

# Advanced statistical methods used in practice

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## ABSTRACT

*New methodology for evaluating calibration is proposed in this paper. This method consists of assumption that calibration is provided by comparative method, where we assume that the value measured by etalon is uncertain. This article provides detailed soffit on a specific selected method of calibration. This method is narrowly elaborated on example of ejection of the tricept's telescopic rod, which is the prototype of parallel kinematic structure type machine designed at the Faculty of Mechanical Engineering in Bratislava.*

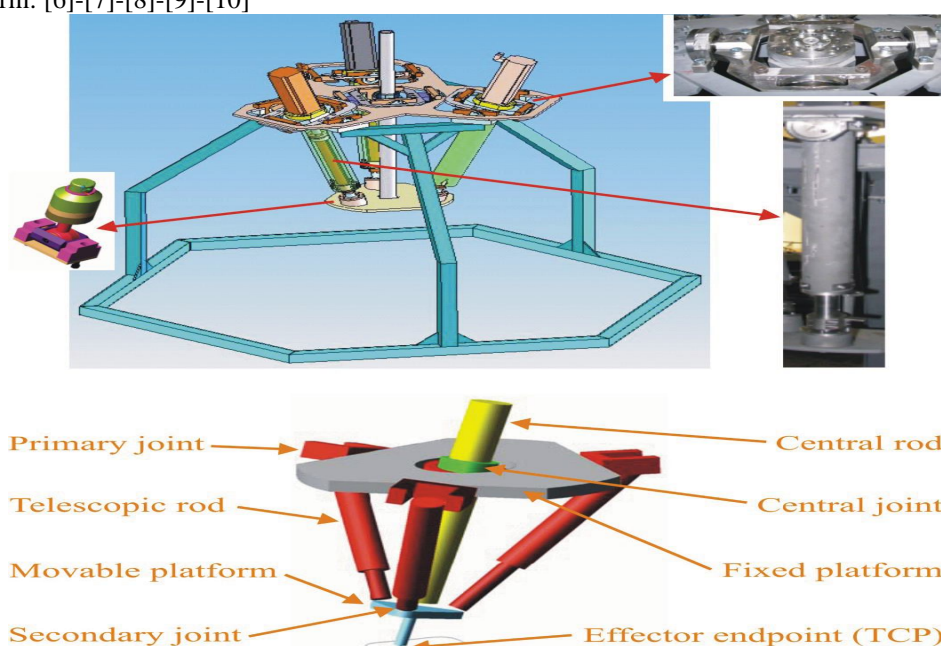
**Keywords:** Tricept, calibration of telescopic rod, measurement model, MATLAB

## 1. INTRODUCTION

Nowadays, we place great attention to the development of new methods of analysis of measurement results. [1]-[2]-[3]-[4] These include especially the uniformity of measurement, which is provided by establishing of measurement results with the etalon, which is used for calibration. This issue can be applied also to parallel kinematic structure type tricept, particularly when calibrating the telescopic rods. The proposed approach of evaluation of measured or simulated data is in accordance with GUM. [5]

## 2. PKS TYPE TRICEPT

The tricept at STU (Fig. 1) belongs to a group of parallel respectively partly hybrid kinematic structures. It consists of a static platform on which they are attached with three primary joints of telescopic rods and one central rod. On the other side there is a movable platform, on which are mounted telescopic rods with secondary joints. The central rod passes through the center of static and movable platform and it is fixed firmly to the mobile platform and articulated to the static platform. Telescopic rods have variable length. Ejecting of the individual telescopic rods is allowed by shift of the ball screw. Joints allow shooting of telescopic rods towards static platform and thus shift of the mobile platform. Telescopic rods of the tricept are actuators, which are driven autonomous by a servomotors. The actuators are controlled by a frequency converter and pledged by an engine, which has integrated pressure brake. A parallel kinematic structure is formed by the two platforms and three telescopic rods. The end effector may be attached on the moveable platform. [6]-[7]-[8]-[9]-[10]



**Figure 1** Parallel kinematic structure of the Tricept type. The design shown represents the device developed at the Slovak University of Technology, Faculty of Mechanical Engineering in Bratislava [7]

**4. CALIBRATION PROCESS OF TELESCOPIC ROD OF THE TRICEPT**

At first we need to calibrate individual telescopic rods during the calibration of the tricept. Secondly we insert the correction functions for the individual telescopic rod to the control system. Only then it is recommended to carry out calibration of the tricept as a whole.

