A New Approach to Solve Type-2 Fuzzy Linear Programming Problem Using Possibility, Necessity, and Credibility Measures

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ABSTRACT

In this paper, a type -2 fuzzy linear programming model based on the possibility, necessity and credibility relation is introduced. By using the degree of measures, the satisfaction of constraints can be measured. With this ranking index, the bound of optimal(opt) solution is obtained at different degree of possibility and necessity measures. To validate the proposed method, Optimal solution is obtained for type -2 fuzzy linear programming problem at different degree of satisfaction by using simplex method with the help of MATLAB. Finally, the optimal solution procedure is illustrated with numerical example.

Keywords: Fuzzy Linear programming, Type-2 Fuzzy Set, Interval Type-2 Fuzzy Set, Interval Type-2 Fuzzy Number, Perfectly normal

1. INTRODUCTION

Management Science includes all rational approaches to management decision-making that are based on an application of scientific and systematic procedures. The analysis process employed by the decision maker may take two basic forms: qualitative and quantitative. The qualitative analysis approach is based primarily upon the manager's judgment and experience. In the quantitative approach to the problem, an analyst will concentrate on the quantitative facts or data associated with the problem and develop mathematical expressions that describe the objectives, constraints, and relationships that exist in the problem.

There are several important tools, or techniques, that have been found useful in the quantitative analysis phase of the decision making process. Among them Linear programming is an unsurpassed technique to achieve the best outcome. The quantitative analysis contains five phases: 1. problem definition. 2. Model development. 3. Data preparation 4. Model solution, and 5. Report generation. In model development phase, the models are representations of real objects. These representations, can be presented in various forms alike Iconic models, Analog models, and Mathematical models. The mathematical models represent the real situation by a system of symbols and mathematical relationships or expressions. The success of the mathematical model and quantitative approach will depend heavily upon how accurately the objective and constraints can be expressed in terms of mathematical equations or relationships.

Such environmental factors, which can affect both the objective function and constraints, are referred to as the Uncontrollable input to the model. The input which are controlled or determined by the decision maker are referred to as the controllable inputs to the model, thus are referred to as the decision variables of the model.

Once all controllable and uncontrollable inputs are specified, the objective function and constraints can be evaluated and the output of the model determined [1]. In this sense, the output of the model is simply the projection of what would happen if the particular environmental factors and decisions occurred in the real situation. The uncontrollable inputs can either be known exactly or be uncertain and subject to variation. If all uncontrollable inputs to a model are known and cannot vary, the model is referred to as a deterministic model. If any of the uncontrollable inputs are uncertain and subjective to variation, uncertainties can be categorized as probabilistic or stochastic uncertainty and fuzziness as pointed out by Zimmerman stochastic uncertainty can be modeled and solved by stochastic mathematical programming techniques and problems with fuzziness can be modeled and solved by fuzzy mathematical programming techniques. For the uncontrollable input analysis, give an example concerning a mathematical model. In the production model, the number of man-hours required per unit of production could vary from 3 to 6 hours depending upon the quality of the raw material, the model would have been stochastic.

Fuzzy set provides a rudimentary mathematical framework for dealing with incomplete uncertain information. It has long been proposed by Zadeh[2] as an extension of the classical theory of crisp sets. Bellman and Zadeh [3] primarily make known to the notion of fuzzy decision making which was extensively developed later by researchers. Fuzzy linear programming with fuzzy coefficients has been formulated by Negoita and Stan [4] and followed by Zimmermann [5], and Tanaka and Asai [6]. Since then, work on Fuzzy linear programming grew continuously its application. Like, Wu [7] presented possibility and necessity measures fuzzy optimization problems based on the embedding theorem. Xu and

Zhou [8] discussed possibility, necessity, and credibility measures for fuzzy optimization. Figueroa[9] presents some definitions about of Interval Type-2 Fuzzy Constraints regarding Interval Type-2 Fuzzy Linear Programming models which can be solved by classical algorithms. In [10] he shows the use of interval optimization models to solve linear programming problems with Interval Type-2 fuzzy constraints and the concept of α -cut of an Interval Type-2 fuzzy set is used to find optimal solutions to uncertain optimization problems. In [11] he presents a general model for Linear Programming where its technological coefficients are assumed as Interval Type-2 fuzzy sets and it is solved through an α -cuts approach. And [12] he shows a method for solving linear programming problems that includes Interval Type-2 fuzzy constraints and he proposed method finds an optimal solution in these conditions using convex optimization techniques. Jindong Qin and Xinwang Liu [13] investigates an approach to multiple attribute group decision-making problems, in which the individual assessments are in the form of triangle interval type-2 fuzzy numbers.

With our best knowledge, however, none of them introduced Type-2 fuzzy linear programming model based on possibility, necessity, and credibility measures on interval type-2 fuzzy set for upper and lower membership functions. The Possibility, necessity, and credibility measures have a significant role in fuzzy and fuzzy optimization. The possibility measure is much suitable for the optimistic decision maker. If the decision-maker is pessimistic, he may use the necessity measure as a tool to make decision and credibility measure as the average of possibility measure and necessity measure. In this paper, The uncontrollable input, due to environmental factors or inputs that cannot be specified by decision maker, uncertainty and imprecision involved in decision maker knowledge can be well addressed using type-2 fuzzy set. We consider the both membership function of perfectly normal Interval type-2fuzzy set are trapezoidal membership function and normal and convex. The advantages of the Perfectly normal Interval Type-2 fuzzy sets in Linear programming instead of Type-1 fuzzy sets allows us to handle higher uncertainty levels which come from typical scenarios where the problem is being defined by decision makers, and they are not in agreement of using a single fuzzy set for representing their perceptions about the problem. The rest of this paper is organized as follows. In Section 2 and 3, we recall some preliminary knowledge about fuzzy and its arithmetic operation. In section 4, the representation for PnIT2TrFN, its properties, and some arithmetic operations of PnIT2TrFN based on type 1 fuzzy number are presented. Section 5 has provided possibility, necessity, and credibility measures of PnIT2TrFN.In Section 6, we have proposed type-2 fuzzy linear programming models based on possibility, necessity, and credibility measures. The solution methodology of the proposed models using possibility, necessity, and credibility measures has been discussed in Section 7.In Section 8,a numerical example is presented to validate the proposed method. The numerical and graphical results at different possibility and necessity levels of the given problems have also been discussed here. Section 9 summarizes the paper and also discusses about the scope of future work.

2. PRELIMINARIES

In this section some basic definitions of fuzzy set theory are reviewed [14].

Definition 2.1

Let X be a non-empty set. A fuzzy set $\tilde{A}$ in X is characterized by its membership function $\mu_{\tilde{A}}: X \to [0,1]$ and $\mu_{\tilde{A}}(x)$ is interpreted as the degree of membership of element x in fuzzy set $\tilde{A}$ for each $x \in X$. It is clear that $\tilde{A}$ is completely determined by the set of tuples

$$\tilde{A} = \left\{ \left(x, \mu_{\tilde{A}}(x) \right) \middle| x \in X \right\}.$$

The family of all fuzzy (sub) sets in X is denoted by $F(X)$. Fuzzy subsets of the real line are called fuzzy quantities.

Definition 2.2

Let $\tilde{A}$ be a fuzzy subset of X : the support of $\tilde{A}$, denoted $Supp(\tilde{A})$, is the crisp subset of X whose elements all have non-zero membership grades in $\tilde{A}$.

$$Supp(\tilde{A}) = \left\{ x \in X \middle| \mu_{\tilde{A}}(x) > 0 \right\}.$$

Definition 2.3

A fuzzy subset $\tilde{A}$ of a classical set X is called normal if there exists an $x \in X$ such that $\mu_{\tilde{A}}(x) = 1$. Otherwise $\tilde{A}$ is subnormal.

Definition 2.4

An $\alpha - level$ set (or $\alpha - cut$) of a fuzzy set $\tilde{A}$ of X is a non-fuzzy set denoted by $\tilde{A}_\alpha$ and defined by

$$\tilde{A}_\alpha = \begin{cases} \left\{ x \in X \,\middle|\, \mu_{\tilde{A}}(x) \geq \alpha \right\} & if\, \alpha > 0 \\ cl\left(Supp\tilde{A}\right) & if\, \alpha = 0 \end{cases}$$

Where $cl\left(Supp\tilde{A}\right)$ denotes the closure of the support of $\tilde{A}$.

Definition 2.5

A fuzzy set $\tilde{A}$ of X is called convex if $\tilde{A}_\alpha$ is a convex subset of X for all $\alpha \in [0,1]$.

Definition 2.6

The complement of a fuzzy set $\tilde{A}$ is defined as $\mu_{\tilde{A}^c}(x) = 1 - \mu_{\tilde{A}}(x)$

Definition 2.7[15]

Let A,B be fuzzy sets with the membership function $\mu_{\tilde{A}} = \mathbb{R} \to [0,1]$, $\mu_{\tilde{B}} = \mathbb{R} \to [0,1]$, respectively,

$$Pos\left(\tilde{A} * \tilde{B}\right) = \sup\left\{\min\left(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(y)\right) \middle| x * y, x, y \in \mathbb{R}\right\}$$

$$Nec\left(\tilde{A} * \tilde{B}\right) = \inf\left\{\max\left(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(y)\right) \middle| x * y, x, y \in \mathbb{R}\right\}$$

Pos and Nec represent possibility of membership and necessity of membership function respectively. * is any of the relations $\prec, \succ, =, \pm$, and $=$. The dual relationship of possibility and necessity gives

$$Nec\left(\tilde{A} * \tilde{B}\right) = 1 - Pos\overline{\left(\tilde{A} * \tilde{B}\right)},$$

where $\overline{\left(\tilde{A} * \tilde{B}\right)}$ represents complement of the event $\bar{\tilde{A}} * \bar{\tilde{B}}$.

Definition 2.8[16]

Let $\tilde{A}$ be a fuzzy set. Then the fuzzy measures of $\tilde{A}$ for membership function is

$$Me_\mu\left\{\tilde{A}\right\} = \lambda Pos\left\{\tilde{A}\right\} + (1-\lambda) Nec\left\{\tilde{A}\right\}$$

where Me_μ represent measures of membership functions and $\lambda\left(0 \leq \lambda \leq 1\right)$ is the optimistic-pessimistic parameter to determine the combined attitude of a decision maker.

