Inventory policies for deteriorating item with stock dependent demand and variable holding costs under permissible delay in payment

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ABSTRACT

The existing inventory models for deteriorating items under the facility of allowable delay in payment considers holding cost per unit time to be constant. However, in business activities nowadays customers are given some delay period within which to settle for the goods supplied to them. During the delay period they can use the ensued money from sales of the supplied goods to earn interest within the allowed delay period. Therefore, customers will order more quantity to earn more money. This fact lays the base of variable holding cost because as well as stock of decaying item increase the holding cost of item increases due to its decaying nature. Also, in many practical situations on-hand stock affects the demand rate of the product. Therefore, in this study we have considered an economic ordered quantity model for deteriorating item with stock dependent demand and holding cost per unit time is considered as a function of two factors: deterioration and storage period. Shortages are allowed and partial backordering is taken into account in the present study. The concept of the model is illustrated with numerical examples and sensitivity analyses are also carried out.

1.INTRODUCTION

The traditional economic order quantity (EOQ) model assumes that retailer's capitals are adequate and must pay for the items as soon as the items are received. In practice, supplier will offer retailer a delay period, which is the trade credit period, in paying for the amount of purchase. Before end of the trade credit period, retailer can sell goods and accumulate revenue and earn interest. A higher interest is charged if the payment is not settled by end of the trade credit period. In the real world, supplier often makes use of this policy to promote commodities. Many researchers discussed this topic that investigates inventory problems under varying conditions.

In this regard, a number of research papers appeared which deal with the EOQ problem under fixed credit period. Goyal (1985) analyzed effects of trade credit on the optimal inventory policy. Chand and Ward (1987) analyzed Goyal's problem under assumptions of the classical economic order model, obtaining different results. Chung (1998) developed an alternative approach to determine the economic order quantity under the condition of permissible delay in payments. Aggarwal and Jaggi (1995) considered the inventory model with an exponential deterioration rate under the condition of permissible delay in payments. Chu et al. (1998) extended Goyal's model to the case of deterioration. Jamal et al. (1997) and Chang and Dye (2001) further generalized the model allow for shortages. Many related articles can be found in Hwang and Shinn (1997), Jamal et al. (2000), Arcelus et al. (2003), Abad and Jaggi (2003), Chang (2004) and Chung (2010) and their references.

Recently, Khanra et al. (2011) developed EOQ (Economic Order Quantity) model for a deteriorating item having time dependent demand when delay in payment is permissible. The deterioration rate is assumed to be constant and the time varying demand rate is taken to be a quadratic function of time. Musa and Sani (2012) developed a mathematical model on the inventory of deteriorating items that do not start deteriorating immediately they are stocked.

The characteristic of all of the above articles is that the unsatisfied demand (due to shortages) is completely backlogged. However, in reality, demands for foods, medicines, etc. are usually lost during the shortage period. Montgomery et al. (1973) studied both deterministic and stochastic demand inventory models with a mixture of backorder and lost sales. Later, Rosenberg (1979) provided a new analysis of partial backorders. Park (1982) reformulated the cost function and established the solution. Mak (1987) modified the model by incorporating a uniform replenishment rate to determine the optimal production-inventory control policies. For fashionable commodities and high-tech products with short product life cycle, the willingness for a customer to wait for backlogging during a shortage period is diminishing with the length of the waiting time. Hence, the longer the waiting time, the smaller the backlogging rate. To reflect this phenomenon, Chang and Dye (1999) developed an inventory model in which the proportion of customers who would like to accept backlogging is the reciprocal of a linear function of the waiting time. Concurrently, Papachristos and Skouri (2000) established a partially backlogged inventory model in which the backlogging rate decreases exponentially as the waiting time increases. Teng et al. (2002, 2003) then extended the fraction of unsatisfied demand back ordered to any decreasing function of the waiting time up to the next replenishment. Teng and Yang (2004) further generalized the partial backlogging EOQ model to allow for time-varying purchase cost. Yang (2005) made a comparison among various partial backlogging inventory lot-size models for deteriorating items on the basis of maximum profit. Lately, Recently, Pentico and Drake (2011) provided a prominent survey of deterministic models for the EOQ with partial backlogging.