The first step during calibration of the tricept telescopic rod that can be regarded as a continuous scale is to design the transformation curve. The second step is to establish a process for the application of this result (of the transformation curve) by calibrated etalon. This object is achieved by determining the calibration curve, which is the functional relationship between the actual value (represented in calibration gauge) and the true value eject telescopic rod (generally calibrated meter) with associated uncertainty. Transformation process of determining transformational and calibration curve are mathematically identical terms. It is necessary to provide an estimate of the true value eject telescopic rod including uncertainties during the measurement. This task is achieved by a calibration curve, which relates the true value of the measured value (resistance value) and the actual value of ejection of telescopic rod (in general calibrated meter). Each measurement, hence also the calibration experiment, is loaded with errors, and therefore we know only estimate transformation functions with associated uncertainties. [7, 8]

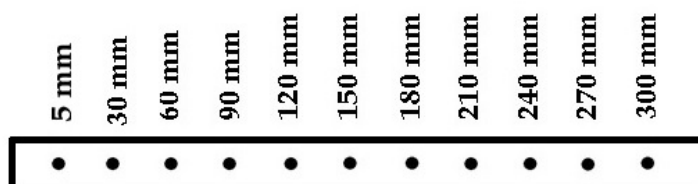
Specific calibration procedure consists of gradually retracting telescopic rod into our chosen location. Overall, we chose  $m = 11$  measuring points evenly spaced along the length of a telescopic rod and at all points we did  $n = 10$  repeated measurements.

Telescopic rod (Fig. 2) may be considered to be numerically controlled linear axis, that can be calibrated according to the standard STN ISO 230:2014 [11]. This standard provides information about the behavior of the rods only in measured points. This paper contains procedure to evaluate the measured data and the one that gives us information on the position of the ejected rod throughout its whole operating range, including uncertainties.



**Figure 2** Telescopic rod of Tricept [8]

The diagram (Fig. 3) shows the select measurement points on the telescopic rod of the prototype of tricept at STU. These points are uniformly spaced along the length of the eject rod. The rod can be pulled out to a maximum distance of 300 mm, except for endpoints as to the construction and safe operation process of the tricept. The paper used the simulated data so we can choose the endpoints on a stick.



**Figure 3** Representation of the measuring points on the telescopic rod of the tricept

**3.1 Polynomial stochastic model of calibration with continuous function**

We measure  $m$  (different) values by the calibrated gauger symbolize as A and calibrating scale (etalon) symbolized as B. In our case the calibrated gauger A is the telescopic rod tricept. Surrounding impacts on calibrated gauge and etalon are regarded as the same.

Values measured with a calibrated scale A (readings eject telescopic rod) will be labelled as:

$$\mu = (\mu_{1,1}, \mu_{1,2}, \dots, \mu_{1,m}, \dots, \mu_{n,m}) \tag{1}$$

The values measured by an etalon B shall be labelled as [10]:

$$v = (v_{1,1}, v_{1,2}, \dots, v_{1,m}, \dots, v_{n,m}) \tag{2}$$

It is proposed measurement model (transformation function) in the form of a polynomial:

$$\gamma_i = b_0 + b_1 v_1 + \dots + b_k v_i^k \tag{3}$$

Where

$$b_i (i = 0, \dots, k)$$

$k$  – the degree of a polynomial

$v_i$  - estimation of measured length of telescopic rod ejection (transformation function) which is calculated as the arithmetic average of the measured values at certain point.

Selection of the degree of the polynomial is based on Akaike's information criterion (AIC). This criterium estimates the degree of the polynomial, which should be chosen. It is looking for condition when the value of the Akaike information criterion is to a minimum [4].

$$AIC = m \ln \sum_{i=1}^m e_i^2 + 2k \tag{4}$$

Where

$m$  - number of measurement points,

$e_i$  - deviation from the point through which pass the polynomial

$k$  – the degree of a polynomial

We transform this transformation function  $\gamma_i$  to be the calibration function [3]-[4]. We consider that measurements are independent with normal probability distribution and they are not deviated.

