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Full Order Observer Controller Design for Two Interacting Tank System Based on State Space Approach

Amitava Biswas[1], Gargee Chakraborty[2]

[1]Department of Applied Physics, Calcutta University, 92, A. P.C Road, Kolkata-700009, INDIA

[2] Department of Applied Physics, Calcutta University, 92, A. P.C Road, Kolkata-700009, INDIA

ABSTRACT

The control of liquid level in tank system and flow between tanks is main problems in process industries. Thus, the control of liquid level in tank system and flow between tanks which is must be controlled. The control of level of tank in the interacting system is major task. The main objective of this paper is to determine the mathematical model of a coupled tank system which will be useful for designing and to implement full-order observer using the software packages for computer aided control system design in MATLAB. A state feedback gain matrix is designed for the interacting tank systems with the help of pole-placement technique. Observer estimation errors are presented by choosing the observer(s) initial conditions. The proposed study of this observers and observer based controllers may find application in numerous engineering and scientific industrial problems.

Keywords: Liquid level system, state space, state observer, pole-placement

1. INTRODUCTION

Liquid level control [1] is needed in various industrial applications, e.g., in food processing, water purification systems, filtration, pharmaceutical industries, etc. is well known that the state coupled two-tank liquid level system. The state space approach is a generalized time domain method for modeling [2], analyzing and designing a wide range of control systems and is particularly well suited to digital computational technique.

The state observers [1] are used not only for the purpose of feedback control, but also in their own right to observe state variables of a dynamic system [3], which can be an experiment in progress whose state has to be monitored at all times. In this paper we shall present design methodology of closed loop control in state space domain using state feedback gain matrix [4]. This concept of technique is called pole placement technique. It will be shown that if system is completely states controllable then poles of closed loop system may be placed at desired location with the help feedback gain matrix $\boldsymbol{K}$.

In pole placement technique, an assumption is made that all state variables are available for feedback to design the control system. In practice, all states are not available during measurement. So it may be required to estimate unavailable variables. So the process of estimation of unmeasured states is called observation [5]. To build an observer gain matrix $\boldsymbol{K_e}$ is required. So the state observer gain matrix $\boldsymbol{K_e}$ can be designed if the system is completely sate observable [6, 7]. After designing the observer it will be shown that the response of measured and unmeasured sate of variables. A control algorithm is implemented in MATLAB software.

2. MATHEMATICAL MODELING OF TWO TANK INTERACTING LEVEL PROCESS

The process consisting of two interacting liquid tanks shows in Fig. 1. The height of the liquid level is h_1 (cm) in tank-1 and h_2 (cm) is tank-2. Volumetric flow into tank-1 is q_{in} (cm^3/sec), the volumetric flow rate from q_1 (cm^3/sec), and the volumetric flow rate from tank-2 is q_o (cm^3/sec). Cross sectional area of tank-1 is A_1 (cm^2) and area of tank-2 is A_2 (cm^2). R_1 and R_2 are the resistance parameter (valve) in flow line shown in Figure 1.

The differential equations related to two interacting tank system are given below:

(*i*) ***For tank-1***:

$$A_1 \frac{dh_1}{dt} = q_{in} - q_1$$

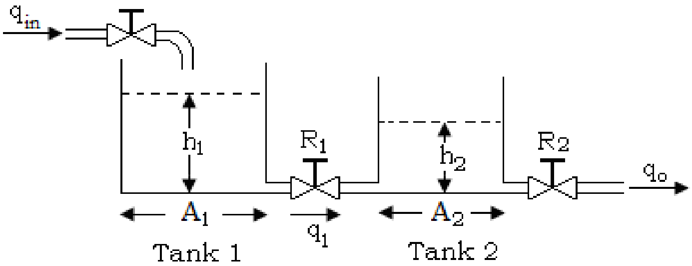

Figure 1 Two interacting tank system

Assume linear resistance to flow,

$$q_1 = \frac{h_1 - h_2}{R_1}$$

$$\Rightarrow \quad A_1 \frac{dh_1}{dt} = q_{in} - \frac{h_1 - h_2}{R_1}$$

$$\therefore \quad \frac{dh_1}{dt} = -\frac{h_1}{R_1 A_1} + \frac{h_2}{R_1 A_1} + \frac{q_{in}}{A_1} \qquad (1)$$

