadow. The shadowed area can be divided into two regions: the umbra is the area that is totally eclipsed by the Earth and the penumbra is the area that is only partially obscured by Earth. [7] For high accuracy models, knowledge of the umbra and penumbra would be necessary to better model SRP. However, since the Sun is very far from the Earth, it can be assumed that the sun is infinitely far away and that the sun rays are parallel, leading to the cylindrical Earth shadow model. [7]. Geometry can be used to determine if the object is in shade or sunlight.

The equations and methods for determining the perturbations due to SRP that have been described thus far are representative of a fairly simplified model. As previously stated, knowledge and modeling of the object surface properties and attitude, the true solar flux, the penumbra and umbra, and many other contributions, increases the fidelity of the model. For the purposes of the current study, a simple model is used, as described in the Current Study Section. It is not inappropriate to use a simple model when addressing HAMR objects since so much about HAMR objects is still unknown, resulting in missing components for higher fidelity models. However, some previous studies have utilized higher fidelity SRP models. The study by Chandnumsin, et al., has separate coefficients for specular and diffuse reflection, a surface normal unit vector, a solar incidence vector, and models a satellite with $n$ number of flat plates.

3. MOTIVATION

The difficult task of modelling SRP is only part of the challenge in modelling HAMR objects. It is also difficult to determine the shape and surface properties of HAMR objects. There does not appear to be any correlation between the apparent brightness and the shape or surface properties. [6] It is also nearly impossible to maintain a full catalogue of all HAMR objects. Variations in area-to-mass ratio result in varying brightness so that it is difficult to keep track of a single object or to differentiate new objects without regular observations at short time intervals. [2] Better models for HAMR objects that account for all perturbations and capture the true dynamics of these objects are needed.

Space situational awareness (SSA) is an ever more prominent area in the field of space debris mitigation. An accurate assessment of the space environment, particularly in the highly trafficked GEO, LEO, and GPS regions is critical in assuring the safety of space assets. HAMR objects do pose a collision hazard due to their close proximity and periodic overlap with the heavily populated GEO regime. In fact, due to the larger orbits of HAMR objects, they would encounter GEO orbits with a higher speed than if they were still in a GEO orbit themselves. [5] They also can impact more than just the GEO orbits that they likely originated from. HAMR orbits can also interfere with higher altitude orbits such as mid-Earth orbits for GPS, GLONASS, and Galileo. [5] Improvements in SSA depend heavily on better tracking and modelling of both natural and man-made space debris. Accurate orbit determination as well as better predictions of HAMR objects is critical in assessing risk and avoiding collisions in the GEO regime.

4. METHODOLOGY

The current study uses a simple model for acceleration due to solar radiation pressure, as described by Equation (3). Acceleration due to the J2 of the Earth is also included and two-body motion is assumed. It is intended for the simple model to show similar patterns in the evolution of orbital elements for HAMR objects as some of the studies previously introduced. In addition to replicating previous studies, the area-to-mass ratio of the object will be varied with time to represent the dynamic nature of the shape and orientation of HAMR objects. The evolution of the orbital elements for a varying area-to-mass ratio will be compared with the results for a constant area-to-mass ratio as well as previous studies.

The earth orbit around the sun must be modelled to determine the vector sum from the satellite to the sun and if the satellite is in the shadow of the earth. The orbital elements of the earth’s orbit are determined by the planetary ephemerides. [7] The starting epoch was chosen in a way that earth is at perihelion so that the true anomaly is zero. The earth orbit about the sun is then propagated for the desired time span using MATLAB® built-in Runge Kutta integrator, ode45, with time steps of one day i.e., 86400 seconds.

Equation (3) is used to find the acceleration due to solar radiation pressure. The area-to-mass ratio in this equation is representative of the area receiving sunlight and is assumed to be 75% of the input ratio. A cylindrical model is used to determine if the object is in shade or shade. If it is determined to be in shade, the acceleration due to SRP is set to zero. The initial orbital elements vary by trial, but are representative of a typical HAMR orbit near GEO. The propagation
time also varies based on the trial, but the time step is always one minute i.e., 60 seconds. Once again, MATLAB® built-in Runge Kutta integrator, ode45, is used to propagate the orbit of the object around earth.

5. RESULTS
Several runs of the model were conducted based on the initial orbital elements summarized in Table 1. Trials conducted are meant to study the effect on \(a, e, i, \) and \(\Omega\), since these are the elements most impacted by SRP. The trials are summarized in subsections below.

<table>
<thead>
<tr>
<th>Table 1: Initial orbital elements. [1]</th>
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5.1 Trial 1
This trial is a baseline trial to determine how orbital elements evolve without the effect of SRP. The elements have periodic variation about a mean value due to the effects of the earth’s \(J_2\), but the orbit ultimately does not change over time. Figure 7 shows the elements for 150 days.