If $\lambda = 1$ then $Me_\mu = Pos$; it means the decision maker is optimistic and maximum chance of $\tilde{A}$ holds.

If $\lambda = 0$ then $Me_\mu = Nec$; it means the decision maker is pessimistic and maximum chance of $\tilde{A}$ holds.\newline

If $\lambda = 0.5$ then $Me_\mu = Cre$; where Cre is the credibility measure; it means the decision maker takes compromise attitude.

Definition 2.9[14][17]

A fuzzy number $\tilde{A}$ is a fuzzy set of the real line with a normal,(fuzzy)convex and continuous membership function of bounded support. Alternatively, the fuzzy subset $\tilde{A}$ of $\mathbb{R}$ is called a fuzzy number if the following conditions are satisfied:

1. $\tilde{A}$ is normal, i.e., there exist an $x \in \mathbb{R}$ such that $\mu_{\tilde{A}}(x) = 1$;
2. The membership function $\mu_{\tilde{A}}(x)$ is quasi-concave,\\ i.e., $\mu_{\tilde{A}}\left(\lambda x_1 + (1-\lambda) x_2\right) \geq \min\left\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\right\}$ for all $\alpha \in [0,1]$;
3. The membership function $\mu_{\tilde{A}}(x)$ is upper semi continuous, i.e., $\left\{x \in \mathbb{R} : \mu_{\tilde{A}}(x) \geq \alpha\right\}$ is a closed subset of $\mathbb{R}$ for all $\alpha \in [0,1]$;
4. The $0-level$ set $\tilde{A}_{\alpha=0}$ is compact (closed and bounded in $\mathbb{R}$)

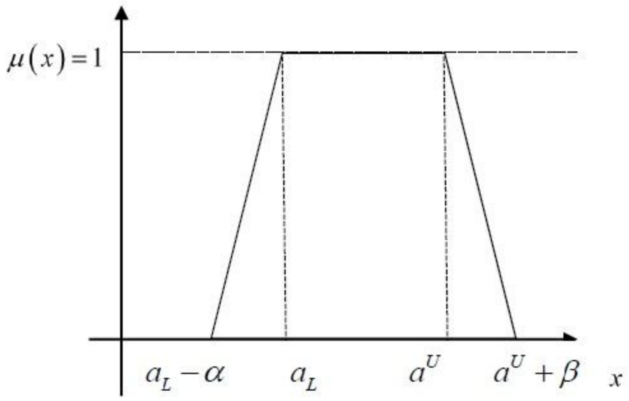

Figure 1 Trapezoidal Fuzzy Number

We denote by $F(\mathbb{R})$ the set of all fuzzy numbers. If $\tilde{A}$ is a fuzzy number, then, from Zadeh[2]. $\alpha-level$ set $\tilde{A}_\alpha$ is a convex set from condition (ii). Combining this fact with condition (iii), the $\alpha-level$ set $\tilde{A}_\alpha$ is a compact and convex set for all $\alpha\in[0,1]$ (since $\tilde{A}_{\alpha=0}$ is bounded, it says that $\tilde{A}_\alpha\subseteq\tilde{A}_{\alpha=0}$ is also bounded for all $\alpha\in(0,1]$). Therefore, we can write $\tilde{A}_\alpha=\left[a_\alpha^L,a_\alpha^U\right]$.

Definition 2.10

The trapezoidal fuzzy number is fully determined by quadruples $\left(a^L,a^U,\alpha,\beta\right)$ of crisp numbers such that $a^L\le a^U,\alpha\ge 0,\beta\ge 0$, whose membership function can be denoted by

$$\mu_{\tilde{A}}(x)=\begin{cases}\left(x-a^L+\alpha\right)/\alpha, & a^L-\alpha\le x\le a^L,\\ 1, & a^L\le x\le a^U,\\ -\left(x-a^U-\beta\right)/\beta & a^U\le x\le a^U+\beta\\ 0 & otherwise\end{cases}$$

when $a^L=a^U$,the trapezoidal fuzzy number becomes a triangular fuzzy number. If $\alpha=\beta$, the trapezoidal fuzzy number becomes a symmetrical trapezoidal fuzzy number. $\left[a^L,a^U\right]$ is the core of $\tilde{A}$, and $\alpha\ge 0,\beta\ge 0$ are the left-hand and right-hand spreads(See Figure \ref{fig:fig1}).It can easily be shown that

$$\tilde{A}_\alpha=\left[\alpha\left(a_L-\left(a_L-\alpha\right)\right)+\left(a_L-\alpha\right),-\alpha\left(\left(a^U+\beta\right)-a^U\right)+\left(a^U+\beta\right)\right]$$

The support of $\tilde{A}$ is $\left(a_L-\alpha,a^U+\beta\right)$

a. Arithmetic operations

In this sub section addition, subtraction and scalar multiplication operation of trapezoidal fuzzy numbers are reviewed [14][18]

Let $\tilde{A}=\left(a^L,a^U,\alpha,\beta\right)$ and $\tilde{B}=\left(b^L,b^U,\theta,\gamma\right)$ be two trapezoidal fuzzy numbers then

$$\tilde{A}=\left(a_L,a^U,\alpha,\beta\right)\ and\ \tilde{B}=\left(b_L,b^U,\gamma,\theta\right)$$

$$\tilde{A}+\tilde{B}=\left(a_L+b_L,a^U+b^U,\alpha+\gamma,\beta+\theta\right),$$

$$\tilde{A}-\tilde{B}=\left(a_L-b^U,a^U-b_L,\alpha+\theta,\beta+\gamma\right)$$

$$\lambda\tilde{A}=\begin{cases}\left(\lambda a_L,\lambda a^U,\lambda\alpha,\lambda\beta\right), & \lambda>0,\\ \left(\lambda a^U,\lambda a_L,-\lambda\beta,-\lambda\alpha\right), & \lambda<0.\end{cases}$$

3 INTERVAL TYPE-2 FUZZY SETS

An Interval type-2 fuzzy set (IT2FS) is a special case of type-2 fuzzy set, which play an important role in management and engineering applications. These fuzzy sets are characterized by their footprints of uncertainty.

Definition 3.1[19]

A Type-2 fuzzy set $\tilde{A}$ in the universe of discourse X can be represented by a type-2 membership function $\mu_{\tilde{A}}(x,u)$ as follows:

$$\tilde{A}=\left\{\left((x,u),\mu_{\tilde{A}}(x,u)\right)\middle|\forall x\in X,\forall u\in J_x\subseteq[0,1],0\le\mu_{\tilde{A}}(x,u)\le1\right\}$$

where $J_x\subseteq[0,1]$ is the primary membership function at x, and

$$\int_{u\in J_x}\mu_{\tilde{A}}(x,u)/u$$

indicates the second membership at x. For discreet situations, $\int$ is replaced by $\sum$.

Definition 3.2[20]

Let $\tilde{A}$ be a type-2 fuzzy set in the universe of discourse X represented by a type-2 membership function $\mu_A(x,u)$. If all $\mu_A(x,u)=1$, then $\tilde{A}$ is called an interval type-2 fuzzy set . An interval type-2 fuzzy set can be regarded as a special case of the type-2 fuzzy set, which is defined as

$$\tilde{A}=\int_{x\in X}\int_{u\in J_x}1/(x,u)=\int_{x\in X}\left[\int_{u\in J_x}1/u\right]/x$$

where x is the primary variable, $J_x\subseteq[0,1]$ is the primary membership of x,u is the secondary variable, and $\int_{u\in J_x}1/u$ is the secondary membership function at x.

It is obvious that the interval type-2 fuzzy set $\tilde{A}$ defined on X is completely determined by the primary membership which is called the footprint of uncertainty, and the footprint of uncertainty can be expressed as follows:

$$FOU(\tilde{A})=\bigcup_{x\in X}J_x=\bigcup_{x\in X}\left\{(x,u)\middle|u\in J_x\subseteq[0,1]\right\}$$

Definition 3.3[21][22]

Let $\tilde{A}$ be an interval type-2 fuzzy set, uncertainty in the primary membership of a type-2 fuzzy set consists of a bounded region called the footprint of uncertainty, which is the union of all primary memberships. Footprint of uncertainty is characterized by upper membership function and lower membership function. Both of the membership functions are type-1 fuzzy sets. Upper membership function is denoted by $0\le\overline{\mu}_{\tilde{A}}\le1$ and lower membership function is denoted by $0\le\underline{\mu}_{\tilde{A}}\le1$ respectively.

Definition 3.4[22]

An interval type-2 fuzzy number is called trapezoidal interval type-2 fuzzy number where the upper membership function and lower membership function are both trapezoidal fuzzy numbers, i.e.,

$$\tilde{A}=\left(A^L,A^U\right)=\left(\left(a_1^L,a_2^L,a_3^L,a_4^L;H_1\left(A^L\right),H_2\left(A^L\right)\right),\left(a_1^U,a_2^U,a_3^U,a_4^U;H_1\left(A^U\right),H_2\left(A^U\right)\right)\right),$$

where $H_j\left(A^L\right)$ and $H_j\left(A^U\right),(j=1,2)$ denote membership values of the corresponding elements a_{j+1}^L and a_{j+1}^U, $(j=1,2)$, respectively.