Time constant for tank-1 is $\tau_1 = R_1 A_1$

(*ii*) ***For tank-2***:

$$A_2 \frac{dh_2}{dt} = q_1 - q_o$$

$$\Rightarrow \quad A_2 \frac{dh_2}{dt} = \frac{h_1 - h_2}{R_1} - \frac{h_2}{R_2}$$

$$\therefore \quad \frac{dh_2}{dt} = \left(\frac{1}{R_1 A_2}\right) h_1 - \frac{1}{R_2 A_2}\left(1 + \frac{R_2}{R_1}\right) h_2 \qquad (2)$$

Time constant for tank-2 is $\tau_2 = R_2 A_2$

After rearranging the equations objective transfer is

$$\frac{H_2}{H_1} = \frac{R_2}{A_1 A_2 R_1 R_2 s^2 + (A_1 R_1 + A_2 A_2 + R_2 A_1)s + 1} \qquad (3)$$

Table 1: Specification of two tank interacting process and experimental result taken from real time system

Condition	Flow in LPH	Height of tank-1 (mm)	Height of tank-2 (mm)	Area of tank-1 = area of tank-2	Time constant of tank-1	Time constant of tank-2
Final state after step change	300	100	50	100 cm^2 = 0.01m^2	$\tau_1 = 9s$	$\tau_2 = 3.6s$
Initial state	100	50	30	100 cm^2 = 0.01m^2	$\tau_1 = 9s$	$\tau_2 = 3.6s$

Now, Equations (1) and (2) becomes

$$R_1 = \frac{dh_1}{dt} = \frac{(100-50)\,mm}{(100-100)\,LPH} = 900 \ s/m^2$$

$$R_2 = \frac{dh_2}{dt} = \frac{(50-30)\,mm}{(300-100)\,LPH} = 360 \ s/m^2$$

3. TWO TANK INTERACTING LEVEL PROCESS USING STATE SPACE ANALYSIS

Rearranging the equations (1) and (2), we represent the differential equations in to state space equation. We choose the state variable

$$x_1 = h_1 \text{ and } x_2 = h_2 = y \qquad (4)$$

Hence, state model of two level interacting tanks is derived from equations (1), (2) and (4), the state equation is

$$\begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{1}{R_1A_1} & \frac{1}{R_1A_1} \\ \frac{1}{R_1A_2} & -\frac{1}{R_2A_2}\left(1+\frac{R_2}{R_1}\right) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{A_1} \\ 0 \end{bmatrix} q_{in} \qquad (5)$$

The output equation is

$$y = x_2 = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad (6)$$

4. STATE FEEDBACK GAIN MATRIX *K* DESIGN FOR TWO INTERACTING TANK PROCESS

Consider a control system

$$\frac{dx_1}{dt} = Ax + Bu \qquad (7)$$

where **x** = state vector (n-vector)
u = control signal (scalar)
A = $n \times n$ constant matrix
B = $n \times 1$ constant matrix
We shall choose the control signal to be

$$u = -Kx \qquad (8)$$

This means that the control signal is determined by an instantaneous state. The ($1\times n$) matrix ***K*** is called the state feedback gain matrix. The steps involves with finding state matrix ***K*** are discussed below:
***Step-1*:** Check controllability condition for the given system:

$$A = \begin{bmatrix} -0.11 & 0.11 \\ 0.27 & -0.0388 \end{bmatrix}; \quad B = \begin{bmatrix} 100 \\ 0 \end{bmatrix}$$

and $C = \begin{bmatrix} 0 & 1 \end{bmatrix}$

Here controllability matrix

$$Q_c = \begin{vmatrix} B & AB \end{vmatrix} = \begin{vmatrix} 100 & 11 \\ 0 & 27 \end{vmatrix} = 2700 \neq 0$$

