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# Tuning of Extended Kalman Filter for Speed Estimation of PMSM Drive using Particle Swarm Optimization

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## ABSTRACT

***This paper presents a speed estimation technique for Permanent Magnet Synchronous Motor (PMSM) based on Extended Kalman Filter (EKF). Particle Swarm Optimization (PSO) method is used to optimize the noise covariance matrices of EKF, thereby ensuring filter stability and accuracy in speed estimation. The proposed method will be performed in two steps; in the first the covariance matrices are optimized in off-line manner and then in the second step these error covariance matrices values are injected into EKF to estimate rotor speed. Simulation results shows that the covariance matrices improve the convergence of estimation process and quality of the system performance.***

**Keywords:** Permanent Magnet Synchronous Motor, Extended Kalman Filter, Particle swarm optimization, Objective function.

## 1. INTRODUCTION

Current industry trends advocate the PMSM as the first preference for motor control application designers. Its unparallel features such as high power density, fast dynamic response and high efficiency in comparison with other motors in its category, together with decreased manufacturing costs and improved magnetic properties, make the PMSM a good recommendation for large-scale product implementation [1-2]. However, conventional motor control needs a speed sensor or an optical encoder to measure the rotor speed with better accuracy. Sensors presents some disadvantages such as drive cost, machine size, reliability and noise immunity; therefore, a sensorless control without position and speed sensors for PMSM drive become a popular research topic in literature [3-4]. Various control algorithms like Sliding Mode Observer (SMO) [ 5], reduced order observer [6], Full order observer [7], Extended Kalman Filter(EKF) [8-9], Model Reference Adaptive System (MRAS) [10], Fuzzy logic [11] and Artificial Neural Networks [12] are proposed in the literature for speed and position estimation of PMSM. Among the proposed algorithms EKF is one of the promising observers, if the noise covariance matrices are known; offers best possible filtering of the noise in measurement and of the system. If the rotor speed considered as an extended state and is incorporated in the dynamic model of a PMSM then the EKF can be used to re-linearize the non-linear state model for each new value of estimate. As a result, EKF is the best solution for the speed estimation of a PMSM. But EKF estimation mainly depends on noise covariance matrices $Q$ & $R$. They can be obtained by considering the stochastic properties of the corresponding noises. Since these are usually unknown, in most cases the EKF matrices are designed and tuned by trial-and-error procedures. But it is a time consuming process, to overcome this problem the covariance matrices are tuned using Genetic algorithm [13].

In this paper, a new alternative method combination of EKF-PSO is used to tune the covariance matrices $Q$ & $R$. In the first step, finding the optimal values of covariance matrices in off-line method and finally these values are placed in the corresponding matrices and run in on-line to estimate rotor speed.

## 2. MATHEMATICAL MODELING

The voltage equations for a PMSM in the rotor reference frame can be expressed as [14].

$$V_d = R_s i_d + L_d \, p \, i_d - \omega_e L_q i_q \quad \text{--- (1)}$$

$$V_q = R_s i_q + L_q \, p \, i_q + \omega_e L_d i_d + \omega_e \psi_f \quad \text{--- (2)}$$

The electromagnetic torque of PMSM is described as

$$T_e = \left[\frac{3}{2}P_n i_q\left(\psi_f - \left(L_q - L_d\right)i_d\right)\right] \quad \text{--- (3)}$$