Definition 3.5[23]
The upper membership function and lower membership function of an interval type-2 fuzzy set are type-1 membership function, respectively.
Definition 3.6[24]
A interval type-2 fuzzy set, $\tilde{A}$, is said to be perfectly normal if both its upper and lower membership function are normal i.e.,

$$\sup \overline{\mu}_{\tilde{A}}(x) = \sup \underline{\mu}_{\tilde{A}} = 1.$$

4 PERFECTLY NORMAL IT2TRFN

In this section, the concepts of Perfectly normal interval type-2 trapezoidal fuzzy number (PnIT2TrFN) have been discussed. It is the extension work of Chiao[25][26].
Definition 4.1
A PnIT2TrFN $\tilde{A} = \left[A^L, A^U\right] = \left(\left(a_2^L, a_3^L, \alpha_L, \beta_L\right), \left(a_2^U, a_3^U, \alpha_U, \beta_U\right)\right)$ of crisp numbers such that $a_2^L \le a_3^L, a_2^U \le a_3^U, \alpha_L, \alpha^U \ge 0$ and $\beta_L, \beta^U \ge 0$. $\left[a_2^L, a_3^L\right]$ is the core of $\tilde{A}^L$, and $\alpha_L \ge 0$, $\beta_L \ge 0$ are the left-hand and right-hand spreads and $\left[a_2^U, a_3^U\right]$ is the core of $\tilde{A}^U$ and $\alpha_U \ge 0$, $\beta_U \ge 0$ are the left-hand and right-hand spreads such that the membership function are as follows(see Figure 2).:

$$\underline{\mu}_{\tilde{A}}(x) = \begin{cases} \left(x - a_2^L + \alpha_L\right)/\alpha_L, & a_2^L - \alpha_L \le x \le a_2^L, \\ 1, & a_2^L \le x \le a_3^L, \\ -\left(x - a_3^L - \beta_L\right)/\beta_L, & a_3^L \le x \le a_3^L + \beta_L, \\ 0, & otherwise. \end{cases}$$

$$\overline{\mu}_{\tilde{A}}(x) = \begin{cases} \left(x - a_2^U + \alpha_U\right)/\alpha_U, & a_2^U - \alpha_U \le x \le a_2^U, \\ 1, & a_2^U \le x \le a_3^U, \\ -\left(x - a_3^U - \beta_U\right)/\beta_U, & a_3^U \le x \le a_3^U + \beta_U, \\ 0, & otherwise. \end{cases}$$

Obviously, If $a_2^L = a_3^L$, $a_2^U = a_3^U$ the perfectly normal interval type-2 trapezoidal fuzzy number reduce to the perfectly normal interval type-2 triangular fuzzy number. If $A^L = A^U$, then the perfectly normal interval type-2 trapezoidal fuzzy number $\tilde{A}$ becomes a type-1 trapezoidal fuzzy number[23][27].

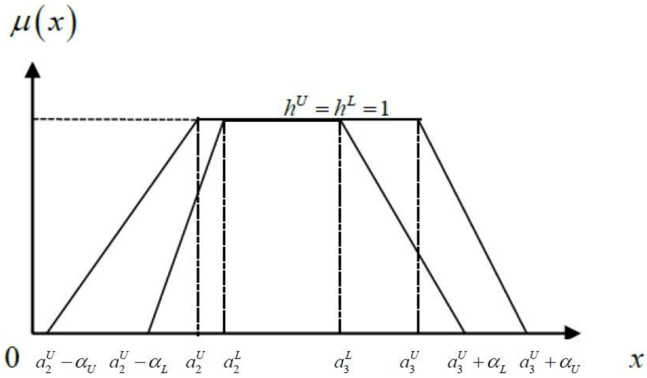

Figure 2 The upper Trapezoidal membership function $\tilde{A}^U$ and the lower trapezoidal membership function $\tilde{A}^L$ of the IT2FS $\tilde{A}$.

Definition 4.2 (Primary $\alpha - cut$ of an PnIT2FS)
The primary $\alpha - cut$ of an PnIT2FS is ${}^{\alpha}\tilde{A} = \left\{(x,u) \middle| J_x \ge \alpha, u \in [0,1]\right\}$ which is bounded by two regions

$$^{\alpha}\underline{\mu}_{\tilde{A}}(x)=\left\{\left(x,\underline{\mu}_{\tilde{A}}(x)\right)\middle|\underline{\mu}_{\tilde{A}}(x)\geq\alpha,\forall\alpha\in[0,1]\right\}$$

and

$$^{\alpha}\overline{\mu}_{\tilde{A}}(x)=\left\{\left(x,\overline{\mu}_{\tilde{A}}(x)\right)\middle|\overline{\mu}_{\tilde{A}}(x)\geq\alpha,\forall\alpha\in[0,1]\right\}.$$

Definition 4.3 (Crisp bounds of PnIT2TrFN)

The crisp bounds of the primary $\alpha-cut$ of the PnIT2TrFN $\tilde{A}=\left(\left(a_2^L,a_3^L,\alpha_L,\beta_L\right),\left(a_2^U,a_3^U,\alpha_U,\beta_U\right)\right)$ is closed interval $^{\alpha}\tilde{A}$ shall be obtained as follows $\forall\alpha\in(0,1]$ thus are defined as the upper and lower membership functions. The A^L and A^U are the lower and upper interval valued bounds of $\tilde{A}$. Also, let the bounds of $^{\alpha}A^L$ and $^{\alpha}A^U$ be defined as the boundaries of the $\alpha-cuts$ of each interval type-1 fuzzy set, as follows:

$$A_{\alpha}^L=\left[\inf_x {}^{\alpha}\underline{\mu}_{\tilde{A}}(x),\sup_x {}^{\alpha}\underline{\mu}_{\tilde{A}}(x)\right]=\left[a_l^L,a_u^L\right]$$

$$A_{\alpha}^U=\left[\inf_x {}^{\alpha}\overline{\mu}_{\tilde{A}}(x),\sup_x {}^{\alpha}\overline{\mu}_{\tilde{A}}(x)\right]=\left[a_l^U,a_u^U\right]$$

$$^{\alpha^{cb}}\tilde{A}=\left[\inf_x\left\{{}^{\alpha}\mu_{\tilde{A}}(x,u)\right\},\sup_x\left\{{}^{\alpha}\mu_{\tilde{A}}(x,u)\right\}\right]=\left[\left[a_l^U,a_l^L\right],\left[a_u^L,a_u^U\right]\right]=\left[\left[{}^{\alpha}\overline{a}^L,{}^{\alpha}\underline{a}^L\right],\left[{}^{\alpha}\underline{a}^R,{}^{\alpha}\overline{a}^R\right]\right]$$

which is equivalent to say

$$^{\alpha^{cb}}\mu_{\tilde{A}}\in\left[{}^{\alpha}\overline{a}^L,{}^{\alpha}\underline{a}^L\right],\left[{}^{\alpha}\underline{a}^R,{}^{\alpha}\overline{a}^R\right].$$

Evidently, from the Figure 3

$$^{\alpha}\overline{a}^L\leq{}^{\alpha}\underline{a}^L\leq{}^{\alpha}\underline{a}^R\leq{}^{\alpha}\overline{a}^R$$

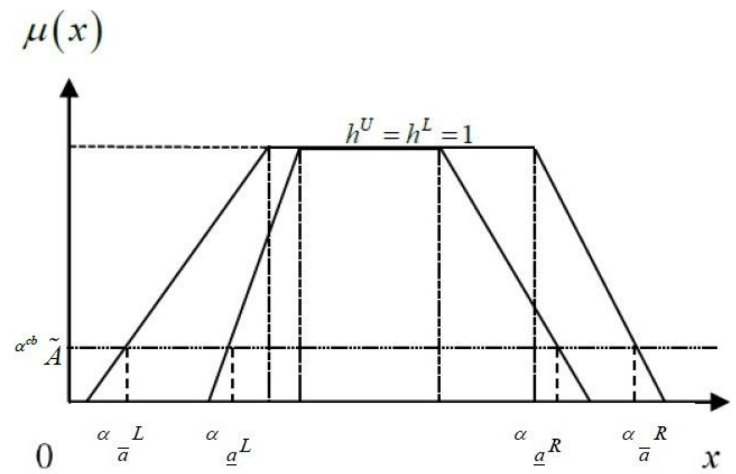

Figure 3 Crisp bounds of PnIT2TrFN

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Definition 4.4

A fuzzy number $\tilde{A}=\left(A^L,A^U\right)=\left(\left(a_2^L,a_3^L,\alpha_L,\beta_L\right),\left(a_2^U,a_3^U,\alpha_U,\beta_U\right)\right)$ is said to be non-negative PnIT2TrFN if $\overline{\mu}_{\tilde{A}}(x)=\underline{\mu}_{\tilde{A}}=0,\forall x>0.$

Definition 4.5

A fuzzy number $\tilde{A}=\left(A^{L},A^{U}\right)=\left(\left(a_{2}^{L},a_{3}^{L},\alpha_{L},\beta_{L}\right),\left(a_{2}^{U},a_{3}^{U},\alpha_{U},\beta_{U}\right)\right)$ is said to be non-positive PnIT2TrFN if $\overline{\mu}_{A}\left(x\right)=\underline{\mu}_{A}=0,\forall x<0.$

Definition 4.6

A fuzzy number $\tilde{A}=\left(A^{L},A^{U}\right)=\left(\left(a_{2}^{L},a_{3}^{L},\alpha_{L},\beta_{L}\right),\left(a_{2}^{U},a_{3}^{U},\alpha_{U},\beta_{U}\right)\right)$ is said to be zero PnIT2TrFN if $a_{2}^{L}=b_{2}^{L}=0,a_{3}^{L}=b_{3}^{L}=0$, $\alpha_{L}=\gamma_{L}=\beta_{L}=\theta_{L}=\alpha_{U}=\gamma_{U}=0$ and $\beta_{U}=\theta_{U}=0.$

Definition 4.7

A PnIT2TrFNs $\tilde{A}$ and $\tilde{B}=\left(B^{L},B^{U}\right)=\left(\left(b_{2}^{L},b_{3}^{L},\gamma_{L},\theta_{L}\right),\left(b_{2}^{U},b_{3}^{U},\gamma_{U},\theta_{U}\right)\right)$ are is said to be identically equal $\tilde{A}=\tilde{B}$ if and only if $a_{2}^{L}=b_{2}^{L},a_{3}^{L}=b_{3}^{L},\alpha_{L}=\gamma_{L},\beta_{L}=\theta_{L},\alpha_{U}=\gamma_{U}$ and $\beta_{U}=\theta_{U}$.

