

# Effectiveness and Efficiency Analysis of Parallel Flow and Counter Flow Heat Exchangers

Roopesh Tiwari<sup>1</sup>, Dr.Govind Maheshwari<sup>2</sup>

<sup>1</sup>Assistant Professor, Department of Mechanical Engineering, Sagar Institute of Research and Technology, Indore-452020  
INDIA

<sup>2</sup>Associate Professor, Department of Mechanical Engineering, Institute of Engineering & Technology, DAVV, Indore- 452017  
INDIA

## Abstract

*This paper provides a theoretical analysis of a heat exchanger to determine the maximum effectiveness of a heat exchanger corresponding to the heat capacity rate ratio. Moreover, thermal efficiency for heat exchanger based on the second law of thermodynamics has been incorporated in the design. Also the efficiency based on fin analogy number which resembles the efficiency of a fin of constant area with tip insulated has been utilized.*

**Key words:** Heat exchanger, Effectiveness, Thermal efficiency, heat capacity rate.

## 1. INTRODUCTION

Heat exchangers have wide applications in modern industries. Heat exchangers are devices that facilitate the exchange of heat between two fluids with high effectiveness and low investment and low maintenance cost. Optimizing the performance of the heat exchanger results in efficient utilization of energy with reduction in total volume and the weight of the heat exchanger. Thus, heat exchanger has always been one of the main research topics in thermodynamics. Several methods and theories have been developed to optimize the heat exchanger performance. Generally the performance of a heat exchanger is measured by its effectiveness, but effectiveness does not provide any information about the efficiency of heat exchange that takes place in a heat exchanger [1], and the measure of irreversibility is entropy generation [2-3]. In order to take into account the irreversibility, the concept of heat exchanger efficiency based on the second law of thermodynamics has been introduced [4-8] and the relation between effectiveness and efficiency has been derived. The main objective of this work is to determine the heat capacity rate ratio accounting for both efficiency and effectiveness of the heat exchanger.

## 2. METHODOLOGY

Considering a heat exchanger with negligible fluid flow pressure drop, the heat exchanger effectiveness in the  $\varepsilon - NTU$  approach is defined as the ratio of actual amount of heat transfer rate to the maximum possible heat transfer rate,

$$\varepsilon = \frac{Q_{actual}}{Q_{max}} = \frac{C_h (T_{h_i} - T_{h_o})}{C_{min} (T_{h_i} - T_{c_i})} = \frac{C_c (T_{c_o} - T_{c_i})}{C_{min} (T_{h_i} - T_{c_i})} \quad (1)$$

Where  $T_{h_i}$  is the hot fluid inlet temperature,  $T_{h_o}$  is the hot fluid outlet temperature,  $T_{c_i}$  and  $T_{c_o}$  are cold fluid inlet and outlet temperatures respectively.  $C_h$  and  $C_c$  are the heat capacity rates of hot and cold fluid respectively  $C_{min}$  is the minimum heat capacity rate. The  $\varepsilon$ -NTU method is generally used when the inlet temperatures and the size of the heat exchanger are known and fluid outlet temperatures and the heat transfer rate are required to be found out.

Now the heat exchanger efficiency is defined as the ratio of the actual heat transfer rate to the optimum heat transfer rate,

$$\eta = \frac{Q_{actual}}{Q_{opt}} = \frac{C_h (T_{h_i} - T_{h_o})}{UA (\bar{T}_h - \bar{T}_c)} = \frac{C_c (T_{c_o} - T_{c_i})}{UA (\bar{T}_h - \bar{T}_c)} \quad (2)$$

The optimum heat transfer is the product of overall heat transfer coefficient (U), heat exchanger surface area (A) and arithmetic mean temperature difference (AMTD). AMTD is the difference between the average temperatures of hot and cold fluids.

