Peridynamic analysis of residual stress around a cold expanded fastener hole

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ABSTRACT

Cold expansion is the most effective technique to improve the strength and fatigue life of engineering components with fastener holes. Cold expansion creates a compressive residual stress field around the fastener hole, consequently reduces the tensile stress field under remotely applied loads and delays the fatigue crack initiations and propagation. The residual stress field plays an essential role in fatigue life of such specimens. Therefore, it is of great importance to have an accurate estimation on residual stress magnitude and distribution. This paper employs the non-ordinary state-based peridynamics to predict the residual stresses distribution in a cold expanded fastener hole. The obtained results are validated through comparison with available experimental and finite element results. The presented modeling method provides a promising technique in modeling cold expansion and effectively can be applied to predict the fracture behavior of cold expanded fastener holes under static and cyclic loading.

Keywords: Cold expansion; Fastener hole; Peridynamics; Residual stress.

1. INTRODUCTION

Fastener holes in aerospace structures are the stress concentrated areas. Cracks often initiates around the faster holes under static and cyclic loading. The cold expansion is the most effective technique in the aerospace industry to improve the static strength and fatigue life of components with fastener holes [1-4], though other techniques are available [5, 6]. The primary concept of the cold expansion method is to insert an oversized mandrel inside the faster hole and then to remove the mandrel. Inserting the mandrel creates a plastic region around the hole surrounded by an elastic region. By removing the mandrel, the elastic region springs back and produces a compressive tangential residual stress around the hole. The produced residual stress plays an essential role in improving strength and fatigue life of the components. Under tensile loading, either static or cyclic, the compressive residual stress considerably reduces the tensile stress field. Obviously, fracture behavior of the cold expanded fastener holes are highly depends of the magnitude and distribution of residual stress. Therefore, it is of great importance to accurately characterize the cold expansion process. There are some destructive and non-destructive experimental techniques to measure the residual stress [7-10]. However, they are costly and also have limitations to predict residual variation of residual stresses through the thickness of the plate. Numerical modeling is an alternative to experimental studies to predict residual stress field in cold expanded fastener holes.

Finite element methods (FEM) are of great importance in numerical modeling complicated systems and structures [11-13]. 2D and 3D finite element analysis (FEA) are widely used to model cold expansion process in the literature. Chakherlou and Vogwell [1] carried out an analytical and experimental study to assess the effectiveness of the cold expansion technique on the fatigue strength of fastener holes. Chakherlou and Yaghoobi [4] developed a 2D finite element simulation to determine the initial tangential and radial residual stress distributions generated by cold expansion and their relaxation under cyclic loading. Yaghoobi [14] provided 2D and 3D finite element method (FEM) simulations on effectiveness of kinematic hardening models in predicting residual stress relation around a cold expanded fastener hole under cyclic loading. Chakherlou et al. [3] used a 3D FEM study to investigate study the role of bolt clamping force on the residual stress distribution, fatigue life and failure mode of bolted cold expanded specimens.