In the inventory researcher papers above, holding cost per unit time is taken as constant. Inventory decisions under the permissible delay assumption are play a vital role because for take the benefits of permissible period of delay in payment everyone will want to order more. However, the holding problems also occur due to the deteriorating nature of the item in stock. For better demonstrating such real life situations, holding cost per unit should be more general as possible. A notable extension of stock-dependent demand models incorporates variable holding costs. Weiss (1982) noted that variable holding costs are appropriate when the value of an item decreases the longer it is in stock; Ferguson et al. (2007) recently indicated that this type of model is suitable for perishable items in which price markdowns or removal of aging product are necessary. Alfares (2007) states that more sophisticated storage facilities and services may be needed for perishable items if they are kept for longer periods of time. Goh (1994) first considered a stock-dependent demand model with variable holding costs, and assumed that the unit holding cost is a nonlinear continuous function of the time the item is in stock or a nonlinear continuous function of the inventory level. Giri and Chaudhuri (1998) extended this model to account for perishable products. Chang (2004) then amended the Giri and Chaudhuri model to utilize a profit-maximization objective and to allow for a positive inventory level at the end of the order cycle. Alfares (2007) recently investigated the situation in which the variable holding costs are discrete in nature a step function of the time in stock with successively increasing costs. He considered two scenarios: (1) the holding cost of the last storage period of an order cycle is applied retroactively to all storage periods, so the same holding cost is applied to all units in the cycle, and (2) the holding cost of a storage period is applied incrementally to each period, so the holding cost for each storage period is applied only to the units held in that period. However, the proposed models and solution algorithms are derived using a cost-minimization objective and impose a terminal condition of zero inventory at the end of each order cycle. Roy (2008) developed an inventory model for deteriorating items with time varying holding cost and demand is price dependent. Mishra and Singh (2011) developed the inventory model for deteriorating items with time dependent linear demand and holding cost.

The above mentioned research papers make clear that the inventory models for decaying products with stock dependent demand and variable holding cost under permissible delay facility are so much rare. Therefore, during the proposed study an inventory model with these realistic and helpful circumstances is developed. Shortages are permitted and partially backordered with a variable backlogging rate. Two different situations of permissible delay period are taken into consideration during this study. Finally, numerical example and sensitivity analysis are also given to exemplify the model.

Notations

The following notation is used throughout the chapter:

$I(t)$	The inventory level at any time t, $t \geq 0$;
T	Constant prescribed scheduling period or cycle length (time units);
W	Maximum inventory level at the start of a cycle (units);
S	Maximum amount of demand backlogged per cycle (units);
θ	Deteriorated portion on hand inventory during cycle time;
t_1	Duration of inventory cycle when there is positive inventory;
Q	Order quantity (units/cycle);
I_e	Interest earned (/$/cycle);
I_k	Interest charges per $ investment in inventory per cycle;
M	The trade credit period length per cycle;
s	Unit selling price per item ($);
c_1	Cost of the inventory items ($);
c_2	Fixed cost per order ($/order);
c_3	Shortage cost per unit back-ordered per unit time ($/unit/unit time);
c_4	Opportunity cost due to lost sales ($/unit).

Assumptions

In developing the mathematical model of the inventory system, the following assumptions are made:

1. The inventory system involves only one perishable item and the planning horizon is infinite.
2. The replenishment occurs instantaneously at an infinite rate.
3. The deteriorating rate, $\theta(0<\theta<1)$, is constant and there is no replacement or repair of deteriorated units during the period under consideration.
4. Holding cost per unit $h(t)$ is continuously variable with storage period of unit. This cost varies exponentially and deterioration of unit governs its variation.
5. The demand rate, $d(t)$ is stock dependent and defined as $d(t)=a+bI(t)$ for $I(t)>0$ and $d(t)=a$ for $I(t)\le 0$. Where, $a(>0)$ is initial demand and $b(0<b<1)$ is a constant parameter.
6. Permissible delay in payment approach is considered. During the time the account is not settled, generated sales revenue is deposited in an interest-bearing account. When $t_1 \le M$, the account is settled at $t_1 = M$, the retailer pays off all units sold and keeps his/her profits and start paying for the higher interest charges on the items in stock. When $t_1 \le M$, the account is settled at $t_1 = M$ and retailer does not need to pay any charge.
7. During the shortage period, the backlogging rate is variable and dependent on the length of the waiting time for the next replenishment. The longer the waiting time is the smaller the backlogging rate would be. Hence, the portion of customers who would like to accept backlogging at time t is decreasing with the waiting time $(T-t)$ for the next replenishment. To take care of this situation we have assumed the backlogging rate to be $1/\{1+\delta(T-t)\}$ when inventory is negative. The backlogging parameter δ is a positive constant.

Model Formulations

Here, the replenishment policy of a deteriorating item with partial backlogging is considered. The objective of the inventory system is to determine the optimal ordering quantity and the length of ordering cycle so as to keep the total relevant cost as low as possible. The behavior of inventory system at any time t is depicted in Figure 1 and 2.