We find that $|Q_c| \neq 0$ and rank of the state controllable test. $|Q_c| \neq 0$ so the system is completely controllable and it is stable.
Step-2*:** By mentioning the desired state feedback gain matrix ***K as $\boldsymbol{K} = \begin{bmatrix} k_1 & k_2 \end{bmatrix}$ and equating $|SI - A + BK|$ with the desired characteristic equation, we obtain

$$|SI - A + BK| = 0$$

$$\Rightarrow \left| \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -0.11 & 0.11 \\ 0.27 & -0.0388 \end{bmatrix} + \begin{bmatrix} 100 \\ 0 \end{bmatrix} \begin{bmatrix} k_1 & k_2 \end{bmatrix} \right| = 0$$

$$\Rightarrow \begin{vmatrix} s+0.11+100k_1 & -0.11+100k_2 \\ -0.27 & s+0.0388 \end{vmatrix} = 0$$

$$\therefore \quad s^2 + (0.49+100k_1)s + (38k_1 + 27k_2 + 0.0715) = 0 \quad (9)$$

***Step-3*:** Suppose the desired location of closed loop poles are at s = -10 and s = -100. Hence, desired characteristics equation is

$$(s+10)(s+1000)=0$$

$$\therefore \quad s^2+110s+1000=0 \qquad (10)$$

***Step-4*:** Comparing equations (9) and (10), we get

$$k_1 = 1.095 \text{ and } k_2 = 35.463$$

The feedback gain matrix is

$$K = \begin{bmatrix}1.0950 & 35.4630\end{bmatrix}$$

Ackerman's formula is used to the state gain matrix ***K*** to write the MATLAB program and find the state feedback gain matrix as

$$K = \begin{bmatrix}1.0950 & 35.4630\end{bmatrix}$$

The simulink diagram of the closed-loop control of two interacting tanks system with $u=-Kx$ is shown Figure 2 and the response of the system with initial condition is shown in Figure 3.

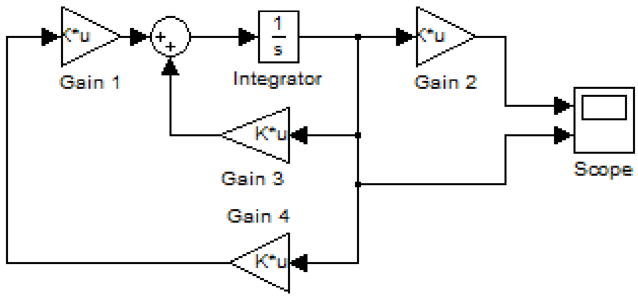

Figure 2 Closed-loop control of two interacting tank system with $u=-Kx$.

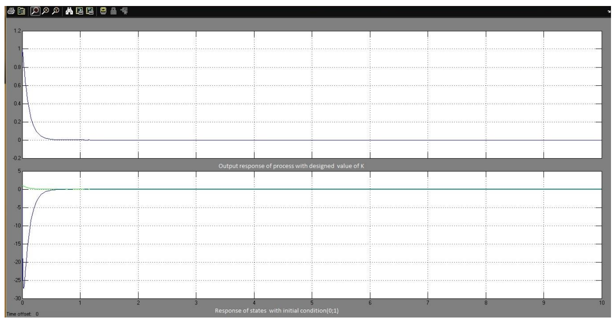

Figure 3 Response curve of closed-loop control system with initial condition [0 1].

5. DESIGN OF FULL ORDER STATE OBSERVER GAIN MATRIX FOR TWO INTERACTING TANK PROCESS

In the pole placement technique to the design of control systems, we assumed that all state variables are available for feedback. In practice, all state variables are not available for feedback measurement [9]. Methods are available to estimate immeasurable state variables without a differentiation process. Estimation of immeasurable state variables is commonly called observation. A device (or a computer program) that estimates or observes the state variables is called a state observer [10], or simply an observer. If the state observer observes all state variables of the system, regardless of whether some state variables are available for direct measurement, it is called a full-order state observer [11, 12].

Assume that the state x is to be approximated by the state x' of the dynamic model of observer to be

$$\dot{x}' = [A-K_eC]x' + Bu + K_e y \qquad (11)$$

where x' is the estimated state and Cx' is the estimated output. The matrix K_e is called observer gain matrix. Hence observer error equation is defined by

$$\dot{x}-\dot{x}' = [A-K_eC](x-x')$$

$$\dot{e} = [A-K_eC]e \qquad (12)$$

where $e = \dot{x} - \dot{x}'$ is error vector and the dynamic behavior of this vector depends upon eigen values of $A - K_e C$. The steps involves with finding state matrix K_e are discussed below:

Step (a): Check observability condition for the given system. The observability matrix

$$Q = [C^T \quad A^T C^T] = \begin{bmatrix} 0 & 0.27 \\ 1 & -0.388 \end{bmatrix}$$

We find that $Q \neq 0$ and therefore, rank of observability matrix $Q = 2$. So the system is completely state observable.