Although FEM can effectively predict the initial distribution of the residual stress, they are incapable of modeling damage evaluation in cold expanded components. This problem goes back to limitations of FEM in modeling discontinuities. Classical FEM solves the system of classical mechanics’ equations of motion. In classical mechanics, the equations of motion take differential forms. Therefore, they are invalid in discontinuities such as crack tips since differential terms produce singularities in such points. More advanced methods based on FEM such as cohesive element zone method (CZM) and extended finite element method (XFEM) are introduced in order to address the limitation of classical FEM in fracture analysis. However, they are computationally more demanding and mesh dependent. Furthermore, extra criteria are required for damage nucleation, growth and coalescence, and for obtaining crack growth direction. Khalili et al. [15-17] introduced WSFE-based UEL to model the wave propagation in structures.
Peridynamics is an alternative continuum mechanics formulation which reformulates the equation of motion by substituting differential terms with integral terms [21,22]. Therefore, the peridynamic equations are valid in discontinuities and consequently is a perfect technique in predicting material failure. Furthermore, the damage modeling is an inherent feature of the peridynamics and there is no need for extra criteria for crack initiations and propagation. Ha and Bobaru [23] studied dynamic crack branching in brittle materials using peridynamics. Yaghoobi and Chorzepa [24] proposed a meshless analysis framework based on peridynamics to predict the response of fiber reinforced concrete (FRC) structures. Chorzepa and Yaghoobi [25] used peridynamics to investigate fracture in FRC beams under dynamic loading. A mesoscale modeling of cementitious composites are conducted by Yaghoobi et al. [26] to investigate damage mechanics using peridynamics. The original peridynamics, so called “bond-based peridynamics” (BBPD), is limited to materials with Poisson’s ratio of 0.25 [22,27]. The most developed form of the peridynamics, non-ordinary state-based peridynamics (NOSBPD), addressed this issue and can cover materials with varying Poisson’s ratio [28,29]. Furthermore, NOSBPD is able to utilize the constitutive equations from the classical continuum mechanics. Therefore, it is a promising technique in fracture behavior of engineering structures.

In the present investigation, NOSBPD modeling is employed to predict the residual stress distribution in a cold expanded fastener hole. In doing so, a 2D plate model with a central hole is provided. Since during the cold expansion, only the material around the hole shows plastic deformation, whereas to increase computational efficiency, surrounding areas are discretized using conventional elements. The FEM-NOSBPD coupling technique introduced by Yaghoobi and Chorzepa [30] is employed to couple two subsystems. The cold expansion process is modeled by applying displacement loading on peridynamics particles around the hole and then removing the displacement condition. The nonlinear material model with von-Mises yield criterion is adopted within the peridynamic formulations in order to obtain plastic deformation due to cold expansion. The presented modeling method provides a promising technique in modeling cold expansion and effectively can be applied to predict the fracture behavior of cold expanded fastener holes under static and cyclic loading.

2.MATERIALS AND THE METHOD

2.1 Non-ordinary state-based peridynamics

Peridynamics is a non-local method first introduced by Silling [21]. In the discretized form, the peridynamic body is represented by a set of material points. In peridynamics, material continuity is assured by interaction between material points. To ensure the computational efficiency, the interaction between material points are limited to a finite distance. In other word, a point located at position $\mathbf{x}$, hereinafter shortly called point $\mathbf{x}$, interacts with its surrounding points within a finite distance of $\mathbf{d}$. The area of influence is called ‘horizon’.

In the original peridynamics (BBPD), the interaction is a central pairwise force which limits BBPD to model material with fixed Poisson’s ratio of 0.25. In NOSBPD, the interaction takes a more general form as seen in Fig. 1. Therefore, NOSBPD is able to cover material models with any thermodynamically accepted values of Poisson’s ratio. The steady-state equilibrium equations for point $\mathbf{x}$ is given as:

$$\int_{H_\mathbf{x}} \{\mathbf{T}[\mathbf{x}](\mathbf{x'}-\mathbf{x}) - \mathbf{T}[\mathbf{x'}](\mathbf{x}-\mathbf{x'})\} dV_{\mathbf{x'}} + \mathbf{b}(\mathbf{x}) = \mathbf{0}, \quad (1)$$

where $\mathbf{T}$ is the force vector-state, $\mathbf{b}$ is the body force applied on the point $\mathbf{x}$ and $dV_{\mathbf{x'}}$ is the volume of point $\mathbf{x'}$. The force vector-state, $\mathbf{T}$, is obtained as a function of stress, $\sigma$. Please refer to Breitenfeld et al. [31] and Yaghoobi and Chorzepa [24] for detailed description on the force vector-state and its formulation.
Furthermore, NOSBPD formulations suffers from an instability problem so called “zero-energy mode”. Breitenfeld et al. [31] described the zero energy mode and introduced treatments. An alternative solution is introduced by Yaghoobi and Chorzepa [32] using a higher-order terms of error in peridynamic deformation gradient tensor. This paper uses the higher-order formulation in order to suppress the zero-energy mode.