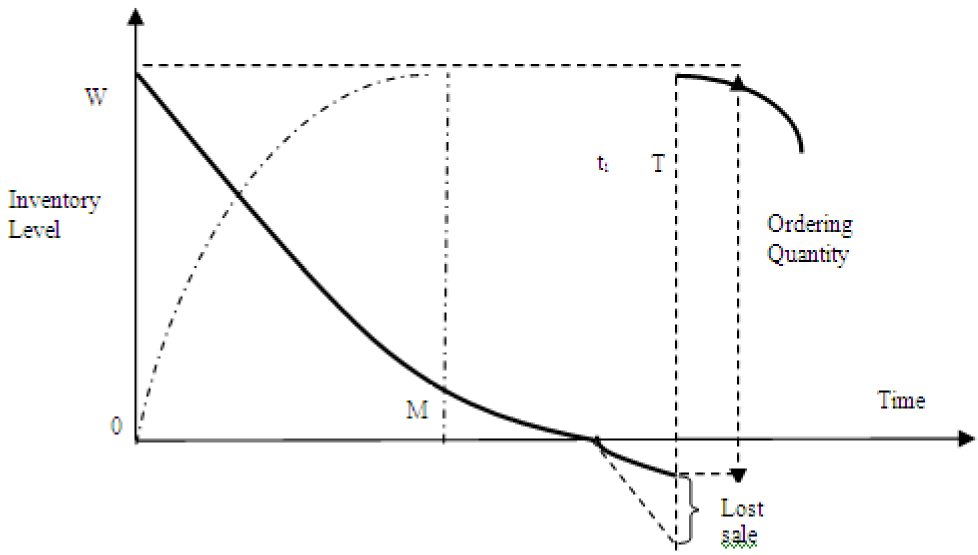

Fig. 2. The graphical representation of inventory system in Study 2: $t_1 < M$

Replenishment is made at time $t=0$ and the inventory level at $t=0$ is maximum (W). Due to both the market demand and deterioration of the item, the inventory level decreases during the period $[0,t_1]$, and ultimately falls to zero at $t=t_1$. After that, shortages are allowed to occur during the time interval $[t_1,T]$, and a fraction of the demand during the period $[t_1,T]$ is backlogged.

As described above, the inventory level decreases owing to demand rate as well as deterioration during inventory interval $[0,t_1]$. Hence, the differential equation representing the inventory status is given by

$$\frac{dI(t)}{dt}+\theta I(t)=-(a+bI(t)) \text{ for } 0\le t\le t_1, \qquad (1)$$

with the boundary condition $I(0)=W, I(t_1)=0$. The solution of Eq. (1) is

$$I(t)=\frac{a}{(b+\theta)}\left[e^{(b+\theta)(t_1-t)}-1\right], \qquad 0\le t\le t_1. \qquad (2)$$

So the maximum inventory level for each cycle can be obtained as

$$W=I(0)=\frac{a}{(b+\theta)}\left[e^{(b+\theta)t_1}-1\right]. \qquad (3)$$

During the shortage period $[t_1,T]$, the demand at time t is partly backlogged at the fraction $1/\{1+\delta(T-t)\}$. Thus, the differential equation with boundary $I(t_1)=0$ condition governing the amount of demand backlogged is as

$$\frac{dI(t)}{dt}=-\frac{a}{\{1+\delta(T-t)\}} \qquad ,t_1<t\le T\ . \qquad (4)$$

The solution of Eq. (4) can be given by

$$I(t)=\frac{a}{\delta}\left[In[1+\delta(T-t)]-In[1+\delta(T-t_1)]\right] \qquad ,\ t_1<t\le T\ . \qquad (5)$$

Let $t=T$ in Eq. (5), we obtain the maximum amount of demand backlogged per cycle as follows

$$S=-I(T)=\frac{a}{\delta}In[1+\delta(T-t_1)]. \qquad (6)$$

Hence, the order quantity per cycle is given by

$$Q=W+S=\frac{a}{(b+\theta)}\left[e^{(b+\theta)t_1}-1\right]+\frac{a}{\delta}In[1+\delta(T-t_1)]. \ (7)$$

Now, **t**he inventory holding cost per cycle represents the capital cost invested in keeping inventory. This cost may fluctuate with the nature of item which is stored, time the item is in stock and obligatory facilities and services to keep stored item in good condition. So there are two phase of stock holding cost: first fixed cost of place at which item is to be stored and second handling cost in keeping item safe. Handling charges are originated by the decline of item in-stock. Handling charges increase as deterioration of item raise and vice-versa. Therefore, holding cost per unit time should be the function of storage time of item and its deterioration rate. To give better strength, we have assumed that the holding cost per unit per unit time $h(t)$ is to be as $R+He^{\theta t}$. Where R and H are the positive fixed rent of warehouse and handling charges respectively. θ is the deterioration rate of inventory.
Therefore, the holding cost per cycle is

$$HC=\int_0^{t_1}h(t)I(t)dt$$

$$=\frac{a}{(b+\theta)}\left[R\left\{\frac{\left(e^{(b+\theta)t_1}-1\right)}{(b+\theta)}-t_1\right\}+H\left\{\frac{e^{\theta t_1}}{b}\left(e^{bt_1}-1\right)-\frac{1}{\theta}(e^{\theta t_1}-1)\right\}\right]. \qquad (8)$$