Step (b): By defining the desired state feedback gain matrix K_e as $K_e = \begin{bmatrix} K_{e1} \\ K_{e2} \end{bmatrix}$ and equating $|SI - A + K_e C|$ with the desired characteristic equation, we obtain

$$|SI - A + K_e C| = 0$$

$$\Rightarrow \left| \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -0.11 & 0.11 \\ 0.27 & -0.0388 \end{bmatrix} + \begin{bmatrix} K_{e1} \\ K_{e2} \end{bmatrix} \begin{bmatrix} 0 & 1 \end{bmatrix} \right| = 0$$

$$\Rightarrow \begin{vmatrix} s + 0.11 & -0.11 + K_{e1} \\ -0.27 & s + 0.0388 + K_{e2} \end{vmatrix} = 0$$

$$\therefore \quad s^2 + (0.49 + K_{e2})s + (0.27K_{e1} + 0.11K_{e2} + 0.012) = 0 \qquad (13)$$

Step (c): Design a full-order state observer, assuming that the system configuration is identical to that shown in Figure 4. Assume that the desired Eigen values of the observer matrix are $s = -1 + j2$ and $s = -1 - j2$. The design of the state observer reduces to the determination of an appropriate observer gain matrix $\boldsymbol{K_e}$.

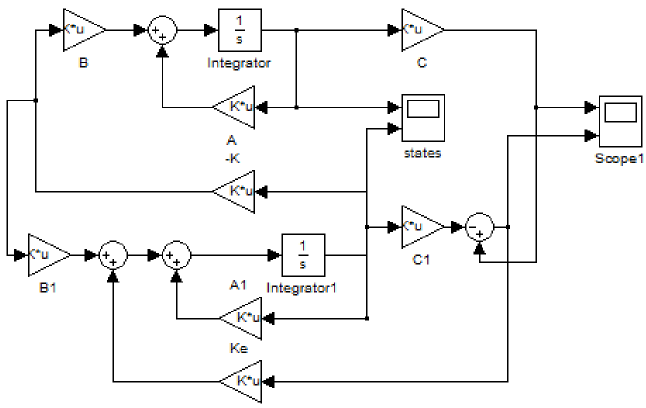

Figure 4 Closed-loop control of two interacting tank system with full order observer.

Hence desired characteristics equation is

$$(s + 1 + j2)(s + 1 - j2) = 0$$

$$\therefore \quad s^2 + 2s + 5 = 0 \qquad (14)$$

Step (d): Comparing equation (13) and (14), we get

$$K_{e1} = 17.8588 \text{ and } K_{e1} = 1.51$$

Ackerman's formula is used to the state gain matrix K_e to write the MATLAB program. The state feedback gain matrix is

$$K_e = \begin{bmatrix} 17.8585 & 1.5100 \end{bmatrix}$$

The simulink diagram of the closed-loop control of two interacting tanks system with full order observer is shown Figure 4 and the response of the system and error with initial condition [0 1] are shown in Figure 5(a) and (b).

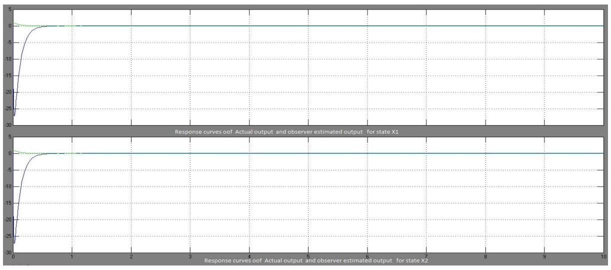

Figure 5(a) Response curve of actual output and estimated output of system with full order observer to initial condition [0 1].

