Square Sum Prime Labeling of Some Path Related Graphs

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Abstract

Square sum prime labeling of a graph is the labeling of the vertices with {0,1,2-------,p-1} and the edges with sum of the squares of the labels of the incident vertices. The greatest common incidence number of a vertex (gcin) of degree greater than one is defined as the greatest common divisor of the labels of the incident edges. If the gcin of each vertex of degree greater than one is one, then the graph admits square sum prime labeling. Here we identify some path related graphs for square sum prime labeling.

Keywords: Graph labeling, square sum, greatest common incidence number, prime labeling

1. Introduction

All graphs in this paper are simple, finite, connected and undirected. The symbol V(G) and E(G) denotes the vertex set and edge set of a graph G. The graph whose cardinality of the vertex set is called the order of G, denoted by p and the cardinality of the edge set is called the size of the graph G, denoted by q. A graph with p vertices and q edges is called a (p,q)- graph.

A graph labeling is an assignment of integers to the vertices or edges. Some basic notations and definitions are taken from [2],[3] and [4] . Some basic concepts are taken from [1] and [2]. The square sum labeling was defined by V Ajitha, S Arumugan and K A Germina in [5]. In this paper we introduced square sum prime labeling using the concept greatest common incidence number of a vertex. We proved that some path related graphs admit square sum prime labeling.

Definition: 1.1 Let G be a graph with p vertices and q edges. The greatest common incidence number (***gcin***) of a vertex of degree greater than or equal to 2, is the greatest common divisor (gcd) of the labels of the incident edges.

2. MAIN RESULTS

Definition 2.1 Let G = (V, E) be a graph with p vertices and q edges. Define a bijection
f : V(G) → {0,1,2,3,----------------,p-1} by $f(v_i) = i-1$, for every i from 1 to p and define a 1-1 mapping f_{sqsp}^{*} : E(G) → set of natural numbers N by $f_{sqsp}^{*}(uv) = \{f(u)\}^2 + \{f(v)\}^2$. The induced function f_{sqsp}^{*} is said to be a square sum prime labeling, if for each vertex of degree at least 2, the ***gcin*** is 1.

Definition 2.2 A graph which admits square sum prime labeling is called a square sum prime graph.

Theorem 2.1 Path graph P_n admits square sum prime labeling.
Proof: Let G = P_n and let v_1,v_2,----------------,v_n are the vertices of G
Here |V(G)| = n and |E(G)| = n-1
Define a function f : V → {0,1,2,3,----------------,n-1 } by
$f(v_i) = i-1$, i = 1,2,------,n
Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling f_{sqsp}^{*} is defined as follows
$f_{sqsp}^{*}(v_i\, v_{i+1}) = 2i^2 - 2i + 1$,
i = 1,2,------------,n-1

Clearly f^*_{sqsp} is an injection.

gcin of (v_{i+1}) $= \gcd$ of $\{f^*_{sqsp}(v_i\ v_{i+1}), f^*_{sqsp}(v_{i+1}\ v_{i+2})\}$
$= \gcd$ of $\{2i^2+2i+1, 2i^2-2i+1\}$
$= \gcd$ of $\{4i, 2i^2-2i+1\}$,
$= \gcd$ of $\{i, 2i^2-2i+1\} = 1$, $i = 1,2,\text{------------},n-2$

So, ***gcin*** of each vertex of degree greater than one is 1.
Hence P_n , admits square sum prime labeling.

Theorem 2.2 $(P_n)^2$ admits square sum prime labeling, when n is not a multiple of 5.
Proof: Let G = $(P_n)^2$ and let $v_1,v_2,\text{----------------},v_n$ are the vertices of G
Here $|V(G)| = n$ and $|E(G)| = 2n-3$
Define a function $f : V \rightarrow \{0,1,2,3,\text{----------------},n-1\}$ by
$f(v_i) = i-1$, $i = 1,2,\text{------},n$
Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling f^*_{sqsp} is defined as follows

$f^*_{sqsp}(v_i\ v_{i+1}) = 2i^2 - 2i + 1$, $i = 1,2,\text{------------},n-1$
$f^*_{sqsp}(v_i\ v_{i+2}) = 2i^2+2$ $i = 1,2,\text{------------},n-2$

Clearly f^*_{sqsp} is an injection.