The motion equation is expressed as follows as

$$J\frac{d\omega_r}{dt} + B\omega_r + T_l = T_e \quad \text{--- (4)}$$

## 3.PROPOSED SCHEME OF EKF BASED SENSOR LESS SPEED CONTROL OF PMSM DRIVE

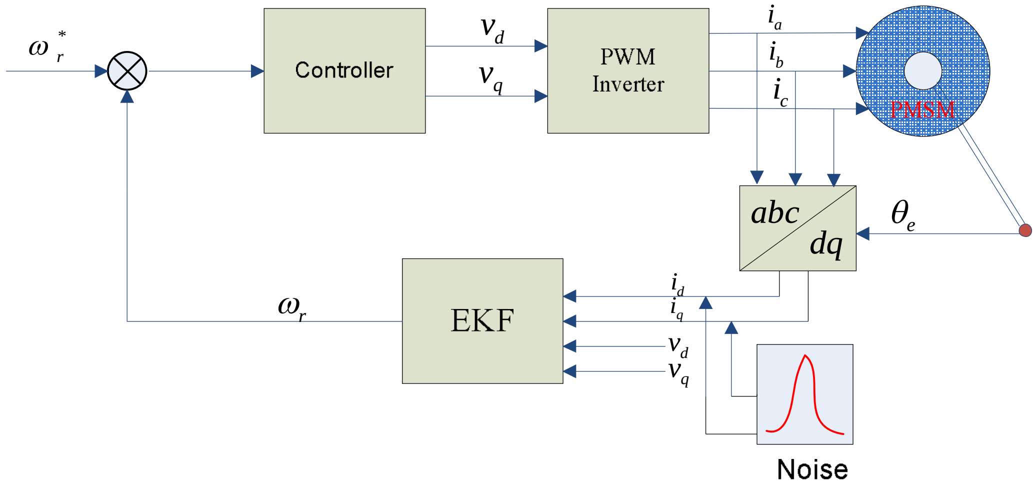


**Figure 1** Block diagram of proposed speed estimation system of PMSM

### 3.1 Extended Kalman Filter (EKF)

Kalman Filter is a mathematical model that runs in parallel to the actual system and provides the estimation of the states of linear systems. But the drawback of Kalman filter is that it is a time consuming process when applied for non-linear systems. For the implementation of nonlinear systems, these functions of the state variables change with every time step, as a result the iteration cannot be pre-computed. Short comings of this model can be overcome by using Extended Kalman filter (EKF). In the model described by equations (1), (2) and (3), the currents of dq-axis and the speed are selected as state variables, and the input variable u and output variable y are defined as follows:

$$x = \begin{bmatrix} i_d \\ i_q \\ \omega_r \end{bmatrix},\ u = \begin{bmatrix} V_d \\ V_q \end{bmatrix},\ y = \begin{bmatrix} i_d \\ i_q \end{bmatrix}$$

The discrete time system equations are

$$x_k = f_{k-1}(x_{k-1}, u_{k-1}, w_{k-1}) \quad \text{--- (5)}$$

$$y_k = h_k(x_k, v_k) \quad \text{--- (6)}$$

Where $x_k$ is state vector, $u_k$ is input vector, $w_k$ is random state noise, $y_k$ is the noisy observation or measured variable vector and $v_k$ is the measurement noise.

### 3.2 EKF Algorithm

Step 1: State vector and covariance matrices are initialized

i.e. $x(0)$, P(0), Q, R

Step 2: Find Jacobian matrices $f_{k-1}$ and $h_k$ using

$$F_{k-1} = \frac{\partial f_{K-1}}{\partial x_k} \quad \text{--- (7)}$$

$$H_k = \frac{\partial h_K}{\partial x_k} \quad \text{--- (8)}$$

Step 3: Prediction of state matrix and error covariance matrices

$$x_{k-1} = f_{k-1}(x_{k-1}, u_{k-1}) + x(0) \quad \text{--- (9)}$$

$$P_{k-1} = F_{k-1}P_{k-1}F_{k-1}^{\ T} + Q_{k-1} \quad \text{--- (10)}$$

Step 4: Correction state

- Calculation of Kalman gain matrix

$$K_k = P_{k-1}H_k^{\ T} / \left(H_k P_{k-1} H_k^{\ T} + R_k\right) \quad \text{--- (11)}$$

- Update state prediction

$$x_k = x_{k-1} + K_k\left(y_k - H_k x_{k-1}\right) \quad \text{--- (12)}$$

- Estimation of error covariance matrix

$$P_k = \left(I - K_k H_k\right)P_{k-1} \quad \text{--- (13)}$$

### 3.3 EKF estimation for PMSM drive

The dynamic state equations of PMSM are

$$\frac{d\,i_d}{dt} = \frac{1}{L_d}\left[-R_s i_d + P_n \omega_r L_q i_q\right] + \frac{V_d}{L_d} \quad \text{--- (14)}$$

$$\frac{diq}{dt} = \frac{1}{L_q}\left[-R_s i_q - P_n W_r L_d i_d - P_n \omega_r \psi_f\right] + \frac{V_q}{L_q} \quad \text{--- (15)}$$

$$\frac{d\omega r}{dt} = \frac{1}{J}\left[\frac{3}{2}P_n i_q\left(\psi_f - \left(L_q - L_d\right)i_d\right)\right] - \frac{B}{J}\omega_r - \frac{T_l}{J} \quad \text{--- (16)}$$