Theoretical shape of calibration function is

$$v_i = \beta_0 + \beta_1 \mu_i + \dots + \beta_k \mu_i^k \tag{5}$$

Where

$\beta_0, \dots, \beta_k$  - parameters of the polynomial where  $\beta_k (i = 0, \dots, k)$

$\mu_1, \dots, \mu_i^k$  - estimates of ejection of telescopic rod

Shape of the calibration function (2) transforms to the system of equations

$$\begin{aligned} v_{1,1} &= \beta_0 + \beta_1 \mu_{1,1} + \beta_2 \mu_{1,2}^2 + \dots + \beta_k \mu_{1,n}^k \\ &\vdots \\ v_{m,n} &= \beta_0 + \beta_1 \mu_{1,n} + \beta_2 \mu_{1,n}^2 + \dots + \beta_k \mu_{m,n}^k \end{aligned} \tag{6}$$

The results of calibration is to determine the parameters of the calibration function. There are several possible approaches to get the calibration function. In practice, for example, directly calibrated. Comparison method calibrated etalon using a measurement standard. Here it is assumed that the value of a standard fixed value, which is in terms of statistics internally. It is therefore appropriate to consider that the Standard is uncertain (that has its own uncertainty). In the present paper we consider the case that measurement with a gauge is uncertain and it is therefore appropriate to the approach of calibration.

### 3.1 Application using the method of maximum likelihood

Denoting  $\zeta_{x_{i,j}}$  the  $j$ -th measurement of the  $i$ -th value of the calibrated scale (telescopic rod) and  $\zeta_{y_{i,j}}$  the  $j$ -th measurement value of the  $i$ -th reference standard B, we get  $2mn$  measurements that are independent, while

$$\zeta_{x_{i,j}} \sim N(\mu_i, \sigma_x^2)$$

And

$$\zeta_{y_{i,j}} \sim N(v_i, \sigma_y^2)$$

Where in  $i = 1, \dots, m$  and  $j = 1, \dots, n$ .  $\sigma_x^2$  and  $\sigma_y^2$  are the dispersions of etalon and telescopic rod that is now considered to be unknown. Result of that calibration is to determine the parameters of the calibration function and determine the value of the dispersions of etalon and telescopic rod. [3]-[4]

Maximum likelihood method is based on maximizing of so-called assurance function. By choosing this method we will receive consistent and asymptotically efficient estimates, while estimates of selected parameters are derived from the normal distribution. We have measurements

$$\zeta_{x_{1,1}}, \zeta_{y_{1,1}}, \dots, \zeta_{x_{i,j}}, \zeta_{y_{i,j}}$$

Independent measurements are derived from a normal probability distribution and can be entered in the form

$$\zeta_{x_{i,j}} \sim N(\nu_i, \sigma_x^2), \zeta_{y_{i,j}} \sim N(\beta_0 + \beta_1\mu_1 + \dots + \beta_k\mu_i^k, \sigma_y^2) \quad (7)$$

Where

$$i = 1, \dots, m$$

$$j = 1, \dots, n$$

$\beta_0, \beta_1, \dots, \beta_k$  - the unknown parameters of the calibration function

Vector of all unknown parameters contains unknown parameters to the polynomial degree, the dispersion of both sensors detected and the positional deviation in the measuring point and is shaped [3]- [4]:

$$\theta = (\beta_0, \beta_1, \dots, \beta_k, \sigma_x^2, \sigma_y^2, \mu_1, \dots, \mu_k)^T \quad (8)$$

The density of the j-th experiment is of the form:

$$j(x_{i,j}, \dots, y_{m,j}, \theta) = \prod_{i=1}^m \frac{1}{\sqrt{2\pi\sigma_x^2}} e^{-\frac{(x_{i,j}-\mu_i)^2}{2\sigma_x^2}} \prod_{i=1}^m \frac{1}{\sqrt{2\pi\sigma_y^2}} e^{-\frac{(y_{i,j}-\beta_0-\beta_1\mu_i-\beta_2\mu_i^2-\dots-\beta_k\mu_i^k)^2}{2\sigma_y^2}} \quad (9)$$