Arithmetic Operations on PnIT2TrFN

Definition 4.8

If $\tilde{A}$ and $\tilde{B}$ are PnIT2TrFNs, then $\tilde{C}=\tilde{A}+\tilde{B}$ is also a PnIT2TrFN and defined by

$$\tilde{C}=\left(\left(a_{2}^{L}+b_{2}^{L},a_{3}^{L}+b_{3}^{L},\alpha_{L}+\gamma_{L},\beta_{L}+\theta_{L}\right),\left(a_{2}^{U}+b_{2}^{U},a_{3}^{U}+b_{3}^{U},\alpha_{U}+\gamma_{U},\beta_{U}+\theta_{U}\right)\right)$$

Definition 4.9

If $\tilde{A}$ and $\tilde{B}$ are PnIT2TrFNs, then $\tilde{C}=\tilde{A}-\tilde{B}$ is also a PnIT2TrFN and defined by

$$\tilde{C}=\left(\left[a_{2}^{L}-b_{3}^{U},a_{3}^{L}-b_{2}^{U},\alpha_{L}+\theta_{U},\beta_{L}+\gamma_{U}\right],\left[a_{2}^{U}-b_{3}^{L},a_{3}^{U}-b_{2}^{L},\alpha_{U}+\theta_{L},\beta_{U}+\gamma_{L}\right]\right)$$

Definition 4.10

Let $\lambda\in R$. If $\tilde{A}$ is also a PnIT2TrFN and is given by

$$\tilde{C}=\lambda\tilde{A}=\begin{cases}\left(\left(\lambda a_{2}^{L},\lambda a_{3}^{L},\lambda\alpha_{L},\lambda\beta_{L}\right),\left(\lambda a_{2}^{U},\lambda a_{3}^{U},\lambda\alpha_{U},\lambda\beta_{U}\right)\right);if\quad\lambda\geq 0\\ \left(\left(\lambda a_{3}^{U},\lambda a_{2}^{U},|\lambda|\beta_{U},|\lambda|\alpha_{U}\right),\left(\lambda a_{3}^{L},\lambda a_{2}^{L},|\lambda|\beta_{L},|\lambda|\alpha_{L}\right)\right);if\quad\lambda<0\end{cases}$$

5. POSSIBILITY, NECESSITY, AND CREDIBILITY MEASURES OF PNIT2TRFN

Definition 5.1

Let $\tilde{A}$ and $\tilde{B}$ be two PnIT2TrFNs with lower and upper membership function, and $\mathbb{R}$ is the set of real numbers. Then the possibility degrees of lower and upper membership functions are defined as follows:

$$\underline{Pos}\left(A^{L}\preceq B^{L}\right)=\max\left(1-\max\left(\frac{\max\left({}^{\alpha}\underline{a}^{L}-{}^{\alpha}\underline{b}^{L},0\right)+\max\left(\left(a_{2}^{L}-b_{2}^{L}\right),0\right)+\max\left(\left(a_{3}^{L}-b_{3}^{L}\right),0\right)+\max\left({}^{\alpha}\underline{a}^{R}-{}^{\alpha}\underline{b}^{R},0\right)+\left({}^{\alpha}\underline{a}^{R}-{}^{\alpha}\underline{b}^{L}\right)}{\left|{}^{\alpha}\underline{a}^{L}-{}^{\alpha}\underline{b}^{L}\right|+\left|a_{2}^{L}-b_{2}^{L}\right|+\left|a_{3}^{L}-b_{3}^{L}\right|+\left|{}^{\alpha}\underline{a}^{R}-{}^{\alpha}\underline{b}^{R}\right|+\left({}^{\alpha}\underline{b}^{R}-{}^{\alpha}\underline{b}^{L}\right)+\left({}^{\alpha}\underline{a}^{R}-{}^{\alpha}\underline{a}^{L}\right)},0\right),0\right)\quad(1)$$

$$\overline{Pos}\left(A^{U}\preceq B^{U}\right)=\max\left(1-\max\left(\frac{\max\left({}^{\alpha}\overline{a}^{L}-{}^{\alpha}\overline{b}^{L},0\right)+\max\left(\left(a_{2}^{U}-b_{2}^{U}\right),0\right)+\max\left(\left(a_{3}^{U}-b_{3}^{U}\right),0\right)+\max\left({}^{\alpha}\overline{a}^{R}-{}^{\alpha}\overline{b}^{R},0\right)+\left({}^{\alpha}\overline{a}^{R}-{}^{\alpha}\overline{b}^{L}\right)}{\left|{}^{\alpha}\overline{a}^{L}-{}^{\alpha}\overline{b}^{L}\right|+\left|a_{2}^{U}-b_{2}^{U}\right|+\left|a_{3}^{U}-b_{3}^{U}\right|+\left|{}^{\alpha}\overline{a}^{R}-{}^{\alpha}\overline{b}^{R}\right|+\left({}^{\alpha}\overline{b}^{R}-{}^{\alpha}\overline{b}^{L}\right)+\left({}^{\alpha}\overline{a}^{R}-{}^{\alpha}\overline{a}^{L}\right)},0\right),0\right)\quad(2)$$

The dual relationship of possibility and necessity gives

$$\underline{Nec}\left(A^{L}\preceq B^{L}\right)=1-\underline{Pos}\left(A^{L}\succ B^{L}\right)\quad(3)$$

$$\overline{Nec}\left(A^{U}\preceq B^{U}\right)=1-\overline{Pos}\left(A^{U}\succ B^{U}\right)\quad(4)$$

Definition 5.2

Let $\tilde{A}$ ba a type-2 fuzzy number. Then the fuzzy measures of $\tilde{A}$ for membership function is

$$\underline{Me_{\mu}}\left\{\tilde{A}\right\}=\lambda\underline{Pos}\left\{\tilde{A}\right\}+\left(1-\lambda\right)\underline{Nec}\left\{\tilde{A}\right\}\quad(5)$$

$$\overline{Me_{\mu}}\left\{\tilde{A}\right\}=\lambda\overline{Pos}\left\{\tilde{A}\right\}+\left(1-\lambda\right)\overline{Nec}\left\{\tilde{A}\right\}\quad(6)$$

where $\underline{Me_\mu}$ and $\overline{Me_\mu}$ are represent measures of lower and upper membership functions and $\lambda\left(0\le\lambda\le1\right)$ is the optimistic-pessimistic parameter to determine the combined attitude of a decision maker.

If $\lambda=1,$ then $\underline{Me_\mu}=\underline{Pos}$, $\overline{Me_\mu}=\overline{Pos}$; it means the decision maker is optimistic and maximum chance of $\tilde{A}$ holds.

If $\lambda=0,$ then $\underline{Me_\mu}=\underline{Nec}$, $\overline{Me_\mu}=\overline{Nec}$; it means the decision maker is pessimistic and maximum chance of $\tilde{A}$ holds.

If $\lambda=0.5,$ then $\underline{Me_\mu}=\underline{Cre}$, $\overline{Me_\mu}=\overline{Cre}$; where Cre is the credibility measure; it means the decision maker takes compromise attitude.

Definition 5.3[14][27][28]

Let $\tilde{A}$ and $\tilde{B}$ be two PnIT2TrFNs,From Definition5.1 the possibility of lower and upper membership functions are as follows:

$$\underline{Pos}\left(A^L\preceq B^L\right)=\begin{cases}1 & a_2^L\le b_3^L\\ \dfrac{b_3^L-a_2^L+\theta_L+\alpha_L}{\theta_L+\alpha_L} & \left(a_2^L-\alpha_L\right)<\left(b_3^L+\theta_L\right),a_2^L>b_3^L\\ 0 & \left(b_3^L+\theta_L\right)\le\left(a_2^L-\alpha_L\right)\end{cases} \tag{7}$$

$$\overline{Pos}\left(A^U\preceq B^U\right)=\begin{cases}1, & a_2^U\le b_3^U,\\ \dfrac{b_3^U-a_2^U+\theta_U+\alpha_u}{\theta_U+\alpha_U}, & \left(a_2^U-\alpha_U\right)<\left(b_3^U+\theta_U\right),a_2^U>b_3^U\\ 0, & \left(b_3^U+\theta_L\right)\le\left(a_2^U-\alpha_L\right)\end{cases} \tag{8}$$

The possibility of the event $\tilde{A}\pm\ \tilde{B}$ for lower and upper membership function are as follows:

$$\underline{Pos}\left(A^L\pm\ B^L\right)=\begin{cases}1 & a_3^L\ge b_2^L\\ \dfrac{a_3^L-b_2^L+\beta_L+\gamma_L}{\beta_L+\gamma_L} & a_3^L<b_2^L,\left(a_3^L+\beta_L\right)>\left(b_2^L-\gamma_L\right)\\ 0 & \left(a_3^L+\beta_L\right)\le\left(b_2^L-\gamma_L\right)\end{cases} \tag{9}$$

$$\overline{Pos}\left(A^U\pm\ B^U\right)=\begin{cases}1, & a_3^U\ge b_2^U\\ \dfrac{a_3^U-b_2^U+\beta_U+\gamma_U}{\beta_L+\gamma_L}, & a_3^U<b_2^U,\left(a_3^U+\beta_U\right)>\left(b_2^U-\gamma_U\right)\\ 0, & \left(a_3^U+\beta_U\right)\le\left(b_2^U-\gamma_U\right)\end{cases} \tag{10}$$

Definition 5.4

Let $\tilde{A}$ and $\tilde{B}$ be two PnIT2TrFNs, From Definition5.1 the necessity of lower and upper membership functions are as follows:

$$\underline{Nec}\left(\tilde{A}^L\preceq\tilde{B}^L\right)=1-\underline{Pos}\left(\tilde{A}^L\succ\tilde{B}^L\right)=\begin{cases}0, & b_3^L\ge\left(a_2^L-\alpha_L\right),\\ \dfrac{b_2^L-a_3^L-\gamma_L}{\beta_L-\gamma_L}, & \left(b_2^L-\gamma_L\right)>a_3^L,\left(a_3^L+\beta_L\right)>b_2^L\\ 1 & b_2^L\ge\left(a_3^L+\beta_L\right).\end{cases} \tag{11}$$

$$\overline{Nec}\left(\tilde{A}^U \preceq \tilde{B}^U\right)=1-\overline{Pos}\left(\tilde{A}^U \succ \tilde{B}^U\right)=\begin{cases} 0, & b_3^U \geq \left(a_2^U-\alpha_U\right), \\ \dfrac{b_2^U-a_3^U-\gamma_U}{\beta_U-\gamma_U}, & \left(b_2^U-\gamma_U\right)>a_3^U, \left(a_3^L+\beta_L\right)>b_2^L \\ 1 & b_2^U \geq \left(a_3^U+\beta_U\right). \end{cases} \quad (12)$$