Heat exchanger efficiency [4] has also been defined on the basis of efficiency of a fin of a constant area with its tip insulated, as

$$\eta = \frac{\tanh(F_a)}{F_a} \quad (3)$$

Where  $F_a$  is the heat exchanger number and is expressed as,

$$F_a = \frac{NTU(1-R)}{2} \text{ for counter flow heat exchanger and for parallel flow } F_a = \frac{NTU(1+R)}{2}$$

where NTU is the number of transfer units.

R is the ratio of minimum heat capacity rate ( $C_{\min}$ ) and maximum ( $C_{\max}$ ) heat capacity rates. From equations (2) and (3) the equation for heat transfer can be written as,

$$Q_{actual} = UA(\bar{T}_h - \bar{T}_c) \frac{\tanh(F_a)}{F_a}$$

On substituting the value of  $F_a$  for counter flow and parallel flow arrangement the equations for heat transfer rate for counter flow and parallel flow heat exchanger are obtained respectively as,

$$Q_{actual} = \frac{2C_{\min}}{1-R} \tanh\left[\frac{UA(1-R)}{2C_{\min}}\right](\bar{T}_h - \bar{T}_c) \quad (4)$$

and

$$Q_{actual} = \frac{2C_{\min}}{1+R} \tanh\left[\frac{UA(1+R)}{2C_{\min}}\right](\bar{T}_h - \bar{T}_c) \quad (5)$$

Now finding the relation between

$$(T_{h_i} - T_{c_i}) \text{ and } (\bar{T}_h - \bar{T}_c),$$

We know that,

$$(\bar{T}_h - \bar{T}_c) = \frac{T_{h_i} + T_{h_o}}{2} - \frac{T_{c_i} + T_{c_o}}{2}$$

Where

$$T_{h_o} = T_{h_i} - \varepsilon \frac{C_{\min}}{C_h} (T_{h_i} - T_{c_i})$$

and

$$T_{c_o} = T_{c_i} + \varepsilon \frac{C_{\min}}{C_c} (T_{h_i} - T_{c_i})$$

On substituting the values of  $T_{h_o}$  and  $T_{c_o}$  we get,

$$\bar{T}_h - \bar{T}_c = \frac{1}{2} (T_{h_i} - T_{c_i}) [2 - \varepsilon(1+R)] \quad (6)$$

Writing the equation (6) in terms of  $\eta$ ,

$$\bar{T}_h - \bar{T}_c = \frac{T_{h_i} - T_{c_i}}{\left[1 + \frac{1}{2}\eta.NTU(1 + R)\right]} \tag{7}$$

On combining equation (2) and equation (7) we get,

$$Q_{actual} = \frac{(T_{h_i} - T_{c_i})C_{min}}{\frac{1}{\eta.NTU} + \frac{1+R}{2}} \tag{8}$$

Now the relationship between  $\varepsilon$  and  $\eta$  can be obtained using equation (1) and (2) as,

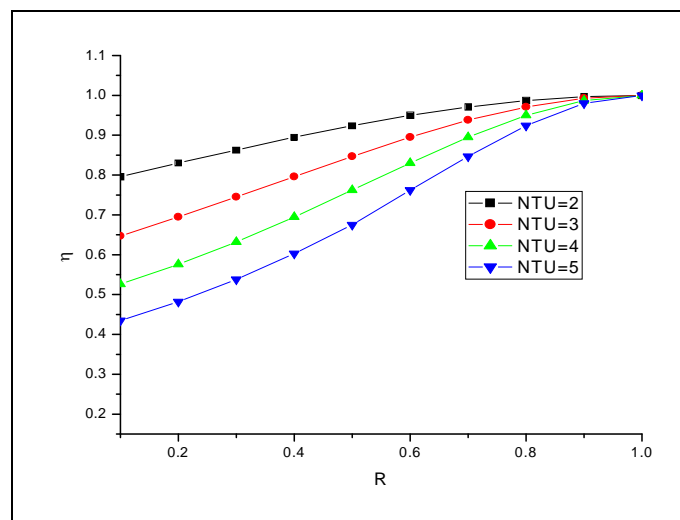
$$\varepsilon C_{min}(T_{h_i} - T_{c_i}) = \eta UA(\bar{T}_h - \bar{T}_c) \tag{9}$$