The result of credible function of the experiment for  $n$  repetitions has the form [3]-[4]:

$$L_{(x_{1,1}, \dots, y_{m,n}, \theta)} = (2\pi)^{-mn} \sigma_x^{\frac{-mn}{2}} \sigma_y^{\frac{-mn}{2}} e^{-\frac{1}{2\sigma_x^2} \sum_{i=1}^m \sum_{j=1}^n (x_{i,j} - \mu_i)^2} \times e^{-\frac{1}{2\sigma_y^2} \sum_{i=1}^m \sum_{j=1}^n (y_{i,j} - \beta_0 - \beta_1\mu_i - \beta_2\mu_i^2 - \dots - \beta_k\mu_i^k)^2} \quad (10)$$

and logarithmic functions credible shape is in the form [3]-[4]:

$$l_{(x_{1,1}, \dots, y_{m,n}, \theta)} = \ln L_{(x_{1,1}, \dots, y_{m,n}, \theta)} = -mn \ln(2\pi) - \frac{mn}{2} \ln \sigma_x^2 - \frac{mn}{2} \ln \sigma_y^2 - \frac{1}{2\sigma_x^2} \sum_{i=1}^m \sum_{j=1}^n (x_{i,j} - \mu_i)^2 - \frac{1}{2\sigma_y^2} \sum_{i=1}^m \sum_{j=1}^n (y_{i,j} - \beta_0 - \beta_1\mu_i - \beta_2\mu_i^2 - \dots - \beta_k\mu_i^k)^2 \quad (11)$$

After calculating the maximum credible estimate  $\theta^*$  of the parameter  $\theta$  and therefore its realization  $\theta^*_{(real)}$  for such a vector is valid

$$L_{(x_{1,1}, \dots, y_{m,n}, \theta)} \leq L_{(x_{1,1}, \dots, y_{m,n}, \theta^*_{(real)})} \quad (12)$$

For each parameter  $\theta$  is asymptotically unbiased estimate [3]-[4]

$$\theta^* \approx N\left(\theta, \frac{1}{n} J_{(\theta)}^{-1}\right) \quad (13)$$

Where  $J_{(\theta)}$  is Fischer's information matrix of the  $i$ -th measurement and next equation is established for it [3]-[4]:

$$\left\{ J_{(\theta)} \right\}_{i,j} = E_{\theta} \left( -\frac{\partial^2 \ln_{\phi_i}(X_{1,1}, \dots, Y_{m,1}, \theta)}{\partial \theta_j \partial \theta_j} \right) \quad (14)$$

After calculating the final form of the matrix we obtain [3]-[4]:

$$\mathbf{J}_{(\theta)} = \begin{pmatrix} \frac{\sum_{i=1}^m \mu_i^0}{\sigma_y^2} & \frac{\sum_{i=1}^m \mu_i^1}{\sigma_y^2} & \dots & \frac{\sum_{i=1}^m \mu_i^k}{\sigma_y^2} & 0 & 0 & \frac{dp_1}{\sigma_y^2} & \frac{dp_2}{\sigma_y^2} & \dots & \frac{dp_m}{\sigma_y^2} \\ \frac{\sum_{i=1}^m \mu_i^1}{\sigma_y^2} & \frac{\sum_{i=1}^m \mu_i^2}{\sigma_y^2} & \dots & \vdots & 0 & 0 & \frac{dp_1 \mu_1}{\sigma_y^2} & \frac{dp_1 \mu_2}{\sigma_y^2} & \dots & \frac{dp_m \mu_m}{\sigma_y^2} \\ \vdots & \vdots & \ddots & \vdots & 0 & 0 & \vdots & \vdots & \dots & \vdots \\ \frac{\sum_{i=1}^m \mu_i^k}{\sigma_y^2} & \frac{\sum_{i=1}^m \mu_i^{k+1}}{\sigma_y^2} & \dots & \frac{\sum_{i=1}^m \mu_i^{2k}}{\sigma_y^2} & 0 & 0 & \frac{dp_1 \mu_1^k}{\sigma_y^2} & \frac{dp_1 \mu_2^k}{\sigma_y^2} & \dots & \frac{dp_m \mu_m^k}{\sigma_y^2} \\ 0 & 0 & 0 & 0 & \frac{m}{2\sigma_x^4} & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{m}{2\sigma_y^4} & 0 & 0 & \dots & 0 \\ \frac{dp_1}{\sigma_y^2} & \frac{dp_1 \mu_1}{\sigma_y^2} & \dots & \frac{dp_1 \mu_1^k}{\sigma_y^2} & 0 & 0 & \frac{1}{\sigma_x^2} + \frac{dp_1^2}{\sigma_y^2} & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 0 & \ddots & \ddots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \ddots & 0 \\ \frac{dp_m}{\sigma_y^2} & \frac{dp_m \mu_m}{\sigma_y^2} & \dots & \frac{dp_m \mu_m^k}{\sigma_y^2} & 0 & 0 & 0 & \dots & 0 & \frac{1}{\sigma_x^2} + \frac{dp_m^2}{\sigma_y^2} \end{pmatrix} \quad (15)$$