Figure 3 Baseline orbit without solar radiation pressure

5.2 Trial 2
This trial is intended to replicate the results of Channumsin, et. al for propagated duration of 150 days with \(A/m = 20\) \(m^2/kg\). [1] The eccentricity increases, but at a faster rate and in a more linear manner. The inclination increases as well and more closely matches the magnitude of the inclination of Channumsin, et. al. the inclination is also periodic, but with a higher frequency. The inclination also increases for about 100 days then begins to decrease, indicating an overall periodic behaviour that Channumsin, et. al does not indicate. Considering that the study by Channumsin, et. al used a higher fidelity model than the current study, the results show similar overall behaviour in the orbital elements.
5.3 Trial 3

This trial extended the propagation for trial 2 from 150 days to 365 days. The object is propagated with \( A/m = 20 \text{ m}^2/\text{kg} \) and \( A/m = 1 \text{ m}^2/\text{kg} \) in order to study the impact on the magnitude of area-to-mass ratio has on the orbital elements. This trial is meant to align with the study by Liou and Weaver. [5] Figure 5 below shows the elements for 365 days for each area-to-mass ratio. Both objects show a periodic motion in eccentricity where one period equals one year. However, the amplitude of the \( A/m = 1 \text{ m}^2/\text{kg} \) object eccentricity is an order of magnitude smaller than the \( A/m = 20 \text{ m}^2/\text{kg} \) object. The magnitude and period of the eccentricity in Figure 9 aligns very well with the eccentricity plots determined by Liou and Weaver. [5] Therefore, the yearly periodic behaviour in eccentricity is captured well by the simple model.

The inclination for the \( A/m = 1 \text{ m}^2/\text{kg} \) object in Figure 5 is shown not to vary much time. This can be said that to align with the results of Liou and Weaver, where the inclination grows very slowly, only reaching 10 degrees after 30 years. This equates to about 0.333 degree per year. While the current study shows inclination decreasing slightly over a year for the \( A/m = 1 \text{ m}^2/\text{kg} \) object, the result ultimately shows that the inclination does not vary significantly. Therefore, the object behaves more like a typical GEO object, which still agrees with the results from Liou and Weaver. For the \( A/m = 20 \text{ m}^2/\text{kg} \) object, the growth in the inclination in one year is much pronounced in Figure 5. The inclination increases up to approximately 2.5 degrees in 365 days. Results from Liou and Weaver shows the inclination reaches a maximum of about 35 degrees after around 8 years. This equates to a little over 4 degrees per year. Therefore, the current model produced a yearly increase in eccentricity of the same magnitude as Liou and Weaver. It can also be seen in both studies for \( A/m = 20 \text{ m}^2/\text{kg} \) object, that the eccentricity changes in a periodic manner with a period of about a year.

![Figure 4](image.png)

**Figure 4** \( A/m = 20 \text{ m}^2/\text{kg} \) object propagated over 150 days
Another result discussed by Liou and Weaver is the precision of the right ascension of the ascending node (RAAN, $\Omega$). Figure 1 depicted this behaviour as the orbit of HAMR objects over time migrate so that the perigee aligns with the sun. The final plot for the $A/m = 20 \text{ m}^2/\text{kg}$ object in Figure 5 indicates a strong precession of the RAAN, starting at 45 degrees, going through 0 degree, and ending at around 250 degrees. This result could correlate with the orbit perigee aligning with the sun as the earth orbits due to SRP. The precession in the RAAN is far noticeable for the $A/m = 1 \text{ m}^2/\text{kg}$ object, and is only about 7 degrees over the year.

5.4 Trial 4

This trial is intended to replicate the results of Kelecy et al. [3]. Figure 6 shows the elements for 25 days. The plots agree extremely well with the results of Kelecy et al, especially considering results from the model (Figure 10) are being compared to results from observation filtering. The variation in semi-major axis, eccentricity, and RAAN in Figure 10 match with the results of Kelecy et al almost perfectly. As expected, the eccentricity increases over time due to SRP. The semi-major axis varies periodically over time about a mean value. The RAAN decreases from 170 degrees to almost 169 degrees. Only the change in inclination does not align between two studies. The current model shows a decrease in inclination, whereas Kelecy et al results showed an increase. However, both the studies showed an inclination change of around 0.05 degrees in 25 days. Overall, the results indicate that while simple, the model used to calculate the acceleration due to SRP in this study can represent realistic SRP effects on HAMR objects.
5.5 Trial 5

This trial begins with a 20 m²/kg object and increases the area-to-mass ratio by 2% for the first 75 days, then decreases the area-to-mass ratio by 2% for the last 75 days of the propagation period. While this is not likely to be representative of actual HAMR object dynamics, the exaggeration in the variation of the area-to-mass ratio will make it easier to see the impact that the ratio has on the orbital elements.