2.2 Constitutive model

A nonlinear elastic-plastic material model is adopted within the peridynamic formulations. The infinitesimal strain tensor for an isotropic material is determined by $\epsilon = 1/2 (F + F^T) - I$, $F$ is the deformation gradient tensor and $I$ is the identity matrix. For an elastic-plastic material an additive decomposition can be assumed to decompose the strain into elastic and plastic parts as: $\epsilon = \epsilon^e + \epsilon^p$. Therefore, the stress $\sigma$, can be described as:

$$\sigma = D^{ep} : \epsilon = D : \epsilon^e = D : (\epsilon - \epsilon^p),$$  (2)

where the superscripts $\epsilon$ and $\varphi$ stand for elastic and plastic, respectively. $D$ is the elastic constitutive matrix and $D^{ep}$ is the elasto-plastic constitutive matrix. In von-Mises plasticity model, a scalar yield function is defined to obtain the elastic part limit as:

$$f(\sigma) = \sigma_{eq} (\sigma) - \sigma_y (\epsilon^p),$$  (3)

where $\sigma_y$ is the uniaxial yield stress and to consider hardening varies as a function of plastic strain. $\sigma_{eq}$ is the equivalent stress where in the von-Mises formulation takes the following form:

$$\sigma_{eq} = \sqrt{\frac{3}{2} S : S},$$  (4)

where $S$ is the deviatoric stress tensor. The boundary $f = 0$ defines the elastic limit so that $f < 0$ represents the elastic region, whereas, $f > 0$ yields to plastic deformation. In the plastic region, the plastic strain is obtained by summation of plastic strain increments:

$$\epsilon^p = \int d\epsilon^p,$$  (5)

where the plastic strain increment can be derived from the so-called associated flow rule:

$$d\epsilon^p = d\lambda \frac{\partial f}{\partial \sigma},$$  (6)

where $d\lambda$ is a positive scalar and determines the magnitude of the increment. Since $f(\sigma) = 0$, then $df = 0$, the $d\lambda$ is obtained as:

$$d\lambda = \left| \frac{\partial f}{\partial \sigma} \right|^{-1} D : \frac{\partial f}{\partial \sigma}.$$

(7)

Assuming isotropic hardening, as increase in the yield stress is assumed during plastic loading:

$$\sigma_y = \sigma_{y0} + H \epsilon_{eq}^p,$$  (8)

where $\epsilon_{eq}^p$ is a scalar and represents the equivalent plastic.

$$\epsilon_{eq}^p = \int d\epsilon_{eq}^p, \quad d\epsilon_{eq}^p = \frac{1}{\sqrt{3}} d\epsilon^p : d\epsilon^p,$$  (9)
and \( H = \frac{E_t}{(1 - E_t/E)} \), where \( E \) is the elastic modulus and \( E_t \) is the tangent modulus of plastic region in the bilinear plastic model shown in Fig. 2.

\[ \text{Figure 2} \quad \text{Uniaxial tension test with hardening [33].} \]

2.3 Specimen and numerical discretization

Many experimental studies are dedicated on cold expansion process [1-3,7-10]. In this research, the experimental research conducted by Ozdemir and Edwards [8] is used to validate the numerical results. In the experiments, the tangential (hoop) residual stress distributions created with cold expansion is experimentally measured. The destructive modified Sachs method is used to measure residual stress field. The specimen and its dimensions are shown in Fig. 3. The specimen’s material was 7050-T76 aluminum alloy with mechanical properties of elastic modulus \( E = 73.2 \text{ MPa} \), Poisson’s ratio \( v = 0.33 \), yield stress \( \sigma_y = 543 \text{ MPa} \), tensile strength \( S_u = 589 \text{ MPa} \) and elongation to failure of 16%. The initial diameter of hole was 8.69 mm that after 4% cold expansion using split sleeve Fatigue Technology Inc. (FTI) method became 8.95 mm. The numerical studies for the same experimental setup is also conducted by Chakerloiu and Yaghoobi [4]. They developed a 2D FE model to characterize the residual stress distribution.