Where

$$p_i = \beta_0 + \beta_1 \mu_1 + \dots + \beta_k \mu_i^k$$

$$dp_i = \frac{\partial p_i}{\partial \mu_i} = \beta_1 + \beta_2 \mu_i + \dots + k \beta_k \mu_i^{k-1}$$

Denoting  $\mathbf{J}_{(\theta)}$  in formula (16) [3]-[4]

$$\mathbf{J}_{(\theta)} = \begin{pmatrix} A & B \\ B' & D \end{pmatrix} \quad (16)$$

Then we get forms of following equations (17, 18, 19) [3]-[4]

$$A = \begin{pmatrix} \frac{\sum_{i=1}^m \mu_i^0}{\sigma_y^2} & \dots & \frac{\sum_{i=1}^m \mu_i^0}{\sigma_y^2} & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ \frac{\sum_{i=1}^m \mu_i^p}{\sigma_y^2} & \dots & \frac{\sum_{i=1}^m \mu_i^{2k}}{\sigma_y^2} & 0 & 0 \\ 0 & \dots & 0 & \frac{m}{2\sigma_x^4} & 0 \\ 0 & \dots & 0 & 0 & \frac{m}{2\sigma_y^4} \end{pmatrix} \quad (17)$$

$$\mathbf{B} = \begin{pmatrix} \frac{dp_1}{\sigma_y^2} & \dots & \frac{dp_m}{\sigma_y^2} \\ \vdots & \vdots & \vdots \\ \frac{dp_1 \mu_1^k}{\sigma_y^2} & \dots & \frac{dp_m \mu_m^k}{\sigma_y^2} \\ 0 & \dots & 0 \\ 0 & \dots & 0 \end{pmatrix} \quad (18)$$

$$\mathbf{D} = \begin{pmatrix} \frac{1}{\sigma_x^2} + \frac{dp_1^2}{\sigma_y^2} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \frac{1}{\sigma_x^2} + \frac{dp_m^2}{\sigma_y^2} \end{pmatrix} \quad (19)$$

The formula (16) is used for evaluation of the inverse matrix (20) [3]-[4]:

$$\begin{pmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B}' & \mathbf{D} \end{pmatrix}^{-1} = \begin{pmatrix} (\mathbf{A} - \mathbf{B}\mathbf{D}^{-1}\mathbf{B}')^{-1} & -(\mathbf{A} - \mathbf{B}\mathbf{D}^{-1}\mathbf{B}')^{-1}\mathbf{B}\mathbf{D}^{-1} \\ -\mathbf{D}^{-1}\mathbf{B}'(\mathbf{A} - \mathbf{B}\mathbf{D}^{-1}\mathbf{B}')^{-1} & \mathbf{D}^{-1} + \mathbf{D}^{-1}\mathbf{B}'(\mathbf{A} - \mathbf{B}\mathbf{D}^{-1}\mathbf{B}')^{-1}\mathbf{B}\mathbf{D}^{-1} \end{pmatrix} \quad (20)$$

$$\mathbf{J}(\boldsymbol{\beta}) = \begin{pmatrix} \sum_{i=1}^m \frac{1}{\sigma_y^2 + \sigma_x^2 + dp_i^2} & \sum_{i=1}^m \frac{\mu_i}{\sigma_y^2 + \sigma_x^2 + dp_i^2} & \sum_{i=1}^m \frac{\mu_i^2}{\sigma_y^2 + \sigma_x^2 + dp_i^2} & \dots & \sum_{i=1}^m \frac{\mu_i^k}{\sigma_y^2 + \sigma_x^2 + dp_i^2} & 0 & 0 \\ \sum_{i=1}^m \frac{\mu_i}{\sigma_y^2 + \sigma_x^2 + dp_i^2} & \sum_{i=1}^m \frac{\mu_i^2}{\sigma_y^2 + \sigma_x^2 + dp_i^2} & \sum_{i=1}^m \frac{\mu_i^3}{\sigma_y^2 + \sigma_x^2 + dp_i^2} & \vdots & \vdots & 0 & 0 \\ \sum_{i=1}^m \frac{\mu_i^2}{\sigma_y^2 + \sigma_x^2 + dp_i^2} & \sum_{i=1}^m \frac{\mu_i^3}{\sigma_y^2 + \sigma_x^2 + dp_i^2} & \sum_{i=1}^m \frac{\mu_i^4}{\sigma_y^2 + \sigma_x^2 + dp_i^2} & \vdots & \vdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & 0 & 0 \\ \sum_{i=1}^m \frac{\mu_i^k}{\sigma_y^2 + \sigma_x^2 + dp_i^2} & \dots & \dots & \dots & \sum_{i=1}^m \frac{\mu_i^{2k}}{\sigma_y^2 + \sigma_x^2 + dp_i^2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{m}{2\sigma_x^4} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{m}{2\sigma_y^4} \end{pmatrix} \quad (21)$$