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For the vertex labeling f, the induced edge labeling f^*_{sqsp} is defined as follows
$f^*_{sqsp}(v_i\ v_{i+1}) = 2i^2 - 2i + 1$, $i = 1,2,\text{------------},n-1$
$f^*_{sqsp}(v_i\ v_{i+2}) = 2i^2+2$ $i = 1,2,\text{------------},n-2$
Clearly f^*_{sqsp} is an injection.

gcin of (v_{i+1}) $= 1$, $i = 1,2,\text{------------},n-2$
gcin of (v_1) $= \gcd$ of $\{f^*_{sqsp}(v_1\ v_2), f^*_{sqsp}(v_1\ v_3)\}$
$= \gcd$ of $\{1, 4\} = 1$.
gcin of (v_n) $= \gcd$ of $\{f^*_{sqsp}(v_n\ v_{n-1}), f^*_{sqsp}(v_n\ v_{n-2})\}$
$= \gcd$ of $\{2n^2-6n+5, 2n^2-8n+10\}$
$= \gcd$ of $\{2n-5, 2n^2-8n+10\} = 1$
$= \gcd$ of $\{2n-5, n\} =$ g c d of $\{n-5, n\}$
$= \gcd$ of $\{n-5, 5\} = 1$.

So, ***gcin*** of each vertex of degree greater than one is 1.
Hence $(P_n)^2$, admits square sum prime labeling.

Theorem 2.3 Middle graph of path P_n admits square sum prime labeling.
Proof: Let G = $M(P_n)$ and let $v_1,v_2,\text{----------------},v_{2n-1}$ are the vertices of G
Here $|V(G)| = 2n-1$ and $|E(G)| = 3n-4$
Define a function $f : V \rightarrow \{0,1,2,3,\text{----------------},2n-2\}$ by
$f(v_i) = i-1$, $i = 1,2,\text{------},2n-1$
Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling f^*_{sqsp} is defined as follows

$f^*_{sqsp}(v_i\ v_{i+1}) = 2i^2-2i+1$, $i = 1,2,\text{------------},2n-2$
$f^*_{sqsp}(v_{2i}\ v_{2i+2}) = 8i^2+2$, $i = 1,2,\text{------------},n-2$

Clearly f^*_{sqsp} is an injection.

gcin of (v_{i+1}) $= 1$, $i = 1,2,\text{------------},2n-3$

So, ***gcin*** of each vertex of degree greater than one is 1.
Hence $M(P_n)$, admits square sum prime labeling.

Theorem 2.4 Total graph of path P_n admits square sum prime labeling, when n+2 is not a multiple of 5.
Proof: Let G = $T(P_n)$ and let $v_1, v_2, \ldots, v_{2n-1}$ are the vertices of G
Here |V(G)| = 2n-1 and |E(G)| = 4n-5.
Define a function $f : V \rightarrow \{0,1,2,3,\ldots,2n-2\}$ by
$f(v_i) = i-1$, i = 1,2,------,2n-1
Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling f^*_{sqsp} is defined as follows

$f^*_{sqsp}(v_i\, v_{i+1}) = 2i^2-2i+1$, i = 1,2,-----------,2n-2
$f^*_{sqsp}(v_{2i}\, v_{2i+2}) = 8i^2+2$, i = 1,2,-----------,n-2
$f^*_{sqsp}(v_{2i-1}\, v_{2i+1}) = 8i^2-8i+4$, i = 1,2,-----------,n-1
Clearly f^*_{sqsp} is an injection.
gcin of (v_{i+1}) = 1, i = 1,2,-----------,2n-3
gcin of (v_1) = gcd of $\{f^*_{sqsp}(v_1\, v_2), f^*_{sqsp}(v_1\, v_3)\}$
= gcd of {1, 4}
gcin of (v_{2n-1}) = gcd of $\{f^*_{sqsp}(v_{2n-3}\, v_{2n-1}), f^*_{sqsp}(v_{2n-1}\, v_{2n-2})\}$
= gcd of $\{8n^2-24n+20, 8n^2-20n+13\}$,
= gcd of $\{4n-7, 8n^2-24n+20\}$
= gcd of {4n-7, 2n-1}= g c d of {2n-6, 2n-1},
= gcd of {2n-6, 5}= 1.
So, *gcin* of each vertex of degree greater than one is 1.
Hence $T(P_n)$, admits square sum prime labeling.