The ordering cost per cycle per order is

$$OC=c_2 \qquad (9)$$

The deterioration cost per cycle is

$$DC=c_1\left[W-\int_0^{t_1}d(t)dt\right]$$

$$=\frac{c_1a\theta}{(b+\theta)^2}\left[(e^{(b+\theta)t_1}-1)-(b+\theta)t_1\right]. \qquad (10)$$

The shortage cost per cycle is

$$SC=c_3\left[-\int_{t_1}^{T}I(t)dt\right]$$

$$=c_3a\left[\frac{(T-t_1)}{\delta}-\frac{(1+\delta T)In[1+\delta(T-t_1)]}{\delta^2}\right]. \qquad (11)$$

The opportunity cost due to lost sales per cycle is

$$OPC=c_4\int_{t_1}^{T}\left[1-\frac{1}{1+\delta(T-t)}\right]adt$$

$$=c_4 a\left[(T-t_1)-\frac{In[1+\delta(T-t_1)]}{\delta}\right]. \quad (12)$$

Now, consider the supplier's credit period M in settling the accounts. There are two studies. Study1: $M \le t_1$ and study 2: $M > t_1$. We shall discuss these two studies one by one.

Study 1: Suppose $M \le t_1$

Since, the length of the period with positive inventory stock of the items is greater than the credit period; the buyer can use the sale revenue with an annual rate I_e in $[0, t_1)$.

The interest earned denoted by IE_1 is

$$IE_1 = sI_e \int_0^{t_1}(t_1-t)\,d(t)\,dt$$

$$= sI_e a\left[\frac{b}{(b+\theta)^3}\left(1-e^{(b+\theta)t_1}\right)+\frac{bt_1 e^{(b+\theta)t_1}}{(b+\theta)^2}+\frac{\theta t_1^{\,2}}{(b+\theta)}-\frac{\theta t_1^{\,2}}{2}\right]. \quad (13)$$

After, the credit period the buyer has to pay the interest for the goods still in stock with annual rate I_k. we can find the interest payable denoted by IP follows:

$$IP = c_1 I_k \int_M^{t_1} I(t)\,dt$$

$$=\frac{c_1 I_k a}{(b+\theta)}\left[\frac{\left(e^{(b+\theta)(t_1-M)}-1\right)}{(b+\theta)}-(t_1-M)\right]. \quad (14)$$

Thus, the average total cost per unit time for study 1 is

$$ATC_1(t_1,T)=(OC+HC+DC+SC+OPC+IP-IE_1)/T$$

$$\begin{aligned}
&=\frac{1}{T}\Bigg[\frac{a}{(b+\theta)}\left[R\left\{\frac{\left(e^{(b+\theta)t_1}-1\right)}{(b+\theta)}-t_1\right\}+H\left\{\frac{e^{\theta t_1}}{b}\left(e^{bt_1}-1\right)-\frac{1}{\theta}(e^{\theta t_1}-1)\right\}\right]+c_2\\
&+\frac{c_1 a\theta}{(b+\theta)^2}\left[(e^{(b+\theta)t_1}-1)-(b+\theta)t_1\right]+c_3 a\left[\frac{(T-t_1)}{\delta}-\frac{(1+\delta T)In[1+\delta(T-t_1)]}{\delta^2}\right]\\
&+c_4 a\left[(T-t_1)-\frac{In[1+\delta(T-t_1)]}{\delta}\right]+\frac{c_1 I_k a}{(b+\theta)}\left[\frac{\left(e^{(b+\theta)(t_1-M)}-1\right)}{(b+\theta)}-(t_1-M)\right]\\
&-sI_e\left[\frac{ab}{(b+\theta)^3}\left(1-e^{(b+\theta)t_1}\right)+\frac{abt_1 e^{(b+\theta)t_1}}{(b+\theta)^2}+\frac{a\theta t_1^{\,2}}{(b+\theta)}-\frac{a\theta t_1^{\,2}}{2}\right]\Bigg]
\end{aligned} \quad (15)$$

The objective of the model is to determine the optimal values of t_1 and T, when $M \le t_1$, in order to minimize the average total cost per unit time $ATC_1(t_1,T)$. The optimal solution t_1^* and T^* can be obtained by solving the following equations simultaneously

$$\frac{\partial ATC_1}{\partial t_1}=0, \quad (16)$$

$$\frac{\partial ATC_1}{\partial T}=0 \quad (17)$$

Provided, they satisfy the following conditions

$$\frac{\partial^2 ATC_1(t_1,T)}{\partial t_1^2}>0,\ \frac{\partial^2 ATC_1(t_1,T)}{\partial T^2}>0,\ \frac{\partial^2 ATC_1(t_1,T)}{\partial t_1^2}\frac{\partial^2 ATC_1(t_1,T)}{\partial T^2}-\left(\frac{\partial^2 ATC_1(t_1,T)}{\partial t_1\partial T}\right)^2>0$$

Next, by using t_1^* and T^*, one can obtain the optimal maximum inventory level and the minimum average total cost per unit time from (3) and (15), respectively (we denote these values by W^* and $ATC_1(t_1^*, T^*)$). Furthermore, one can also obtain the optimal order quantity (we denote it by Q^*) from (7).