\[ \text{Figure 3} \quad \text{Specimen geometry and dimensions (mm) [4].} \]

This paper provides a numerical studies based on non-ordinary state based peridynamics to model the cold expansion process in specimens with fastener hole and also to determine the produced compressive residual stress field. For this purpose, a 2D model is provided as seen in Fig. 3. It is worth mentioning that the plastic field only created around the cold expanded fastener hole. Furthermore, experimental studies reveal that cracks generates from the fastener hole. For these reasons, the area surrounds the fastener hole is modeled and discretized using peridynamic points. However, for the remaining area with elastic deformation field, the faster FEM is utilized and the domain is discretized using 4-node conventional elements. The coupling method introduced by Yaghoobi and Chorzepa [30] is used herein to couple the peridynamics and FEM subdomains. The coupling is insured through embedded points into the conventional element as seen in Fig. 4(b). The FEM subdomain is discretized using quadrilateral 4-node conventional elements. The spacing between the points in the peridynamic subdomain is \( 0.25 \text{ mm} \times 0.25 \text{ mm} \). Furthermore in the numerical study, the bilinear elastic-plastic model is assumed as Fig. 2 with \( E_t = 0.15 \).

The experimental process of cold expansion includes mandrel entrance follows by mandrel exit. In the first step to simulate the mandrel entrance, radial displacement of \( 0.17 \text{ mm} \) is applied at the peridynamic points around the hole surface. The \( 0.17 \text{ mm} \) radial displacement is associated with 4% experimental cold expansion. Also, note that applying displacement loading creates boundary forces in the peridynamic points around the hole. In the second step, the mandrel exit is simulated with removing boundary force produced in the peridynamic points.
3. RESULTS

During the mandrel entrance, numerically modeled by displacement loading, the material undergoes plastic deformation in the region around the hole. During mandrel exit, numerically modeled by removing boundary forces, the elastic region springs back and residual stress field is provided around the hole. The results for peridynamic simulation of the cold expanded hole was obtained and compared with available experimental and FE results. Fig. 5 presents the comparison for tangential residual stress (Hoop residual stress). A good agreement is observed between the peridynamic results and available experimental results. It shows that peridynamics is an effective theory in modeling plastic deformation field from the cold expansion process. The peridynamic simulation is more accurate than FEM model in predicting residual stress field around a cold expanded fastener hole.

Fig. 6 also presents the comparison of the obtained results for radial residual stress distribution with available FE results of [4]. The comparison reveals that there is good agreement for the case of radial stress distribution too. It is noted that only the tangential residual stress is available in the experimental reference paper.
4. Conclusion

The non-ordinary state-based peridynamic formulation is used to simulate the cold expansion process in engineering components with fastener hole. A nonlinear elastic-plastic material model is incorporated within the peridynamic formulation. The numerical model is utilized to determine the residual stress field around the cold expanded hole. The peridynamic results are compared with available experimental and FE result and good agreement is observed. NOSBPD enhances the accuracy of discrete damage modeling. Therefore, the proposed framework can be used effectively for damage analysis and fatigue crack growth of cold expanded specimens to determine their fatigue life. Also, the method can be extended for a 3D model to cover the variation of the residual stress through the thickness of the plate.

References


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Asghar Sami received the B.S. degree in Engineering from the Islamic Azad University, Tabriz Branch in 2012. He is pursuing his MBA degree at the Islamic Azad University, Tabriz Branch.