Theorem 2.5 Duplicate graph of path P_n admits sum square prime labeling.
Proof: Let G = $D(P_n)$ and let $v_1, v_2, \ldots, v_{2n}$ are the vertices of G
Here |V(G)| = 2n and |E(G)| = 2n-2
Define a function $f : V \rightarrow \{0,1,2,3,\ldots,2n-1\}$ by
$f(v_i) = i-1$, i = 1,2,------, 2n
Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling f^*_{sqsp} is defined as follows

$f^*_{sqsp}(v_i\, v_{i+1}) = 2i^2-2i+1$, i = 1,2,-----------,n-1
$f^*_{sqsp}(v_{n+i}\, v_{n+i+1}) = (n+i)^2 + (n+i-1)^2$, i = 1,2,-----------,n-1
Clearly f^*_{ssp} is an injection.
gcin of (v_{i+1}) = 1, i = 1,2,----------- ,n-2
gcin of (v_{n+i+1}) = 1, i = 1,2,-----------,n-2
So, *gcin* of each vertex of degree greater than one is 1.
Hence $D(P_n)$, admits square sum prime labeling.

Theorem 2.6 Strong Shadow graph of path P_n admits square sum prime labeling.
Proof: Let G = $S\{D_2(P_n)$ and let $v_1, v_2, \ldots, v_{2n}$ are the vertices of G
Here |V(G)| = 2n and |E(G)| = 5n-4.
Define a function $f : V \rightarrow \{0,1,2,3,\ldots,2n-1\}$ by
$f(v_i) = i-1$, i = 1,2,------,2n.
Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling f^*_{sqsp} is defined as follows

$f^*_{sqsp}(v_i\, v_{i+1}) = 2i^2-2i+1$, i = 1,2,-----------,2n-1.
$f^*_{sqsp}(v_{2i}\, v_{2i+2}) = 8i^2+2$, i = 1,2,-----------,n-1.
$f^*_{sqsp}(v_{2i-1}\, v_{2i+1}) = 8i^2-8i+4$, i = 1,2,-----------,n-1
$f^*_{sqsp}(v_{2i-1}\, v_{2i+2}) = 8i^2-4i+5$, i = 1,2,-----------,n-1
Clearly f^*_{sqsp} is an injection.
gcin of (v_{i+1}) = 1, i = 1,2,-----------,2n-2
gcin of (v_1) = gcd of $\{f^*_{sqsp}(v_1\, v_2), f^*_{sqsp}(v_1\, v_3)\}$
= gcd of {1, 4}= 1
gcin of (v_{2n}) = gcd of $\{f^*_{sqsp}(v_{2n}\, v_{2n-1}), f^*_{sqsp}(v_{2n}\, v_{2n-2}), f^*_{sqsp}(v_{2n}\, v_{2n-3})\}$
= gcd of $\{8n^2-12n+5, 8n^2-16n+10, 8n^2-20n+17\}$
So, *gcin* of each vertex of degree greater than one is 1.
Hence $T(P_n)$, admits square sum prime labeling.

Theorem 2.7 Z graph of path P_n admits square sum prime labeling, when n is even.

Proof: Let G = $Z(P_n)$ and let v_1, v_2,----------------,v_{2n} are the vertices of G

Here |V(G)| = 2n and |E(G)| = 3n-3.

Define a function f : V → {0,1,2,3,----------------,2n-1 } by

$f(v_i) = i-1$, i = 1,2,------,2n

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{sqsp} is defined as follows

$f^*_{sqsp}(v_i\ v_{i+1})$ = $2i^2-2i+1$, i = 1,2,------------,2n-1.

$f^*_{sqsp}(v_{4i-2}\ v_{4i+1})$ = $32i^2-24i+9$, i = 1,2,------------,$\frac{n-2}{2}$.

$f^*_{sqsp}(v_{4i}\ v_{4i+3})$ = $32i^2+8i+5$, i = 1,2,------------,$\frac{n-2}{2}$.

Clearly f^*_{sqsp} is an injection.

gcin of (v_{i+1}) = 1,

i = 1,2,------------,2n-2

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence $Z(P_n)$, admits square sum prime labeling.

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