Study 2: Suppose $t_1 < M$

In this study, the buyer need not pay interest and he earns the interest with annual interest rate I_e. The interest earned denoted by IE_2 is given by

$$IE_2 = sI_e \left\{ \int_0^{t_1} (t_1 - t) d(t) + (M - t_1) \int_0^{t_1} d(t) dt \right\}$$

$$= sI_e a \left[\frac{b\left(1 - e^{(b+\theta)t_1}\right)}{(b+\theta)^3} + \frac{bt_1 e^{(b+\theta)t_1}}{(b+\theta)^2} + \frac{M\theta t_1}{(b+\theta)} - \frac{\theta t_1^2}{2} + \frac{b(M-t_1)\left(e^{(b+\theta)t_1} - 1\right)}{(b+\theta)^2} \right] \quad (18)$$

We have the average total cost in this study

$$ATC_2(t_1, T) = (OC + HC + DC + SC + OPC - IE_2)/T \quad (19)$$

$$= \frac{1}{T}\left[\frac{a}{(b+\theta)} \left[R\left\{ \frac{\left(e^{(b+\theta)t_1} - 1\right)}{(b+\theta)} - t_1 \right\} + H\left\{ \frac{e^{\theta t_1}}{b}\left(e^{bt_1} - 1\right) - \frac{1}{\theta}(e^{\theta t_1} - 1) \right\} \right] + c_2 \right.$$

$$+ \frac{c_1 a\theta}{(b+\theta)^2}\left[(e^{(b+\theta)t_1} - 1) - (b+\theta)t_1 \right] + c_3 a\left[\frac{(T-t_1)}{\delta} - \frac{(1+\delta T) In[1+\delta(T-t_1)]}{\delta^2} \right]$$

$$+ c_4 a\left[(T-t_1) - \frac{In[1+\delta(T-t_1)]}{\delta} \right] + \frac{c_1 I_k a}{(b+\theta)}\left[\frac{\left(e^{(b+\theta)(t_1-M)} - 1\right)}{(b+\theta)} - (t_1 - M) \right]$$

$$\left. - sI_e a\left[\frac{b\left(1 - e^{(b+\theta)t_1}\right)}{(b+\theta)^3} + \frac{bt_1 e^{(b+\theta)t_1}}{(b+\theta)^2} + \frac{M\theta t_1}{(b+\theta)} - \frac{\theta t_1^2}{2} + \frac{b(M-t_1)\left(e^{(b+\theta)t_1} - 1\right)}{(b+\theta)^2} \right] \right] \quad (20)$$

Again, the objective of the model is to determine the optimal values of t_1 and T, when $t_1 < M$, in order to minimize the average total cost per unit time $ATC_2(t_1, T)$. The optimal solution t_1^* and T^* can be obtained by solving the following equations simultaneously

$$\frac{\partial ATC_2}{\partial t_1} = 0, \quad (21)$$

$$\frac{\partial ATC_2}{\partial T} = 0 \quad (22)$$

Provided, they satisfy the following conditions

$$\frac{\partial^2 ATC_2(t_1,T)}{\partial t_1^2} > 0, \frac{\partial^2 ATC_2(t_1,T)}{\partial T^2} > 0, \frac{\partial^2 ATC_2(t_1,T)}{\partial t_1^2}\frac{\partial^2 ATC_2(t_1,T)}{\partial T^2} - \left(\frac{\partial^2 ATC_2(t_1,T)}{\partial t_1 \partial T}\right)^2 > 0$$

Next, by using t_1^* and T^*, one can obtain the optimal maximum inventory level and the minimum average total cost per unit time from (3) and (20), respectively (we denote these values by W^* and $ATC_2(t_1^*, T^*)$). Furthermore, one can also obtain the optimal order quantity (we denote it by Q^*) from (7).

Numerical Examples

For illustration of the both studies discussed above in developed model, two separate numerical examples are presented for a single produced product. To perform the numerical analysis, data have been taken randomly from literatures in appropriate units.