The above equations written in the form of state space represntation

$$\begin{bmatrix} \dfrac{di_d}{dt} \\ \dfrac{di_q}{dt} \\ \dfrac{d\omega_r}{dt} \end{bmatrix} = \begin{bmatrix} \dfrac{-R_s}{L_d} & \dfrac{P_n\omega_r L_q}{L_d} & 0 \\ \dfrac{-P_n\omega_r L_d}{Lq} & \dfrac{-R_s}{L_q} & \dfrac{-P_n\psi_f}{L_q} \\ \dfrac{-3P_n\left(L_q - L_d\right)i_q}{2J} & \dfrac{3P_n\psi_f}{2J} & \dfrac{-B}{J} \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ \omega_r \end{bmatrix} + \begin{bmatrix} \dfrac{1}{L_d} & 0 \\ 0 & \dfrac{1}{L_q} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_d \\ V_q \end{bmatrix} \quad \text{--- (17)}$$

Discrete time representation of above equation is

$$f_{k-1}\left(x_{k-1}, u_{k-1}, w_k\right) = \begin{bmatrix} 1 - \dfrac{R_s}{L_d}T_s & \dfrac{P_n\omega_r L_q}{L_d}T_s & 0 \\ \dfrac{-P_n\omega_r L_d}{Lq}T_s & 1 - \dfrac{R_s}{L_q}T_s & \dfrac{-P_n\psi_f}{L_q}T_s \\ \dfrac{-3P_n\left(L_q - L_d\right)i_q}{2J}T_s & \dfrac{3P_n\psi_f}{2J}T_s & 1 - \dfrac{B}{J}T_s \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ \omega_r \end{bmatrix} + \begin{bmatrix} \dfrac{1}{L_d}T_s & 0 \\ 0 & \dfrac{1}{L_q}T_s \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_d \\ V_q \end{bmatrix} \quad \text{--- (18)}$$

The Gradient matrix is given as

$$F_{k-1} = \frac{\partial f_{K-1}}{\partial x_k} = \begin{bmatrix} 1 - \dfrac{R_s}{L_d}T_s & \dfrac{P_n\omega_r L_q}{L_d}T_s & 0 \\ \dfrac{-P_n\omega_r L_d}{Lq}T_s & 1 - \dfrac{R_s}{L_q}T_s & \dfrac{-P_n\psi_f}{L_q}T_s \\ \dfrac{-3P_n\left(L_q - L_d\right)i_q}{2J}T_s & \dfrac{3P_n\psi_f}{2J}T_s & 1 - \dfrac{B}{J}T_s \end{bmatrix} \quad \text{--- (19)}$$

$$H_k = \frac{\partial h_K}{\partial x_k} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad \text{--- (20)}$$

## 4. PARTICLE SWARM OPTIMIZATION (PSO)

Particle swarm optimization, a swarm intelligence-based global random search algorithm, is proposed by Kennedy and Eberhart inspired from artificial life research results [15]. It regards all individuals in the population as particles without mass and volume in the D-dimensional search space and each particle moves at a certain speed to the best position of its own history $P_{best}$ and the best position of its neighborhood history $g_{best}$ in the solution space, in order to achieve the evolution of candidate solutions.