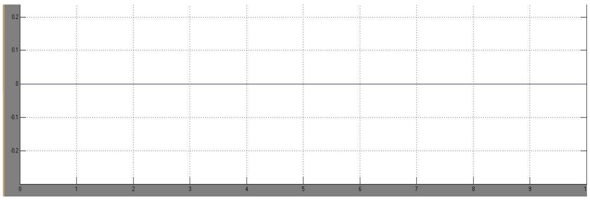

Figure 5(b) Response curve of error of actual output and estimated output of system with full order observer with initial condition [0 1].

6. OBSERVER BASED CONTROLLER DESIGN FOR TWO INTERACTING TANK PROCESS

The transfer function of observer based controller is given by

$$\frac{U(s)}{-Y(s)} = K[SI - A - K_e C + BK]^{-1} K_e$$

This observer based controller represented by the block diagram as shown in Fig.6.

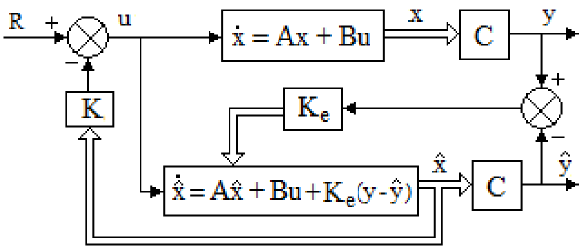

Figure 6 Block diagram of the observer based controller.

The MATLAB program [8] is used to find the transfer function of observer controller for different value of K and K_e. The program is given below:

***Case-A*:** We taking closed loop pole location and observer pole location are $J = [-10 \quad -100]$ and $L = [-1 + j2 \quad -1 - j2]$ which gives the value of $K = [1.0950 \quad 35.4630]$ and $K_e = [17.8585 \quad 1.5100]$

Hence, the Transfer function of observer controller is

```
   73.1 s + 177.7
---------------------------
 s^2 + 111.2 s + 1132
```

This controller transfer function has poles and zeros location all are left hand side of S plane. So this controller is stable controller.

***Case-B*:** We taking closed loop pole location and observer pole location are $J' = [-1 + j2 \quad -1 - j2]$ and $L' = [-40 \quad -100]$ which gives the value of $K = [0.015 \quad 0.163]$ and $K_e = [-1.479 \quad -0.006]$

Transfer function:

```
 - 0.02316 s - 0.07382
-----------------------------
 s^2 + 1.992 s + 4.587
```

This controller Transfer function has pole location $P_{1,2} = -0.9960 + j1.8960; \; -0.9960 - j1.8960$. That means they situated at imaginary axis of s-plane. So this controller is unstable controller. This type control system is not accepted.

6.1Response curve of Observer Based Controller with Two Interacting Tank Process

In this section effect of stable and unstable observer based controller on system will be discussed. After getting the transfer function of observer based controller, transfer function of controller is implemented in simulink block as shown in Figure 7.

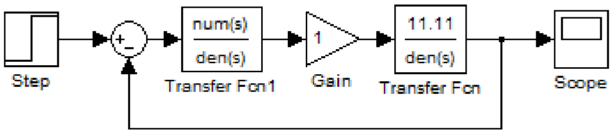

Figure 7 Closed loop control system with observer based controller

By using simulink a closed loop system is designed. Where process is connected with controller and used step as input. Here we just compare the system response with respect to different transfer function of observer based controller as shown in Figure 8(a) and (b).

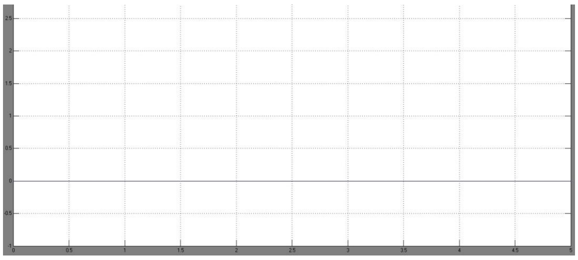

Figure 8(a) Response of Closed loop control system with stable observer based controller