The necessity of the event $\tilde{A} \pm \tilde{B}$ for lower and upper membership function are as follows:

$$\underline{Nec}\left(A^L \pm B^L\right)=1-\underline{Pos}\left(A^L \prec B^L\right)=\begin{cases} 0, & a_2^L \leq \left(b_3^L+\theta_L\right), \\ \dfrac{a_2^L-b_3^L-\theta_L}{\alpha_L-\theta_L}, & a_2^L > \left(b_3^L+\theta_L\right), \left(a_2^L-\alpha_L\right)<b_3^L \\ 1 & \left(a_2^L-\alpha_L\right) \geq b_3^L. \end{cases} \quad (13)$$

$$\overline{Nec}\left(A^U \pm B^U\right)=1-\overline{Pos}\left(A^U \prec B^U\right)=\begin{cases} 0, & a_2^U \leq \left(b_3^U+\theta_U\right) \\ \dfrac{a_2^U-b_3^U-\theta_U}{\alpha_U-\theta_U}, & a_2^U > \left(b_3^U+\theta_U\right), \left(a_2^U-\alpha_U\right)<b_3^U \\ 1 & \left(a_2^U-\alpha_U\right) \geq b_3^U \end{cases} \quad (14)$$

Definition 5.5

Let $\tilde{A}$ and $\tilde{B}$ be two PnIT2TrFNs, By Definition 5.1 measures of the event upper and lower membership functions are as follows:

$$\underline{Me_\mu}\left(A^L \preceq B^L\right)=\lambda \underline{Pos}\left(A^L \preceq B^L\right)+(1-\lambda)\underline{Nec}\left(A^L \preceq B^L\right)$$

$$=\begin{cases} 0, & \left(b_3^L+\theta_L\right) \leq \left(a_2^L-\alpha_L\right), \\ \lambda \dfrac{b_3^L-a_2^L+\alpha_L+\theta_L}{\alpha_L+\theta_L}, & b_3^L > a_2^L, \left(b_2^L-\gamma_L\right)<a_2^L, \\ \lambda, & b_3^L > a_2^L, \left(b_2^L-\gamma_L\right)<a_3^L, \\ \lambda+(1-\lambda)\dfrac{b_2^L-a_3^L-\gamma_L}{\beta_L-\gamma_L}, & \left(b_2^L-\gamma_L\right)>a_3^L, \left(a_3^L+\theta_L\right)>b_2^L \\ 1, & b_2^L \geq \left(a_3^L+\beta_L\right) \end{cases} \quad (15)$$

$$\overline{Me_\mu}\left(A^U \preceq B^U\right)=\lambda \overline{Pos}\left(A^U \preceq B^U\right)+(1-\lambda)\overline{Nec}\left(A^U \preceq B^U\right)$$

$$=\begin{cases} 0, & \left(b_3^U+\theta_U\right) \leq \left(a_2^U-\alpha_U\right), \\ \lambda \dfrac{b_3^U-a_2^U+\alpha_U+\theta_U}{\alpha_U+\theta_U}, & b_3^U > a_2^U, \left(b_2^U-\gamma_U\right)<a_2^U, \\ \lambda, & b_3^U > a_2^U, \left(b_2^U-\gamma_U\right)<a_3^U, \\ \lambda+(1-\lambda)\dfrac{b_2^U-a_3^U-\gamma_U}{\beta_U-\gamma_U}, & \left(b_2^U-\gamma_U\right)>a_3^U, \left(a_3^U+\theta_U\right)>b_2^U, \\ 1, & b_2^U \geq \left(a_3^U+\beta_U\right) \end{cases} \quad (16)$$

The measures of event $\tilde{A} \pm \tilde{B}$ for lower and upper membership function are as follows:

$$\underline{Me_{\mu}}\left(A^{L} \pm B^{L}\right)=\lambda \underline{Pos}\left(A^{L} \pm B^{L}\right)+(1-\lambda)\underline{Nec}\left(A^{L} \pm B^{L}\right)$$

$$=\begin{cases} 1, & b_3^L \le a_2^L, \\ (1-\lambda)\dfrac{a_2^L-b_3^L-\theta_L}{\alpha_L-\theta_L}, & b_3^L > \left(a_2^L-\alpha_L\right), \left(b_3^L+\theta_L\right) < a_2^L, \\ \lambda, & \left(b_3^L+\theta_L\right) > a_2^L, b_2^L < a_3^L, \\ \lambda\dfrac{a_3^L-b_2^L+\alpha_L+\beta_L}{\beta_L+\gamma_L}, & a_3^L < a_2^L, \left(a_3^L+\beta_L\right) > \left(b_2^L-\gamma_L\right) \\ 0, & \left(b_2^L-\gamma_L\right) \ge \left(a_3^L+\beta_L\right) \end{cases} \quad (17)$$

$$\overline{Me_{\mu}}\left(A^{U} \pm B^{U}\right)=\lambda \overline{Pos}\left(A^{U} \pm B^{U}\right)+(1-\lambda)\overline{Nec}\left(A^{U} \pm B^{U}\right)$$

$$=\begin{cases} 1, & b_3^U \le a_2^U, \\ (1-\lambda)\dfrac{a_2^U-b_3^U-\theta_U}{\alpha_U-\theta_U}, & b_3^U > \left(a_2^U-\alpha_U\right), \left(b_3^U+\theta_U\right) < a_2^U, \\ \lambda, & \left(b_3^U+\theta_U\right) > a_2^U, b_2^U < a_3^U, \\ \lambda\dfrac{a_3^U-b_2^U+\alpha_U+\beta_U}{\beta_U+\gamma_U}, & a_3^U < a_2^U, \left(a_3^U+\beta_U\right) > \left(b_2^U-\gamma_U\right) \\ 0, & \left(b_2^U-\gamma_U\right) \ge \left(a_3^U+\beta_U\right) \end{cases} \quad (18)$$

For $\lambda = 0.5$

$$\underline{Cre}\left(A^{L} \preceq B^{L}\right)=\begin{cases} 0, & \left(b_2^L-\gamma_L\right) \le \left(a_2^L-\alpha_L\right), \\ \dfrac{b_3^L-a_2^L+\alpha_L+\theta_L}{2\left(\alpha_L+\theta_L\right)}, & \left(b_3^L+\theta_L\right) > \left(a_2^L-\alpha_L\right), a_2^L > b_3^L, \\ \dfrac{1}{2}, & b_3^L > a_2^L, \left(b_2^L-\gamma_L\right) < a_3^L, \\ \dfrac{\beta_L-a_3^L+b_2^L-2\gamma_L}{2\left(\beta_L-\gamma_L\right)}, & \left(b_2^L-\gamma_L\right) > a_3^L, \left(a_3^L+\beta_L\right) > b_2^L, \\ 1, & b_2^L \ge \left(a_3^L+\beta_L\right) \end{cases} \quad \textbf{(19)}$$

$$\overline{Cre}\left(A^{U} \preceq B^{U}\right)=\begin{cases} 0, & \left(b_2^U-\gamma_U\right) \le \left(a_2^U-\alpha_U\right), \\ \dfrac{b_3^U-a_2^U+\alpha_U+\theta_U}{2\left(\alpha_U+\theta_U\right)}, & \left(b_3^U+\theta_U\right) > \left(a_2^U-\alpha_U\right), a_2^U > b_3^U, \\ \dfrac{1}{2}, & b_3^U > a_2^U, \left(b_2^U-\gamma_U\right) < a_3^U, \\ \dfrac{\beta_U-a_3^U+b_2^U-2\gamma_U}{2\left(\beta_U-\gamma_U\right)}, & \left(b_2^U-\gamma_U\right) > a_3^U, \left(a_3^U+\beta_U\right) > b_2^U, \\ 1, & b_2^U \ge \left(a_3^U+\beta_U\right) \end{cases} \quad \textbf{(20)}$$

$$\underline{Cre}\left(A^L \pm B^L\right)=\begin{cases} 1, & b_3^L \le a_2^L, \\ \dfrac{a_2^L - b_3^L - 2\theta_L + \alpha_L}{2\left(\alpha_L - \theta_L\right)}, & b_3^L > \left(a_2^L - \alpha_L\right), a_2^L > \left(b_3^L + \theta_L\right), \\ \dfrac{1}{2}, & \left(b_3^L + \theta_L\right) > a_2^L, b_2^L < a_3^L, \\ \dfrac{a_3^L - b_2^L + \beta_L + \gamma_L}{2\left(\beta_L + \gamma_L\right)}, & b_2^L > a_3^L, \left(a_3^L + \beta_L\right) > \left(b_2^L - \gamma_L\right) \\ 0, & \left(b_2^L - \gamma_L\right) \ge \left(a_3^L + \beta_L\right) \end{cases} \quad (21)$$

$$\overline{Cre}\left(A^U \pm B^U\right)=\begin{cases} 1, & b_3^U \le a_2^U, \\ \dfrac{a_2^U - b_3^U - 2\theta_U + \alpha_U}{2\left(\alpha_U - \theta_U\right)}, & b_3^U > \left(a_2^U - \alpha_U\right), a_2^U > \left(b_3^U + \theta_U\right), \\ \dfrac{1}{2}, & \left(b_3^U + \theta_U\right) > a_2^U, b_2^U < a_3^U, \\ \dfrac{a_3^U - b_2^U + \beta_U + \gamma_U}{2\left(\beta_U + \gamma_U\right)}, & b_2^U > a_3^U, \left(a_3^U + \beta_U\right) > \left(b_2^U - \gamma_U\right) \\ 0, & \left(b_2^U - \gamma_U\right) \ge \left(a_3^U + \beta_U\right) \end{cases} \quad (22)$$

Theorem 5.6

Let $\tilde{A}$ and $\tilde{B}$ be two PnIT2TrFNs, $p \in (0,1], \underline{Pos}\left(\tilde{A}^L \preceq \tilde{B}^L\right) \ge p,$ if and only if $b_3^L - a_2^L \ge \left(p-1\right)\left(\theta_L + \alpha_L\right)$ and $\overline{Pos}\left(\tilde{A}^U \preceq \tilde{B}^U\right) \ge p,$ if and only if $b_3^U - a_2^U \ge \left(p-1\right)\left(\theta_U + \alpha_U\right).$

Proof :

If $p=1,$ then from $\underline{Pos}\left(\tilde{A}^L \preceq \tilde{B}^L\right) \ge 1$ one can get that $b_3^L \ge a_2^L$, and vice versa. If $0 < p < 1,$ then $b_3^L < a_2^L$ and $b_3^L + \theta_L > a_2^L - \alpha_L, \underline{Pos}\left(\tilde{A}^L \preceq \tilde{B}^L\right) \ge p$ if and only if $\dfrac{b_3^L - a_2^L + \theta_L + \alpha_L}{\theta_L + \alpha_L} \ge p$, that is $b_3^L - a_2^L \ge \left(p-1\right)\left(\theta_L + \alpha_L\right)$. Similarly for $\overline{Pos}\left(\tilde{A}^U \preceq \tilde{B}^U\right) \ge p,$ if and only if $b_3^U - a_2^U \ge \left(p-1\right)\left(\theta_U + \alpha_U\right).$