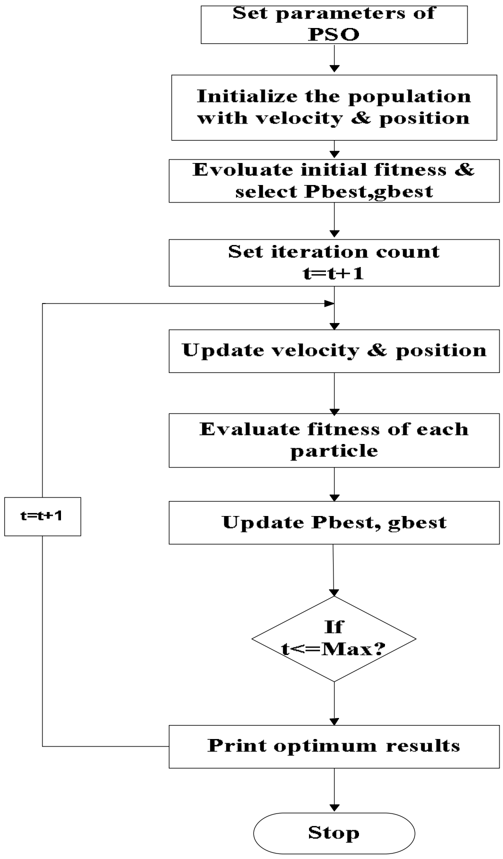


**Figure 2** Flow chart of particle swarm optimization

## 5. TUNING OF EKF USING PARTICLE SWARM OPTIMIZATION

The critical step in a Kalman filter design is to obtain a numerical evaluation of the filter parameters specified by the initial state *x* (0)**,** and the covariance matrices *P*(0), *Q* and *R***.** This process is called tuning and it involves an iterative search for the coefficient values that yield the best estimation performance possible. Changing the covariance matrices *Q* and *R* affects both the transient and the steady state operation of the filter. Increasing *Q* would indicate increase in either noise driving the system or uncertainty in the model. This will increase the values of the state covariance elements. The filter gains will also increase thereby weighting the measurements more heavily, and the filter transient performance is faster. Similarly, increasing the covariance *R* indicates that the measurements are subjected to a stronger corruptive noise and should be weighted less by the filter. Consequently the values of the gain matrix *K* will decrease, and the transient performance is slower. For the initial state covariance matrix *Po,* the diagonal terms represent variances or mean squared errors in knowledge of the initial conditions.

Varying *P* (0) yields a different magnitude transient characteristic. The transient duration will be the same and the steady state conditions are unaffected. The covariance matrices *Q, R* and *P* (0) are assumed to be diagonal due to lack of sufficient statistical information to evaluate their off-diagonal terms. The main objective function of this paper is selection of optimum values of *Q* and *R*. These values are selected manually by using trial & error method. But this is very time consuming process. To surmount this problem, covariance matrices are tuned by using Particle Swarm Optimization (PSO).

The objective function $F= w_1e_1+w_2e_2+w_3e_3$

Where

$$e_1 = \int \left(i_{d-act} - i_{d-est}\right)^2$$
$$e_2 = \int \left(i_{q-act} - i_{q-est}\right)^2$$
$$e_3 = \int \left(\omega_{r-act} - \omega_{r-est}\right)^2$$

Proper selection of weights is essential in tuning else these weights may lead to large errors. These weights are selected as follows $w_1$=0.125, $w_2$=0.05, $w_3$=0.00167.