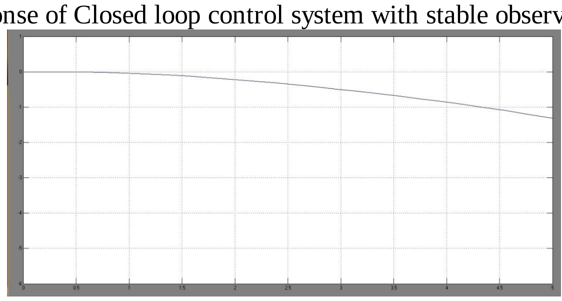

Figure 8(b) Response of Closed loop control system with unstable observer based controller

6.2Comparison of Response curve to Initial condition with Designed *K* and *K*$_e$ values

Case-A: We taking of $K=[1.0950 \quad 35.4630]$ and $K_e=[17.8585 \quad 1.5100]$. The response curve of the system is shown in Figure 9.

Case-B: We taking $K=[0.015 \quad 0.163]$ and $K_e=[-1.479 \quad -0.006]$. The response curve of the system is shown in Figure 10.

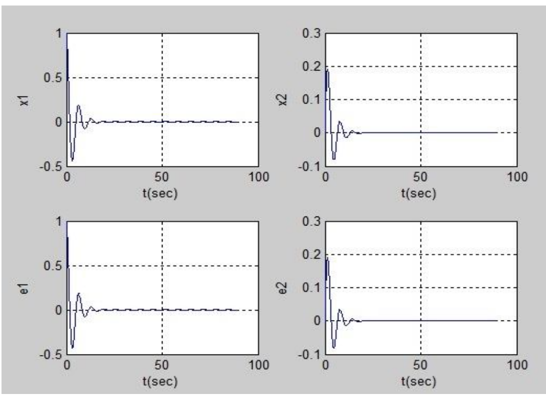

Figure 9 : Response curve for *case-A*.

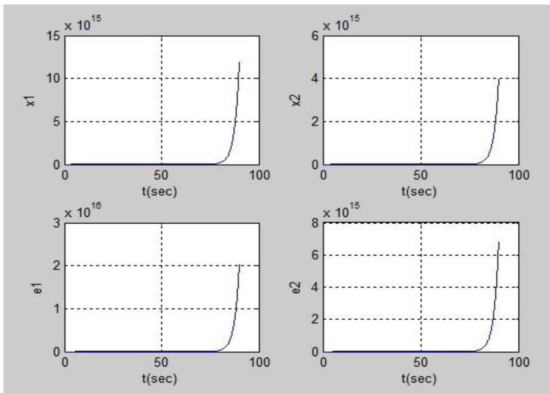

Figure 10 : Response curve for *case-B*.

7. CONCLUSION

State model of interacting two-tank liquid level system has been developed. The state feedback gain matrix (***K***) and the observer gain matrix (K_e) has been determined by direct substation method and verified using MATLAB programming [8]. After designing a observer and getting the response from observer it was that estimated variables [7] takes quite more time to reach at its steady value than actual variables. Comparison study has made on observer based controller has design [10] by using different value of ***K*** and K_e. Two observer based controller has design for interacting two tanks system. Case-A type observer is more acceptable because it is stable in open loop as well as closed loop system.

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AUTHORS

Amitava Biswas received the B.S. degree in Electrical Engineering from Maulana Azad College of Technology (REC, Bhopal) in 1999 and M.S. degree in Electrical Engineering from University of Calcutta in 2002, respectively. He received Ph.D.(Tech) degree from University of Calcutta in 2015. He started his career as a Lecturer (July 2002) in the JIS College of Engineering. Dr. Biswas is presently attached with the Department of Applied Physics, University of Calcutta as Assistant Professor since 2016. Apart from teaching, he is engaged in research in the field of control engineering and application of piecewise constant basis functions in system and control. He is specially working on piecewise linear orthogonal functions and their suitability in analyzing and synthesizing control systems and solving related differential equations. Till now, he has published ten (10) research papers in national and international journals.

Ms. Gargee Chakraborty received the B.S. degree in Instrumentation and Control Engineering from Academy of Technology in 2010 and M.S. degree in the Department of Applied Physics from University of Calcutta in 2012, respectively. During 2012-2016, she worked as an Assistant professor in the department of Instrumentation Engineering of Supreme Knowledge Foundation. Presently she worked as guest lecturer of University of Calcutta and University of Kalyani. She has published two research papers in international conference.