and the covariance matrix is in the form [3]-[4]:

$$u_{\begin{pmatrix} \hat{\sigma}_x^2 \\ \hat{\sigma}_y^2 \end{pmatrix}} = \frac{1}{n} \begin{pmatrix} \frac{mn}{2\sigma_x^4} & 0 \\ 0 & \frac{mn}{2\sigma_y^4} \end{pmatrix}^{-1} = \begin{pmatrix} \frac{2\sigma_x^4}{mn} & 0 \\ 0 & \frac{2\sigma_y^4}{mn} \end{pmatrix} \quad (22)$$

$$*C_{(1-\alpha)}^{ML} = \left\{ \begin{pmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_k \end{pmatrix} : \begin{pmatrix} \hat{\beta}_0 - \beta_0 \\ \hat{\beta}_1 - \beta_1 \\ \vdots \\ \hat{\beta}_k - \beta_k \end{pmatrix}' n\mathbf{J}_{(\hat{\beta})} \begin{pmatrix} \hat{\beta}_0 - \beta_0 \\ \hat{\beta}_1 - \beta_1 \\ \vdots \\ \hat{\beta}_k - \beta_k \end{pmatrix} \leq \chi_{k+1}^2(1-\alpha) \right\} \quad (23)$$

$$I'_\mu \hat{\boldsymbol{\beta}} \approx N = \left( \begin{pmatrix} 1 & \mu & \mu^2 & \dots & \mu^k \end{pmatrix} \begin{pmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_k \end{pmatrix}, I'_\mu \frac{1}{n} \mathbf{J}'_{(\hat{\beta})} I_\mu \right) \quad (24)$$

and then gets

$$P \left\{ l'_\mu \hat{\beta} - u \left( 1 - \frac{\alpha}{2} \right) \sqrt{l'_\mu \frac{1}{n} \hat{J}^{-1}(\hat{\beta})} < l'_\mu \hat{\beta} < l'_\mu \hat{\beta} + u \left( 1 - \frac{\alpha}{2} \right) \sqrt{l'_\mu \frac{1}{n} \hat{J}^{-1}(\hat{\beta})} \right\} \cong 1 - \alpha \quad (25)$$

Then, for each mean value  $\mu$  we calculate confidence intervals  $(1 - \alpha)$ . For convenience, upper  $h$  and the lower interval  $d$  is calculated for [3]-[4]:

$$v_i = \beta_0 + \beta_1 \mu_1 + \dots + \beta_k \mu_i^k$$

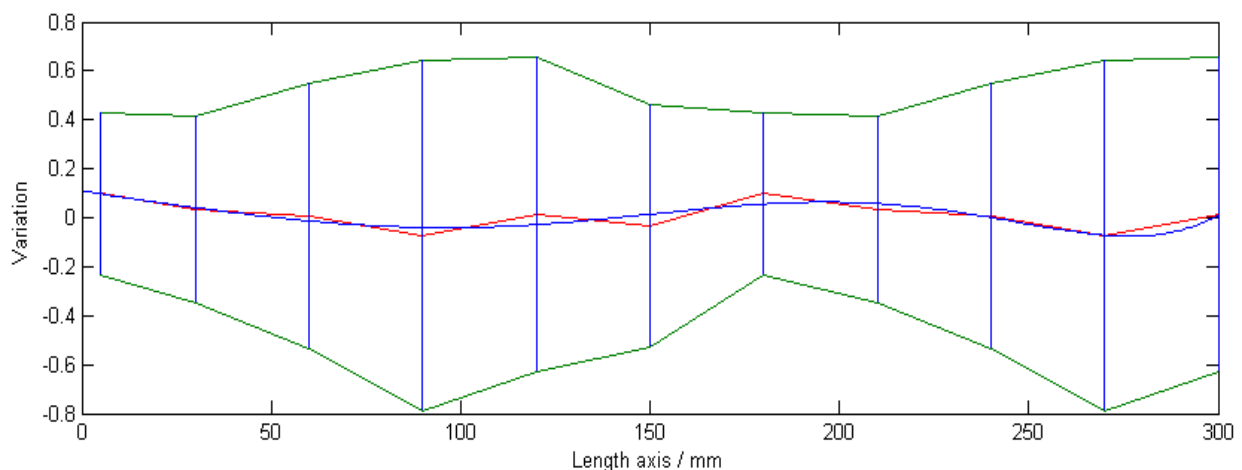
$$d = \omega \in \left( x - \hat{\sigma}_x t_v \left( 1 - \frac{\gamma}{2} \right), x + \hat{\sigma}_x t_v \left( 1 - \frac{\gamma}{2} \right) \right) \hat{\beta}_0 + \hat{\beta}_1 \omega + \hat{\beta}_2 \omega^2 + \dots + \hat{\beta}_k \omega^k \quad (26)$$

$$h = \omega \in \left( x - \hat{\sigma}_x t_v \left( 1 - \frac{\gamma}{2} \right), x + \hat{\sigma}_x t_v \left( 1 - \frac{\gamma}{2} \right) \right) \hat{\beta}_0 + \hat{\beta}_1 \omega + \hat{\beta}_2 \omega^2 + \dots + \hat{\beta}_k \omega^k \quad (27)$$

After using Bonferroni inequality it gets [3]-[4]

$$P \left\{ \hat{\beta}_0 + \hat{\beta}_1 d + \hat{\beta}_2 d^2 + \dots + \hat{\beta}_k d^k - u(1 - \alpha) \sqrt{\left( 1 \ h \ h^2 \ \dots \ h^k \right) \frac{1}{n} \hat{J}^{-1}(\hat{\beta}) \begin{pmatrix} 1 \\ d \\ d^2 \\ \vdots \\ d^k \end{pmatrix}} < \right.$$

$$v_\mu = \beta_0 + \beta_1 \mu + \dots + \beta_k \mu^k < \beta_0 + \beta_1 h + \dots + \beta_k h^k + u \left( 1 - \frac{\alpha}{2} \right) \sqrt{\left( 1 \ h \ h^2 \ \dots \ h^k \right) \frac{1}{n} \hat{J}^{-1}(\hat{\beta}) \begin{pmatrix} 1 \\ h \\ h^2 \\ \vdots \\ h^k \end{pmatrix}} \geq 1 - \alpha - \gamma \quad (28)$$



**Figure 4** Evaluation using the software Matlab. The green color is evaluating the ISO 230-2, the red color is the evaluation using advanced statistical methods and blue they show the confidence intervals for 95% reliability

Figure 4 shows the calibration evaluation of ejected telescopic rod of the tricept using advanced statistical methods. Simulated data were implemented MATLAB [12], where the matrix of input variables was formed. This matrix should have dimensions 11x10, where rows of the matrix contain repeated measurements at the same measuring point (in our case it was the length of the telescopic rod ejection of tricept) and columns of the matrix represent the number of measurement points. Based on AIC, the sixth degree of a polynomial was chosen. Further the figure shows confidence

intervals for the 95% reliability value for the transformation function (3). From there we get estimates of parameters to be entered in the vector form.

$$\beta = \begin{pmatrix} 3,0253e^{-14} \\ -1,7157e^{-11} \\ 1,5328e^{-9} \\ 5,9621e^{-7} \\ -1,11654e^{-4} \\ 5,3025 \end{pmatrix} \quad (29)$$

#### 4. Conclusion

In the present paper it is implemented new evaluation methods using advanced statistical methods. This method was applied on parallel kinematic structure machine type tricept, developed at the Faculty of Mechanical Engineering in Bratislava. The proposed evaluation method provides detailed overview of the possibilities of evaluation of measurement data. Using the proposed approach is the ability to evaluate data over the entire length of the tricept's telescopic rod ejection.

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