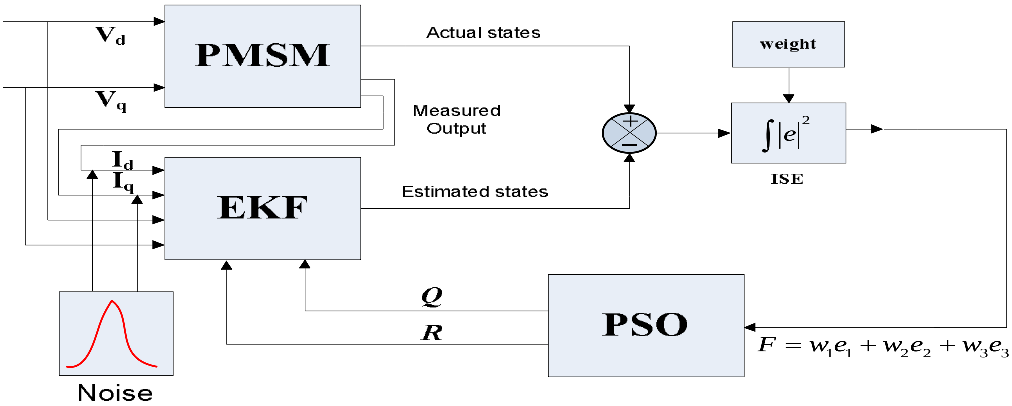


**Figure 3** EKF-PSO block diagram

## 6. RESULTS AND DISCUSSIONS

In the simulation $i_d, i_q, v_d, v_q$ are input variables of EKF algorithm and $i_d, i_q, \omega_r$ are the estimated state variables. In order to mimic the condition of real system Gaussian white noises are added to feedback values of $i_d, i_q$ are set to $3\times10^{-6}$ and sample time of the white noise block is set to $2\times10^{-5}$sec. It should be noted that the convergence of the PSO method to the optimal solution depends on the parameters $c_1$, $c_2$, $w_{min,}$ and $w_{max}$ values. During the simulation, these values are set to $c_1$=2, $c_2$=2, $w_{min,}$=0.5 and $w_{max}$ =0.9 respectively

**Table 1:** PSO-EKF estimations

| Number of iterations = 50(PSO) | | | |
|---|---|---|---|
| Number of Generations | Diagonal matrix *Q* | Diagonal matrix *R* | Objective function value |
| 5 | [18.6932 6.4571 95.5391 ] | [1319.2855 1255.226] | 0.0341 |
| 10 | [21.8921 4.3835 111.8467] | [4294.768 1745.057 ] | 0.0337 |
| 20 | [47.8346 7.2033 131.0535] | [1220.123 2067.458] | 0.0320 |

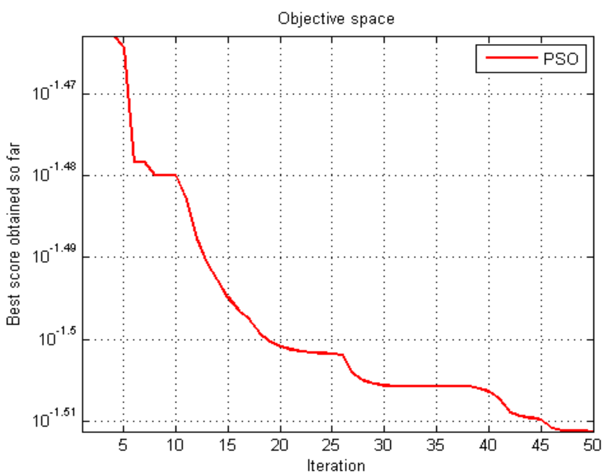


**Figure 4** Evolution of fitness function relative to PSO-EKF

Optimized parameters of matrices *Q* & *R* of EKF with their corresponding ISEs obtained by proposed PSO-EKF method and its performance is given in Figure 4. Table 1 shows the convergence of PSO-EKF process and ISE is decreased with increasing of generation count. Here best objective function value is obtained for generation count 20, corresponding values are injected into EKF and run in online manner. Finally the states of the EKF is estimated as shown in below figures.