Theorem 5.7

Let $\tilde{A}$ and $\tilde{B}$ be two PnIT2TrFNs, $p \in (0,1], \underline{Nec}\left(A^L \preceq B^L\right) \ge p,$ if and only if $b_2^L - a_3^L \ge p\left(\beta_L - \gamma_L\right) + \gamma_L$ and $\underline{Nec}\left(A^U \preceq B^U\right) \ge p,$ if and only if $b_2^U - a_3^U \ge p\left(\beta_U - \gamma_U\right) + \gamma_U.$

Proof: If $p=1,$ then from $\underline{Nec}\left(A^L \preceq B^L\right) \ge 1$ we can get that $b_2^L \ge a_3^L,$ and vice versa. If $0 < p < 1,$ then $b_2^L - a_3^L > \gamma_L$ and $a_3^L + \beta_L < b_2^L, Nec\left(A^L \preceq B^L\right) \ge p$ if and only if $\dfrac{b_2^L - a_3^L - \gamma_L}{\beta_L - \gamma_L} \ge p,$

i.e., $b_2^L - a_3^L \ge p\left(\beta_L - \gamma_L\right) + \gamma_L.$ Similarly for

$\overline{Nec}\left(A^U \preceq B^U\right) \ge p,$ if and only if $b_2^U - a_3^U \ge p\left(\beta_U - \gamma_U\right) + \gamma_U.$

Theorem 5.8

If $\tilde{A}$ and $\tilde{B}$ be two PnIT2TrFNs, $p \in (0,1], \underline{Cre}\left(A^L \preceq B^L\right) \ge p,$ if and only if

$b_3^L - a_2^L \ge \left(2p-1\right)\left(\theta_L + \alpha_L\right), b_2^L - a_3^L + \alpha_L - 2\gamma_L \ge 2p\left(\alpha_L - \gamma_L\right).$

Proof:

Let us consider $\underline{Cre}\left(A^L \preceq B^L\right) \geq p,$ Now, from equation 19,

$$\underline{Cre}\left(A^L \preceq B^L\right) \geq p \Leftrightarrow \frac{b_3^L - a_2^L + \theta_L + \alpha_L}{2\left(\theta_L + \alpha_L\right)} \geq p, \frac{b_2^L - a_3^L + \alpha_L - 2\gamma_L}{2\left(\alpha_L - \gamma_L\right)} \geq p.$$

Remark :

$$\underline{Cre}\left(A^L \preceq x\right) \geq p \Leftrightarrow \left(\frac{x - \left(a_2^L - \alpha_L\right)}{2\alpha_L}\right) \geq p, \left(\frac{\beta_L - a_3^L + x}{2\beta_L}\right) \geq p.$$

6. TYPE-2 FUZZY LINEAR PROGRAMMING MODELS

In this section, we propose a type-2 fuzzy linear programming models based on Chance-Constrained Programming Models(CCM)[29] with Type-2 fuzzy parameter. We can use the chance operator (possibility or necessity or credibility measure) to transform the type-2 fuzzy model in to crisp linear programming model. A general single objective linear programming model with Type-2 fuzzy parameter should have the following form:

$$\begin{aligned} & Opt\ \ f\left(x, \tilde{c}_j\right) \\ & Subject\ to\ g_i\left(x, \tilde{a}_{ij}\right) \preceq \tilde{b}_i, x \geq 0, \end{aligned} \qquad (23)$$

where $\tilde{A} = \left(\tilde{a}_{ij}\right)_{m\times n}, \tilde{b} = \left(\tilde{b}_1, \tilde{b}_2, \ldots \tilde{b}_m\right)^T$, $\tilde{c} = \left(\tilde{c}_1, \tilde{c}_2, \ldots, \tilde{c}_n\right)$, $x = \left(x_1, x_2, \ldots, x_n\right)^T$ and $\tilde{a}_{ij}, \tilde{b}_i, \tilde{c}_j$ are an PnIT2FN and $x_j \in □\ \left(i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n\right)$. $\preceq$ is a Type-2 fuzzy partial order.

Definition 6.1

Consider a set of right-hand-side(resources) parameters of a Fuzzy linear programming problem define as an PnIT2FS $\tilde{b}$ defined on the closed interval

$$\tilde{b}_i \in \left[\left[{}^{\alpha}\overline{b}_i^{\,L}, {}^{\alpha}\underline{b}_i^{\,L}\right], \left[{}^{\alpha}\overline{b}_i^{\,R}, {}^{\alpha}\underline{b}_i^{\,R}\right]\right] \in □$$

and $i \in □_n$. The membership function which represents the fuzzy space $Supp\left(\tilde{b}_i\right)$ is

$$\tilde{b}_i = \int_{b_i \in □} \left[\int_{u \in J_{b_i}} 1/u\right] / b_i\ , i \in □_n, J_{b_i} \subseteq [0,1].$$

Here, $\tilde{b}$ is bounded by both lower and upper primary membership function, namely

$${}^{\alpha}\overline{\mu}_{\tilde{b}_i} = \left\{\left(b_i, u\right) \middle| \overline{\mu}_{\tilde{b}_i} \geq \alpha\right\}$$

with parameter ${}^{\alpha}\underline{b}_i^{\,L}$ &${}^{\alpha}\ \underline{b}_i^{\,R}$ and

$${}^{\alpha}\underline{\mu}_{\tilde{b}_i} = \left\{\left(b_i, u\right) \middle| \underline{\mu}_{\tilde{b}_i} \geq \alpha\right\},$$

with parameter ${}^{\alpha}\overline{b}_i^{\,L}$ and ${}^{\alpha}\overline{b}_i^{\,R}$.

Definition 6.2

Consider a technological coefficient of an fuzzy linear programming problem define as an PnIT2FS $\tilde{a}_{ij}$ defined on the closed interval

$$\tilde{a}_{ij} \in \left[\inf_{a_{ij}}\left\{{}^{\alpha}\mu_{\tilde{a}_{ij}}\left(a_{ij}, u\right)\right\}, \sup_{a_{ij}}\left\{{}^{\alpha}\mu_{\tilde{a}_{ij}}\left(a_{ij}, u\right)\right\}\right] = \left[\left[{}^{\alpha}\overline{a}_{ij}^{\,L}, {}^{\alpha}\underline{a}_{ij}^{\,L}\right], \left[{}^{\alpha}\overline{a}_{ij}^{\,R}, {}^{\alpha}\underline{a}_{ij}^{\,R}\right]\right] \in □, i \in □_n, j \in □_m$$

The membership function which represents the fuzzy space $Supp\left(\tilde{a}_{ij}\right)$ is

$$\tilde{a}_{ij} = \int_{\tilde{a}_{ij} \in \mathbb{R}} \left[\int_{u \in J_{\tilde{a}_{ij}}} 1/u \right] / \tilde{a}_{ij} \ , i \in \mathbb{N}_n, j \in \mathbb{N}_m, J_{\tilde{a}_{ij}} \subseteq [0,1].$$

Here, $\tilde{a}_{ij}$ is bounded by both lower and upper primary membership function, namely

$$^{\alpha}\overline{\mu}_{\tilde{a}_{ij}} = \left\{ \left(a_{ij}, u\right) \middle| \overline{\mu}_{\tilde{a}_{ij}} \geq \alpha \right\}$$

with parameter $^{\alpha}\underline{a}_{ij}^{L}$ & $^{\alpha}\underline{a}_{ij}^{R}$ and

$$^{\alpha}\underline{\mu}_{\tilde{a}_{ij}} = \left\{ \left(a_{ij}, u\right) \middle| \underline{\mu}_{\tilde{a}_{ij}} \geq \alpha \right\}$$

with parameter $^{\alpha}\overline{a}_{ij}^{L}$ &$^{\alpha}\,\overline{a}_{ij}^{R}$.

Definition 6.3

Consider a profit or cost coefficient of an fuzzy linear programming problem define as an PnIT2FS $\tilde{c}$ defined on the closed interval

$$\tilde{c}_j \in \left[\left[{}^{\alpha}\overline{c}_j^{L}, {}^{\alpha}\underline{c}_j^{L} \right], \left[{}^{\alpha}\overline{c}_j^{R}, {}^{\alpha}\underline{c}_j^{R} \right] \right] \in \mathbb{R}$$

and $j \in \mathbb{N}_n$. The membership function which represents the fuzzy space $Supp\left(\tilde{c}_j\right)$ is

$$\tilde{c}_j = \int_{c_j \in \mathbb{R}} \left[\int_{u \in J_{c_j}} 1/u \right] / c_j \ , j \in \mathbb{N}_n, J_{c_j} \subseteq [0,1].$$

Here, $\tilde{c}$ is bounded by both lower and upper primary membership function, namely

$$^{\alpha}\overline{\mu}_{\tilde{c}_j} = \left\{ \left(c_j, u\right) \middle| \overline{\mu}_{\tilde{c}_j} \geq \alpha \right\}$$

with parameter $^{\alpha}\underline{c}_j^{L}$ &$^{\alpha}\,\underline{c}_j^{R}$ and

$$^{\alpha}\underline{\mu}_{\tilde{c}_j} = \left\{ \left(\tilde{c}_j, u\right) \middle| \underline{\mu}_{\tilde{c}_j} \geq \alpha \right\},$$

with parameter $^{\alpha}\overline{c}_j^{L}$ and $^{\alpha}\overline{c}_j^{R}$.