Example 1: We consider a inventory system which verifies the described assumptions above. The input data of parameters are taken randomly as

$s = 5, I_e = 0.128, I_k = 0.160, \theta = 0.1, \delta = 0.1, b = 0.4, a = 30\ D = 2, M = 0.1, R = 6, H = 0.2$
$c_1 = 4, c_2 = 10, c_3 = 3$ and $c_4 = 2$. From (15), by using MATHEMATICA 8.0, the minimum average total cost per unit time $ATC_1(t_1^*, T^*)$, along with the optimal value of t_1^* and T^* is calculated for study 1. The optimal ordered quantity (Q^*) is also calculated from (7) for study 1. Therefore, the optimal policy for study $(M \le t_1^*)$ under assumed conditions is $t_1^* = 0.1781034$, $T^* = 3.07927$, $Q^* = 17.00135$ and $ATC_1(t_1^*, T^*) = 385.111$ in their appropriate units.

Example 2: We consider a inventory system which verifies the described assumptions above. The input data of parameters are taken randomly as
$s = 8, I_e = 0.128, I_k = 0.160, \theta = 1, \delta = 0.1, b = 0.5,\ D = 12, M = 0.9, R = 4, H = 0.2, A = 2, c_2 = 20\ c_1 = 4, c_3 = 3$ and $c_4 = 2$. From (23), by using MATHEMATICA 8.0, the minimum average total cost per unit time $ATC_2(t_1^*, T^*)$, along with the optimal value of t_1^* and T^* is calculated for study 2. The optimal ordered quantity (Q^*) is also calculated from (7) for study 2. Therefore, the optimal policies for study $(M > t_1^*)$ under assumed conditions are $t_1^* = 0.6844439$, $T^* = 1.352184$, $Q^* = 20.12493$ and $ATC_2(t_1^*, T^*) = 40.1211$ in their appropriate units.

Sensitivity Analysis

In this section, for both studies, the effects of the changes in the optimal value of the average total cost per unit time, the shortage point, cycle time and the ordered quantity per cycle with respect to changes in some model parameters are discussed. The sensitivity analysis in each scenario is executed by changing the value of each of the parameters by $\pm 5\%$ and $\pm 10\%$, taking one parameter at a time and keeping the remaining parameters unchanged. Example 1 and 2 are used separately.

Sensitivity Analysis for Study 1

In case $(M \le t_1)$, to discuss the effect of changes of model parameters $s, c_1, R, H, I_e, I_k, M$ *and* δ on the optimal value of the average total cost $(ATC_1(t_1^*, T^*))$, the shortage point (t_1^*), the cycle time (T^*) and the ordered quantity per cycle Q^* for study 1, the different values of these parameter according to $\pm 5\%$ and $\pm 10\%$ change in each have taken and its effect on $ATC_1(t_1^*, T^*)$, t_1^*, T^* and Q^* are presented in the following Table 1.

Table 1: Sensitivity Analysis for Study 1

Parameters	%Change in the value of			
	t_1^*	T^*	Q^*	$ATC_1(t_1^*, T^*)$
$s = 5$	+0.95	+0.031	+0.22	+0.40
	+0.48	+0.016	+0.12	+0.17
	-0.45	-0.013	-0.10	-0.18
	-0.91	-0.028	-0.94	-0.37
$c_1 = 4$	+0.81	+2.98	+2.22	+7.02
	+0.41	+1.50	+1.12	+3.56
	-0.42	-1.48	-1.11	-3.63
	-0.87	-2.98	-2.25	-7.35
$R = 6$	+9.57	+0.22	+2.38	+4.37
	+4.57	+0.11	+1.14	+2.02
	-4.17	-0.08	-1.03	-1.73
	-8.01	-0.16	-2.69	-3.23
$H = 0.2$	+0.30	+0.003	+0.08	+0.129
	+0.15	+0.002	+0.04	+0.065
	-0.13	-0.001	-0.02	-0.055
	-0.28	-0.002	-0.05	-0.126
$I_e = 0.128$	+0.94	+0.031	+0.25	+0.40
	+0.48	+0.016	+0.13	+0.21
	-0.46	-0.013	-0.11	-0.18
	-0.91	-0.029	-0.22	-0.37
$I_k = 0.160$	+1.39	+2.99	+2.36	+7.29
	+0.71	+1.50	+1.19	+3.68
	-0.71	-1.49	-1.18	-3.74
	-1.45	-3.00	-2.40	-7.58
$M = 0.1$	+2.33	+3.00	+2.60	-3.38
	+1.18	+1.50	+1.30	-1.72
	-1.17	-1.50	-1.31	+1.74
	-2.37	-3.02	-2.62	+3.34
$\delta = 0.1$	-0.34	+3.29	+1.27	-3.19
	-0.17	+1.62	+0.64	-1.64
	+0.15	-1.54	-0.60	+1.58
	+0.32	-3.05	-1.20	+3.16

Observations:

1. From Table 1 it is clear that $ATC_1(t_1^*, T^*)$ increases with an increase in the values of model parameters s, c_1, R, H, I_e *and* I_k while $ATC_1(t_1^*, T^*)$ decreases with an increase in the value of M and δ. The obtained results show that $ATC_1(t_1^*, T^*)$ is highly sensitive with respect to the changes in c_1, R, I_k, M *and* δ. It is less sensitive to changes in s and I_e; and very less sensitive to the change in H.
2. From Table 1 it is clear that $ATC_1(t_1^*, T^*)$ decreases with a decrease in the values of model parameters s, c_1, R, H, I_e *and* I_k while $ATC_1(t_1^*, T^*)$ increases with decrease in the value of M and δ. The obtained results make clear that $ATC_1(t_1^*, T^*)$ is highly sensitive to changes in c_1, R, I_k, M *and* δ. It is less sensitive to changes in s and I_e; and very less sensitive to change in H.

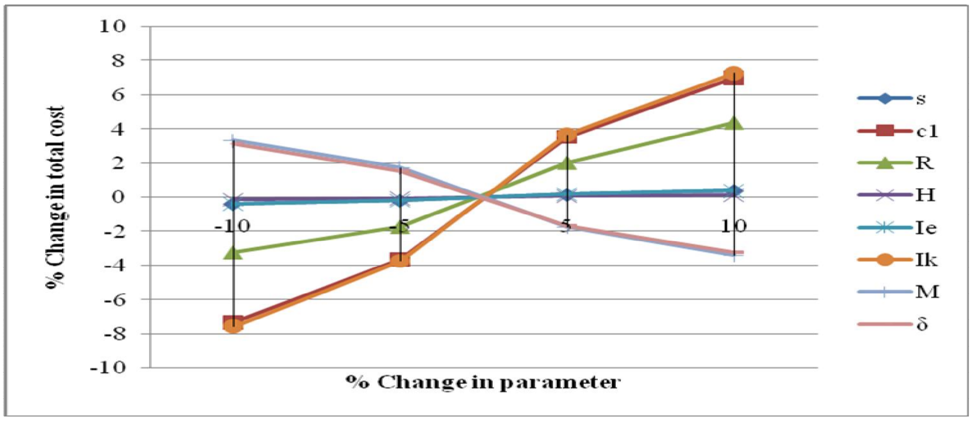

Fig 3. Behavior of optimal average total cost per unit time in Study1

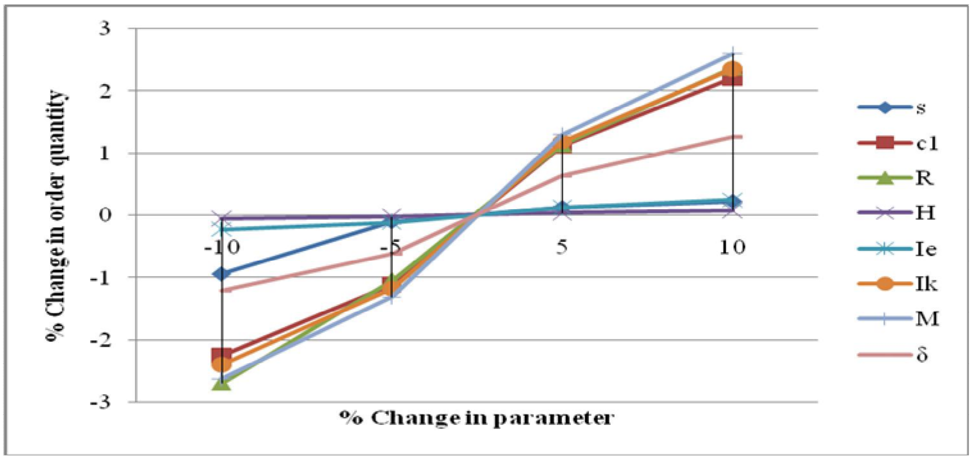

Fig 4. Behavior of optimal ordered quantity in Study1

3. From Table 1 it is clear that Q^* increases with increase in the values of model parameters $s, c_1, R, H, I_e, I_k, M$ and δ. The obtained results show that Q^* is highly sensitive to changes in c_1, R, I_k, M *and* δ. It is less sensitive to changes in s and I_e; and very less sensitive to change in H.
4. From Table 1 it is clear that Q^* decreases with decrease in the values of model parameters $s, c_1, R, H, I_e, I_k, M$ and δ. The obtained results show that Q^* is highly sensitive to changes in c_1, R, I_k, M *and* δ. It is less sensitive to changes in s and I_e; and very less sensitive to change in H.