Combining equations (7) and (9) we get,

$$\varepsilon = \left[ \frac{1}{\frac{1}{\eta.NTU} + \frac{(1+R)}{2}} \right] \tag{10}$$

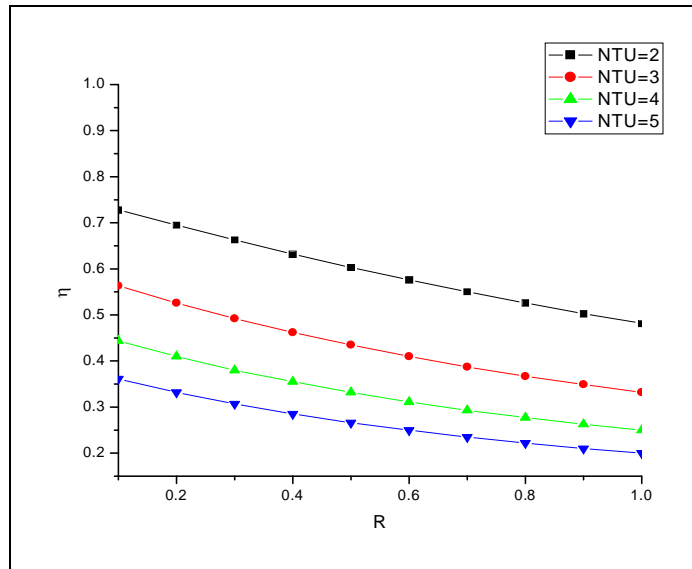
### 3. RESULTS

Using the equation 3, the variation of efficiency is plotted for counter flow arrangement for different values of heat capacity ratio with simultaneously varying the values of number of transfer units. The result is as shown in figure 1.



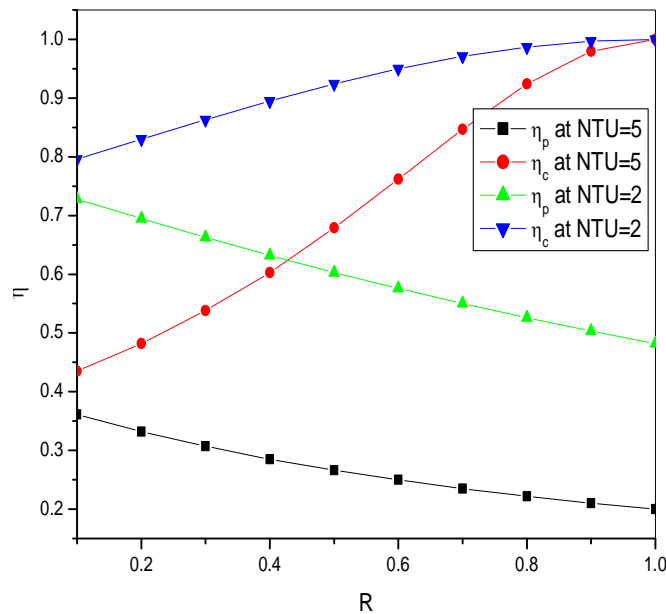
**Figure 1** Variation of  $\eta$  with R for different values of NTU for counter flow arrangement.

Similarly variation of efficiency is plotted for parallel flow arrangement for different values of heat capacity ratio with simultaneously varying the values of number of transfer units. The result is as shown in figure 2.