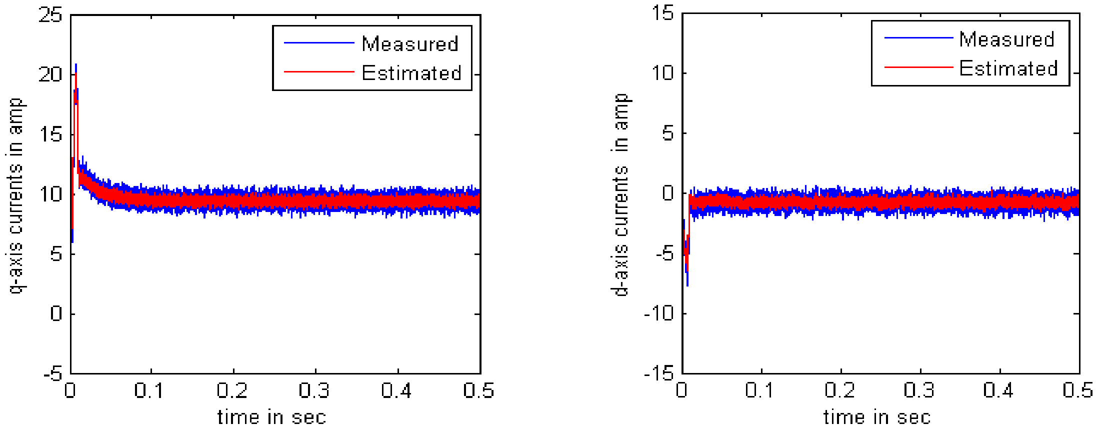


**Figure 5** Measured and estimated current of $i_q$ and $i_d$

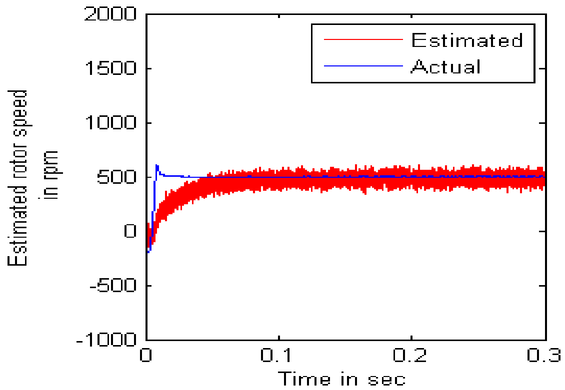


**Figure 6** Actual and estimated rotor speed

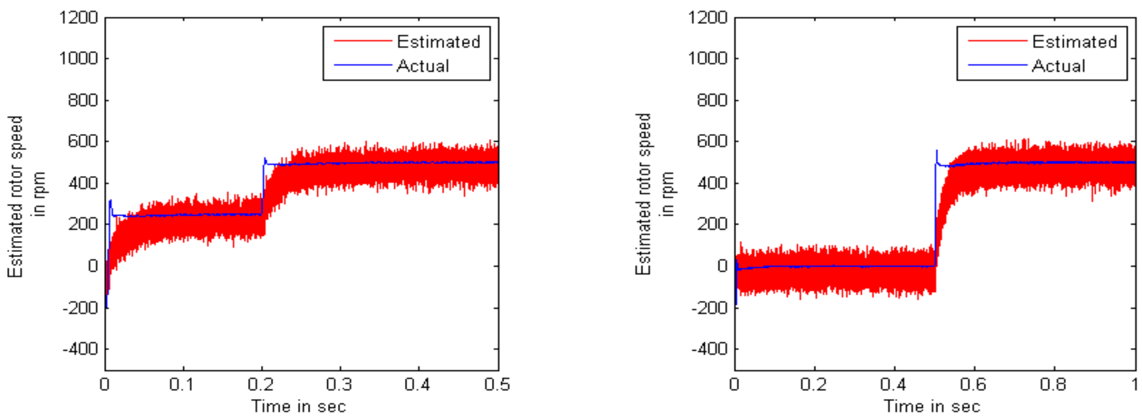


**Figure 7** Actual and estimated rotor speed under variable speeds

The measured and estimated waveforms of $i_d$ and $i_q$ are shown in figure 5. Due to convergence problem of state covariance matrix *P*, the estimated dq-axes currents having large ripples upto0.03 sec. After 0.03 sec matrix *P* is converges, then state variables $i_d$ and $i_q$ are tracks the actual values. From Figure 6 it is evident that the estimated speed matches with the actual speed near 0.001 sec. The effectiveness of PSO-EKF method is evaluated under two cases, in one case the speed is varied from 200 rpm to 500 rpm at 0.5 sec and in another case motor is run at 500 rpm at 0.5 sec as shown in figure 7 and in both cases the estimated speed is converged accordingly due to precise values of matrices *Q* & *R*.

## 7. CONCLUSION

In this paper, EKF based sensorless speed control of PMSM drive has been presented to show the results of estimated values of speed and dq-axes stator currents. The performance of EKF is mainly depends on error covariance matrices Q & R, which are suitably selected. These matrices are improved the system convergence and quality of estimation. The simulation results show the superior performance in terms of settling time, reduction of noise and overall system stability.

**Appendix A:**

Simulation Parameters values of PMSM drive:

| Parameters | Symbol | Numerical value |
|---|---|---|
| Resistance of stator | $R_s$ | 0.675 ohm |
| Direct axis inductance of stator | $L_d$ | 0.0085 H |
| Quadrature axis inductance of stator | $L_q$ | 0.0085 H |
| Flux linkages | $\psi_f$ | 0.12 Wb |
| Inertia of rotor | J | 0.0011 $Kg/m^2$ |
| friction coefficient | B | 0.0014 $Nm/s^2$ |
| Pair of poles | $P_n$ | 3 |
| Rated speed | $\omega$ | 1000 rpm |

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