6.1 Type-2 Fuzzy Linear programming model with PnIT2FN

The general type-2 fuzzy linear programming models based on CCM with PnIT2TrFN for model equation (23) is as follows:

$$\begin{aligned}
&Opt\ \frac{1}{2}(f_L+f_U)\\
&Subject\ to\ \mathrm{Ch}_\mu^L\left\{f\left(x,(\tilde{c}_j)^L\right)\pm\ f_L\right\}\geq p\\
&\mathrm{Ch}_\mu^U\left\{f\left(x,(\tilde{c}_j)^U\right)\pm\ f_U\right\}\geq p\\
&\mathrm{Ch}_\mu^L\left\{g\left(x,(\tilde{a}_{ij})^L\right)\preceq(\tilde{b}_i)^L\right\}\geq p\\
&\mathrm{Ch}_\mu^U\left\{g\left(x,(\tilde{a}_{ij})^U\right)\preceq(\tilde{b}_i)^U\right\}\geq p\\
&x\geq 0, i=1,2,..,m;\ j=1,2,...,n.
\end{aligned} \tag{24}$$

where $\tilde{a}_{ij}=\left((\tilde{a}_{ij})^L,(\tilde{a}_{ij})^U\right)=\left(\left((a_{ij})_2^L,(a_{ij})_3^L,(\alpha_{ij})_L,(\beta_{ij})_L\right),\left((a_{ij})_2^U,(a_{ij})_3^U,(\alpha_{ij})_U,(\beta_{ij})_U\right)\right)$,

$\tilde{b}_i=\left((\tilde{b}_i)^L,(\tilde{b}_i)^U\right)=\left(\left((b_i)_2^L,(b_i)_3^L,(\gamma_i)_L,(\theta_i)_L\right),\left((b_i)_2^U,(b_i)_3^U,(\gamma_i)_U,(\theta_i)_U\right)\right)$ and

$\tilde{c}_i=\left((\tilde{c}_i)^L,(\tilde{c}_i)^U\right)=\left(\left((c_i)_2^L,(c_i)_3^L,(\delta_i)_L,(\eta_i)_L\right),\left((c_i)_2^U,(c_i)_3^U,(\delta_i)_U,(\eta_i)_U\right)\right)$

are PnIT2TrFNs, x_i are the decision variable. The abbreviations Ch_μ^L and Ch_μ^U represent chance operator (i.e., possibility or necessity or credibility measure for) lower and upper membership functions is predetermined confidence levels such that $0\leq p\leq 1$ for $i=1,2,..,m;\ j=1,2,...,n.$

6.1.1 Type-2 fuzzy linear programming model based on possibility measure.

The Type-2 fuzzy linear programming model with PnIT2TrFN (24) based on CCM and possibility measure is as follows:

$$\begin{aligned}
&Opt\frac{1}{2}(f_L+f_U)\\
&Subject\ to\ \underline{Pos}\left\{f\left(x,\tilde{c}_j\right)\pm\ f_L\right\}\geq p\\
&\overline{Pos}\left\{f\left(x,\tilde{c}_j\right)\pm\ f_U\right\}\geq p\\
&\underline{Pos}\left\{g\left(x,\tilde{a}_{ij}\right)\preceq b_i^L\right\}\geq p\\
&\overline{Pos}\left\{g\left(x,\tilde{a}_{ij}\right)\preceq b_i^U\right\}\geq p\\
&x\geq 0, i=1,2,...,m;\ j=1,2,...,n.
\end{aligned} \tag{25}$$

Where p is the predetermined confidence level such that $0\leq p\leq 1$ for $i=1,2,...,m;\ j=1,2,...,n.$

Definition 6.4

A solution x^* of the problem equation (25) satisfies $\underline{Pos}\left\{g\left(x,\tilde{a}_{ij}\right)\leq b_i^L\right\}\geq p$ and $\overline{Pos}\left\{g\left(x,\tilde{a}_{ij}\right)\leq b_i^U\right\}\geq p, i=1,2,...,m;\ j=1,2,...,n.$ is called a feasible solution at $p-cut$ possibility level.

Definition 6.5

A feasible solution at $p-cut$ possibility level, x^*, is said to be $p-cut$ efficient solution for problem (25) if and only if there exists no other feasible solution at $p-cut$ possibility level, such that $\underline{Pos}\left\{f\left(x,\tilde{c}_j\right)\geq f_L\right\}\geq p$ and

$\overline{Pos}\left\{f\left(x,\tilde{c}_j\right)\geq f_U\right\}\geq p$ with $f(x)\geq\frac{1}{2}\left(f_L(x^*)+f_U(x^*)\right)$

6.1.2 Type-2 fuzzy linear programming model based on necessity measure.

The Type-2 fuzzy linear programming model with PnIT2TrFN (24) based on CCM and necessity measure is as follows:

$$\begin{aligned} & Opt\frac{1}{2}\left(f_L + f_U \right) \\ & Subject\ to\ \underline{Nec}\left\{ f\left(x,\tilde{c}_j\right) \pm\ f_L \right\} \geq p \\ & \overline{Nec}\left\{ f\left(x,\tilde{c}_j\right) \pm\ f_U \right\} \geq p \\ & \underline{Nec}\left\{ g\left(x,\tilde{a}_{ij}\right) \preceq b_i^L \right\} \geq p \\ & \overline{Nec}\left\{ g\left(x,\tilde{a}_{ij}\right) \preceq b_i^U \right\} \geq p \\ & x \geq 0, i = 1,2,...,m; j = 1,2,...,n. \end{aligned} \qquad (26)$$

where p is the predetermined confidence level such that $0 \leq p \leq 1$ for $i = 1,2,...,m; j = 1,2,...,n.$

Definition 6.6

A solution x^* of the problem equation (26) satisfies $\underline{Nec}\left\{ g\left(x,\tilde{a}_{ij}\right) \leq b_i^L \right\} \geq p$ and $\overline{Nec}\left\{ g\left(x,\tilde{a}_{ij}\right) \leq b_i^U \right\} \geq p,$ $i = 1,2,...,m; j = 1,2,...,n.$ is called a feasible solution at $p - cut$ necessity level

Definition 6.7

A feasible solution at $p - cut$ necessity level, x^*, is said to be $p - cut$ efficient solution for problem (26) if and only if there exists no other feasible solution at $p - cut$ necessity level, such that $\underline{Nec}\left\{ f\left(x,\tilde{c}_j\right) \geq f_L \right\} \geq \alpha$ and $\overline{Nec}\left\{ f\left(x,\tilde{c}_j\right) \geq f_U \right\} \geq \alpha$ with $f\left(x\right) \geq \frac{1}{2}\left(f_L\left(x^*\right) + f_U\left(x^*\right)\right)$

6.1.3 Type-2 fuzzy linear programming model based on credibility measure.

The Type-2 fuzzy linear programming model with PnIT2TrFN (24) based on CCM and credibility measure is as follows:

$$\begin{aligned} & Opt\frac{1}{2}\left(f_L + f_U \right) \\ & Subject\ to\ \underline{Cre}\left\{ f\left(x,\tilde{c}_j\right) \pm\ f_L \right\} \geq p \\ & \overline{Cre}\left\{ f\left(x,\tilde{c}_j\right) \pm\ f_U \right\} \geq p \\ & \underline{Cre}\left\{ g\left(x,\tilde{a}_{ij}\right) \preceq b_i^L \right\} \geq p \\ & \overline{Cre}\left\{ g\left(x,\tilde{a}_{ij}\right) \preceq b_i^U \right\} \geq p \\ & x \geq 0, i = 1,2,...,m; j = 1,2,...,n. \end{aligned} \qquad (27)$$

Where $ p $ is the predetermined confidence level such that $0 \leq p \leq 1$ for $i = 1,2,...,m; j = 1,2,...,n.$

Definition 6.8

A solution x^* of the problem equation (27) satisfies $\underline{Cre}\left\{ g\left(x,\tilde{a}_{ij}\right) \leq b_i^L \right\} \geq p$ and $\overline{Cre}\left\{ g\left(x,\tilde{a}_{ij}\right) \leq b_i^U \right\} \geq p, i = 1,2,...,m; j = 1,2,...,n.$ is called a feasible solution at $p - cut$ credibility level.

Definition 6.9

A feasible solution at $p - cut$ credibility level, x^*, is said to be $p - cut$ efficient solution for problem (27) if and only if there exists no other feasible solution at $p - cut$ credibility level, such that $\underline{Cre}\left\{ f\left(x,\tilde{c}_j\right) \geq f_L \right\} \geq p$ and

$$\overline{Cre}\left\{ f\left(x,\tilde{c}_j\right) \geq f_U \right\} \geq p \text{ with } f\left(x\right) \geq \frac{1}{2}\left(f_L\left(x^*\right) + f_U\left(x^*\right)\right)$$

7 PROPOSED METHOD TO SOLVE TYPE-2 FUZZY LINEAR PROGRAMMING MODELS:

To solve type 2 fuzzy linear programming model based on possibility or necessity or credibility measures we propose the following method.

Step 1. Apply chance operator possibility/necessity/credibility in type -2 fuzzy linear programming model (23) can be converted into following form.

$$Opt\frac{1}{2}\{f_L+f_U\}$$

$$\begin{aligned}&Subject\ to\ \underline{Pos}\left\{f\left(x,\tilde{c}_j\right)\geq f_L\right\}\geq p\\&\overline{Pos}\left\{f\left(x,\tilde{c}_j\right)\geq f_U\right\}\geq p\\&or\quad \underline{Nec}\left\{f\left(x,\tilde{c}_j\right)\geq f_L\right\}\geq p\\&\overline{Nec}\left\{f\left(x,\tilde{c}_j\right)\geq f_U\right\}\geq p\\&or\quad \underline{Cre}\left\{f\left(x,\tilde{c}_j\right)\geq f_L\right\}\geq p\\&\overline{Cre}\left\{f\left(x,\tilde{c}_j\right)\geq f_U\right\}\geq p\end{aligned}\qquad (28)$$

$$\begin{aligned}&\underline{Pos}\left\{g\left(x,\tilde{a}_{ij}\right)\preceq b_i^L\right\}\geq p\\&\overline{Pos}\left\{g\left(x,\tilde{a}_{ij}\right)\preceq b_i^U\right\}\geq p\\&or\quad \underline{Nec}\left\{g\left(x,\tilde{a}_{ij}\right)\preceq b_i^L\right\}\geq p\\&\overline{Nec}\left\{g\left(x,\tilde{a}_{ij}\right)\preceq b_i^U\right\}\geq p\\&or\quad \underline{Cre}\left\{g\left(x,\tilde{a}_{ij}\right)\preceq b_i^L\right\}\geq p\\&\overline{Cre}\left\{g\left(x,\tilde{a}_{ij}\right)\preceq b_i^U\right\}\geq p\end{aligned}\qquad (29)$$

$$x\geq 0, 0\leq p\leq 1 \qquad (30)$$

for i=1,2,...,m;j=1,2,...,n.where p is the predefined confidence level.