The results are plotted in Figure 7 and are compared to Figure 2, where an object with a constant area-to-mass ratio of 20 m²/kg is propagated for 150 days. Both trials have the same initial orbit elements. Figure 6 clearly shows an inflection point in all plots at 75 days when the variation in area-to-mass ratio switches from increasing with time to decreasing with time. The semi-major axis, while remaining fairly constant with time, varies with a greater amplitude about the mean value when the area-to-mass ratio is at its maximum. The plot of eccentricity also shows exponential-like behaviour for the first 75 days, indicating an increase in the rate of eccentricity continues to grow but at a much slower rate as the area-to-mass ratio continues to decrease. The inclination and RAAN both respond similarly to the varying area-to-mass ratio. Neither respond very much until the area-to-mass ratio reaches around 50 m²/kg then the effects of SRP become noticeable and both grow until the area-to-mass ratio once again decreases to the same threshold value around 50 m²/kg where the SRP effects are less noticeable. It is clear from these results that the area-to-mass ratio has a very strong impact on how SRP affects orbits of HAMR objects.

![Figure 7](image.png)

Figure 7 A/m = 20 m²/kg object with A/m increasing by 2% for 75 days, then decreasing by 2% for 75 days.

5.6 Trial 6

This trial begins with a 20 m²/kg object and decreases the area-to-mass ratio by 2% for the first 75 days, then increases the area-to-mass ratio by 2% for the last 75 days of the propagation period. Once again, this is not likely to be representative of actual HAMR object dynamics, but is intended to emphasize the impact of the area-to-mass ratio on orbital elements.

The results are shown in Figure 8 and are once again compared to Figure 4. They are also compared to Figure 7, where the opposite variation in area-to-mass was implemented. Figure 8 shows the most variations in the orbital elements at the beginning and end of the propagation, where the area-to-mass ratio was largest. In the middle of the propagation, when the area-to-mass ratio reaches around 5 m²/kg, the eccentricity and inclination do not grow as rapidly.

Overall, the plots in Figure 8 resemble Figure 4 in general trends, but the maximum eccentricity and inclination reached in Figure 4 are not attained in Figure 8 since the area-to-mass ratio is less than 20 m²/kg for almost the entire 150 days. The RAAN also precessed by about 1 degree less compared to the constant A/m = 20 m²/kg case. Once again, the effect of area-to-mass ratio on the orbital elements though SRP is demonstrated by these results. The orbital elements vary less with time as the area-to-mass ratio decreases.
5.7 Trial 7

This trial begins with a 20 m²/kg object and alternates daily increasing and decreasing the area-to-mass ratio by 5% for the entire propagation period. This is somewhat more representative of actual HAMR object dynamics, but quite idealized.

The results are shown in Figure 9 and are once again compared to Figure 4. Interestingly, the variation of the area-to-mass ratio seems to have little effect on the orbital elements when the variation is periodic and only decreases slowly with time. The results match Figure 4, reaching the same final values for all orbital elements.

![Graph showing area-to-mass ratio over time](image-url)
6. CONCLUSION

The comparison between the current model results and the results of previous studies confirms that a simple model of solar radiation pressure can produce agreeing effects on the orbital elements of HAMR objects. Overall, the eccentricity variations produced in the current study closely aligned in both magnitude and behaviour with the previous studies. Inclination aligned with previous studies in magnitude, although the behaviour did not align as closely, sometimes showing a decrease in inclination when previous studies indicated a growth.

The results from the variation of area-to-mass ratio, while somewhat exaggerated, showed the dependence that the orbital elements of HAMR objects have on area-to-mass ratio and solar radiation pressure. The plots produced in Trials 6 and 7 particularly help to explain how HAMR objects can develop into such different orbits over time. However, small daily changes in the area-to-mass ratio that result in a gradual change in the area-to-mass ratio over time do not seem to create significantly different orbits than that of an object with a constant area-to-mass ratio. The variations in orbital elements due to SRP are directly related to the area-to-mass ratio of the object. This ratio varies with time, and one contribution not yet discussed is surface charging. As the surface of a HAMR object charges naturally from the space plasma, repulsive or attractive forces can result from an uneven charge distribution. This can cause a dynamic response by the object, resulting in bending and reshaping of thin HAMR objects. This dynamic response alters the area-to-mass ratio, affecting the solar radiation pressure experienced by the object. Accurate dynamic modelling of the charging of HAMR objects could fill one of the voids in the currently inadequate HAMR debris tracking and modelling. Overall, more accurate modelling of the area-to-mass ratio of HAMR objects could improve modelling and tracking methods, leading to better SSA.

References