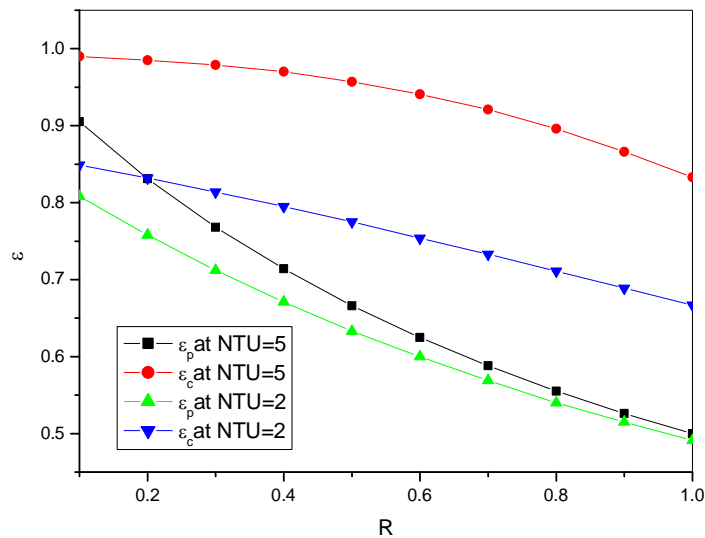
**Figure 2** Variation of  $\eta$  with R for different values of NTU for parallel flow arrangement.

Also a comparison is made of variation of efficiency for counter flow arrangement and parallel flow arrangement for different values of heat capacity ratio as depicted in figure 3.



**Figure 3** Comparison of variation of  $\eta$  with R for different values of NTU, for parallel flow and counter flow arrangement.

By applying the relation between effectiveness and efficiency as given by equation 10, and using the values of efficiency as calculated above, the variation of effectiveness is plotted for counter flow arrangement as well as parallel flow arrangement. The values of NTU along with heat capacity ratio are also varied. The result plotted is as shown in figure 4.



**Figure 4** Variation of  $\epsilon$  with R for different values of NTU , for parallel flow and counter flow arrangement.

#### 4. DISCUSSION

Figure 1 shows the variation of efficiency,  $\eta$  for different values of NTU for counter flow arrangement. From figure 1 it can be seen that  $\eta$  of counter flow arrangement increases with increase in values of heat capacity ratio, R. Figure 1 also shows that for counter flow arrangement efficiency,  $\eta$  is higher for lower values of NTU.

Figure 2 shows the variation of efficiency,  $\eta$  for different values of NTU for parallel flow arrangement. It shows that  $\eta$  decreases with increase in R for parallel flow arrangement and reaches minimum at R=1 for different values of NTU. The decrease in efficiency is maximum for lower value of NTU.

Figure 3 shows the variation of efficiency  $\eta$ , for different values of NTU for parallel flow and counter flow arrangements. From figure 3 it can be seen that  $\eta$  of counter flow arrangement increases and that of parallel flow arrangement decreases with increase in values of R. Figure 3 also shows that  $\eta$  is higher for lower values of NTU for both parallel flow and counter flow arrangements.

Figure 4 shows that  $\epsilon$  decreases with increase in R for both parallel flow and counter flow arrangement and reaches minimum at R=1. Figure 4 also shows that the decrease in  $\epsilon$  is greater for parallel arrangement as R increases and it can also be seen that the  $\epsilon$  is higher for higher values of NTU for both parallel and counter flow as R increases.

#### 5. CONCLUSIONS

1. In order to have the optimum performance of the heat exchanger in terms of effectiveness and efficiency, for parallel flow arrangement, the heat exchanger must operate at lower values of heat capacity ratio R.
2. The heat capacity ratio around R=0.5 gives the optimum performance of counter flow arrangement based on effectiveness and efficiency.

#### FUTURE SCOPE

- The concept of entropy generation analysis can also be applied in the above analysis.
- The above work has been carried out without considering the pressure drop and the heat loss, the same analysis can be done by taking into account the pressure drop and the heat loss.
- The above analysis has been done for counter flow and parallel flow arrangements, the same analysis can be performed for the cross flow arrangement.

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**AUTHOR**



Mr.Roopesh Tiwari presently associated with Sagar Institute of Research and Technology, SAGE University, Indore ,as assistant professor in Mechanical Engineering Department having 12 years of teaching experience. Completed BE from Oriental Institute of Science & Technology, Bhopal and ME from Institute of Engineering & Technology, DAVV, Indore .Pursuing PhD under supervision of Dr.Govind Maheshwari, Associate Professor,IET,DAVV,Indore