Step 2. Using Theorems 5.6, 5.7 and/or Theorem 5.8, the above problem in Step 1 can also be written as

$$\begin{aligned}&Opt\frac{1}{2}\{f_L+f_U\}\\&Subject\ to\ f_L+f_U\geq Z\\&(29)-(30)\end{aligned}\qquad (31)$$

where Z is obtained by applying Theorems.5.6, 5.7 and/or Theorem 5.8 in (28)

Step 3. The above model is equivalent to

$$\begin{aligned}&Opt\ Z\\&Subject\ to\,(29)-(30)\\&for\ i=1,2,...,m;\ j=1,2,...,n.\end{aligned}\qquad (32)$$

Step 4. Crisp programming model obtained in step 2 can be solved using simplex method to get the optimal solution

8 NUMERICAL ILLUSTRATION

A farmer has “about 12" acres of cultivable land and he wanted to grow multiple vegetable crops viz., Brinjal, Ladies finger, Bitter guard and Tomato in a season. Out of his experience, he stated that labour work time available with him is 220 hours and availability of water is 25 acre-inches. The profit coefficients (lakh rupees), required work time and water for each crop for one acre of land are provided in the Table-1. How many acres he has to consider for each crop in order to get guaranteed net returns out of volatility among profit coefficients ?

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Table 1: Profit coefficients, labour requirement and water for entire duration of crop

	Brinjal	Ladies finger	Bitter guard	Tomato
Profit coefficients (lakh rupees)(about)	2.2	1.64	3.12	4.8
Labour requirement per acre('000 hours)(about)	1.760	1.280	1.600	1.840
Water requirement per acre (acre-inch)(about)	27.2	17.5	18.2	18

Here, we illustrate solution of the problem by the working procedure provided in the section-\ref{methodology} . Let x_i for $i=1,2,3,4.$ be the number of acres to be considered for Brinjal, Ladies finger, Bitter guard and Tomato respectively and the undertaken problem is to solve.

$$\begin{aligned} & Max\ Z=\tilde{c}_1x_1+\tilde{c}_2x_2+\tilde{c}_3x_3+\tilde{c}_4x_4 \\ & S.t\ \tilde{a}_{11}x_1+\tilde{a}_{12}x_2+\tilde{a}_{13}x_3+\tilde{a}_{14}x_4\le\tilde{b}_1 \\ & \tilde{a}_{21}x_1+\tilde{a}_{22}x_2+\tilde{a}_{23}x_3+\tilde{a}_{24}x_4\le\tilde{b}_1 \\ & \tilde{a}_{31}x_1+\tilde{a}_{32}x_2+\tilde{a}_{33}x_3+\tilde{a}_{34}x_4\le\tilde{b}_1 \\ & x_j\ge 0,\ j=1,2,3,4. \end{aligned} \quad (33)$$

where\\

$\tilde{c}_1=\left[(0.80,0.84,0,0),(0.80,0.84,0,0)\right]$; $\tilde{c}_2=\left[(0.60,0.64,0,0),(0.60,0.64,0,0)\right]$;

$\tilde{a}_{12}=\left[(1,1,0,0),(1,1,0,0)\right]$; $\tilde{a}_{13}=\left[(1,1,0,0),(1,1,0,0)\right]$; $\tilde{a}_{14}=\left[(1,1,0,0),(1,1,0,0)\right]$,

$\tilde{a}_{21}=\left[(1.72,1.8,0.2,0.2),(1.74,1.78,0.5,0.5)\right]$; $\tilde{a}_{22}=\left[(1.24,1.32,0.2,0.2),(1.26,1.3,0.5,0.5)\right]$;

$\tilde{a}_{23}=\left[(1.56,1.64,0.2,0.2),(1.58,1.62,0.5,0.5)\right]$; $\tilde{a}_{24}=\left[(1.8,1.88,0.2,0.2),(1.82,1.86,0.5,0.5)\right]$,

$\tilde{a}_{31}=\left[(27.16,27.24,0.2,0.2),(27.18,27.22,0.5,0.5)\right]$; $\tilde{c}_3=\left[(1,1.5,0,0),(1,1.5,0,0)\right]$;

$\tilde{a}_{32}=\left[(17.46,17.54,0.2,0.2),(17.48,17.52,0.5,0.5)\right]$; $\tilde{c}_4=\left[(0.7,0.9,0,0),(0.7,0.9,0,0)\right]$;

$\tilde{a}_{33}=\left[(18.16,18.24,0.2,0.2),(18.18,18.22,0.5,0.5)\right]$; $\tilde{a}_{11}=\left[(1,1,0,0),(1,1,0,0)\right]$;

$\tilde{a}_{34}=\left[(17.96,18.04,0.2,0.2),(18.16,18.2,0.5,0.5)\right]$,

$\tilde{b}_1=\left[(11.96,12.04,0.2,0.2),(12.16,12.2,0.5,0.5)\right]$,

$\tilde{b}_2=\left[(24.96,25.04,0.2,0.2),(24.98,25.02,0.5,0.5)\right]$ and $\tilde{b}_3=\left[(180,260,20,20),(180,260,50,50)\right]$.

8. TYPE-2 FUZZY LINEAR PROGRAMMING PROBLEM BASED ON CCM AND POSSIBILITY MEASURE.

Now by using Step 2 of the method explained in Section 7 and theorem 5.6,if we apply the possibility measure in type-2 fuzzy linear programming problem 33, is converted into the following crisp programming problems:

$$Z=\frac{1}{2}\left[Max\underline{Z}+Max\overline{Z}\right]$$

$$\begin{aligned} & Max\underline{Z}=(0.84-0.04p)x_1+(0.64-0.04p)x_2+(1.5-0.5p)x_3+(0.9-0.2p)x_4 \\ & S.t\ x_1+x_2+x_3+x_4\le(12.24-0.2p) \\ & (1.52+0.2p)x_1+(1.04+0.2p)x_2+(1.36+0.2p)x_3+(1.6+0.2p)x_4\le 25.24-0.2p \\ & (26.96+0.2p)x_1+(17.26+0.2p)x_2+(17.96+0.2p)x_3+(17.76+0.2p)x_4\le 280-20p \\ & x_j\ge 0,\ j=1,2,3,4. \end{aligned} \quad (34)$$

$$Max\overline{Z} = (0.84-0.04p)x_1 + (0.64-0.04p)x_2 + (1.5-0.5p)x_3 + (0.9-0.2p)x_4$$
$$S.t\ x_1 + x_2 + x_3 + x_4 \le (12.7-0.5p)$$
$$(1.24+0.5p)x_1 + (0.76+0.5p)x_2 + (1.08+0.5p)x_3 + (1.32+0.5p)x_4 \le 25.52-0.5p \quad (35)$$
$$(26.68+0.5p)x_1 + (16.48+0.5p)x_2 + (17.68+0.5p)x_3 + (17.66+0.5p)x_4 \le 310-50p$$
$$x_j \ge 0,\ j=1,2,3,4.$$

Solving the above crisp problem for efficient levels ($\alpha = 0.1, 0.5$ and 1) and different possibility levels, we get different optimal solutions. Optimal solution of 34 and 35 at different possibility levels are presented in Table 2

Table 2: Optimal solution of 34 and 35 at different possibility levels.

α	Optimal solution and Optimal Value $\underline{Z}$		Optimal solution and Optimal Value $\underline{Z}$		Z
0.1	x_3 =17.719	17.719	x_3 =18.3425	18.3425	18.031
0.5	x_3 =15.175	15.175	x_3 =15.5625	15.5625	15.368
1	x_3 =12.04	12.04	x_3 =15.25	15.25	13.645

3 Type-2 Fuzzy linear programming problem based on CCM and Necessity Measure.

Now by using Step 2 of the method explained in Section 7 and theorem 5.7,if we apply the necessity measure in type-2 fuzzy linear programming problem 33, is converted into the following crisp programming problems:

$$Z = \frac{1}{2}\left[Max\underline{Z} + Max\overline{Z}\right]$$

$$Max\underline{Z} = (0.80)x_1 + (0.60)x_2 + (1)x_3 + (0.7)x_4$$
$$S.t\ x_1 + x_2 + x_3 + x_4 \le (11.76+0.2p)$$
$$(1.8+0.2p)x_1 + (1.32+0.2p)x_2 + (1.64+0.2p)x_3 + (1.88+0.2p)x_4 \le 24.76+0.2p \quad (36)$$
$$(27.24+0.2p)x_1 + (17.54+0.2p)x_2 + (18.24+0.2p)x_3 + (18.04+0.2p)x_4 \le 160+20p$$
$$x_j \ge 0,\ j=1,2,3,4.$$

$$Max\overline{Z} = (0.80)x_1 + (0.60)x_2 + (1)x_3 + (0.7)x_4$$
$$S.t\ x_1 + x_2 + x_3 + x_4 \le (11.66+0.5p)$$
$$(1.78+0.5p)x_1 + (1.3+0.5p)x_2 + (1.62+0.5p)x_3 + (1.86+0.5p)x_4 \le 24.48+0.5p \quad (37)$$
$$(27.22+0.5p)x_1 + (17.52+0.5p)x_2 + (18.22+0.5p)x_3 + (18.2+0.5p)x_4 \le 160=50p$$
$$x_j \ge 0,\ j=1,2,3,4.$$

Table 3: Optimal solution of 36 and 37 at different necessity levels.

α	Optimal solution and Optimal Value $\overline{Z}$		Optimal solution and Optimal Value $\overline{Z}$		Z
0.1	x_3 =8.872	8.872	x_3 =8.484	8.484	8.676
0.5	x_3 =9.269	9.269	x_3 =7.309	7.309	8.289
1	x_3 = 9.761	9.761	x_3 =5.876	5.876	7.819

Solving the above crisp problem for efficient levels ($\alpha = 0.1, 0.5$ and 1) and different necessity levels, we get different optimal solutions. Optimal solution of 36 and 37 at different necessity levels are presented in Table 3.

9. Conclusion

In this paper, we have developed the possibility, necessity, and credibility measures on type-2 fuzzy set. We have also developed the theoretical calculation on possibility, necessity, and credibility measures for defuzzify type-2 fuzzy linear programming model using chance operators. To validate the proposed method, we have discussed three different approaches to defuzzify the type-2 fuzzy relations using possibility, necessity, and credibility measures. Using chance operator we can convert a problem under imprecise models to corresponding crisp models. At different levels of possibility, necessity, and credibility, we have achieved different optimal solution.

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