Sensitivity Analysis for Study 2

In case $(M > t_1)$, to discuss the effect of changes of model parameters s, c_1, R, H, I_e, M *and* δ on the optimal value of the average total cost $(ATC_2(t_1^*, T^*))$, the shortage point (t_1^*), the cycle time (T^*) and the ordered quantity per cycle Q^* for study 2, the different values of these parameter according to $\pm 5\%$ and $\pm 10\%$ change in each have taken and its effect on $ATC_2(t_1^*, T^*), t_1^*, T^*$ and Q^* are presented in the following Table 2.

Table 2: Sensitivity Analysis for Study 2

Parameters	%Change in the value of			
	t_1^*	T^*	Q^*	$ATC_2(t_1^*,T^*)$
$s=8$	+1.36	+3.73	+2.39	+1.04
	+0.68	+1.67	+1.18	+0.50
	-0.69	-1.67	-1.17	-0.50
	-1.38	-3.29	-2.32	-0.95
$c_1=4$	-3.78	-1.55	-1.81	-0.46
	-1.92	-0.79	-0.92	-0.23
	+2.01	+0.84	+0.99	+0.26
	+4.10	+1.71	+2.01	+0.53
$R=4$	-3.78	-1.55	-1.81	-0.45
	-1.91	-0.78	-0.92	-0.35
	+2.01	+0.84	+0.99	+0.26
	+4.10	+1.71	+2.00	+0.53
$H=0.2$	-0.08	+0.38	+0.20	+0.19
	-0.03	+0.18	+0.21	+0.18
	+0.02	-0.20	-0.12	-0.11
	+0.10	-0.40	-0.22	-0.21
$I_e=0.128$	+1.36	+3.46	+2.38	+1.04
	+0.67	+1.70	+1.18	+0.50
	-0.69	-1.67	-1.18	-0.50
	-1.38	-3.28	-2.32	-0.95
$M=0.9$	+2.56	+5.27	+3.74	+1.52
	+1.27	+2.56	+1.83	+0.72
	-1.28	-2.45	-1.79	-0.68
	-2.53	-4.78	-3.50	-1.29
$\delta=0.1$	+0.20	+13.80	+5.42	+3.31
	+0.09	+6.42	+2.61	+1.58
	-0.10	-5.65	-2.40	-1.48
	-0.20	-10.65	-4.61	-2.83

Observations:

1. From Table 2 it is clear that $ATC_2(t_1^*,T^*)$ increases with increase in the values of model parameters s, H, I_e, M *and* δ while $ATC_2(t_1^*,T^*)$ decreases with increase in the value of c_1 and R. The obtained results show that $ATC_2(t_1^*,T^*)$ is highly sensitive to changes in s, I_e, M *and* δ. It is less sensitive to changes in c_1, R and H.
2. From Table 2 it is clear that $ATC_2(t_1^*,T^*)$ decreases with decrease in the values of model parameters s, H, I_e, M *and* δ while $ATC_2(t_1^*,T^*)$ increases with decrease in the value of c_1 and R. The obtained results show that $ATC_2(t_1^*,T^*)$ is highly sensitive to changes in s, I_e, M *and* δ. It is less sensitive to changes in c_1, R and H.

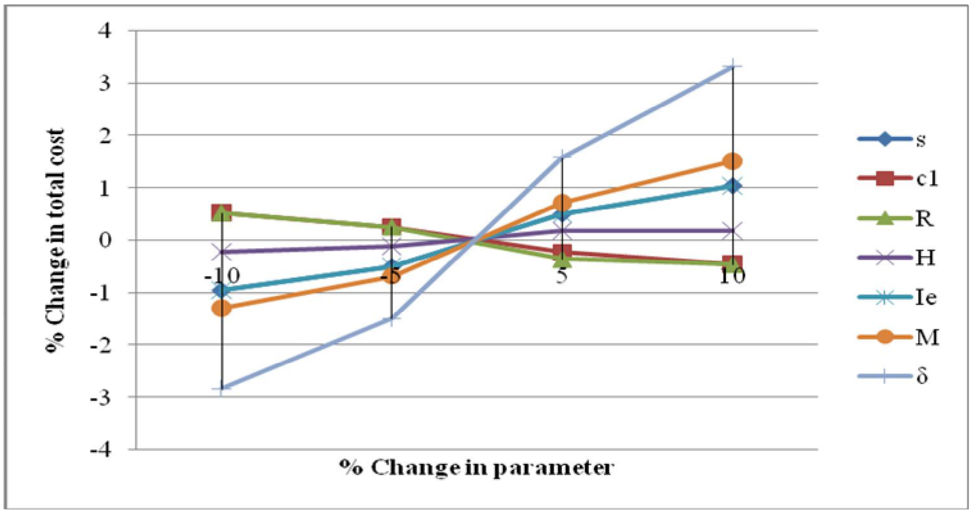

Fig 5. Behavior of optimal average total cost per unit time in Study 2

3. From Table 2 it is clear that Q^* increases with increase in the values of model parameters s, H, I_e, M and δ while Q^* decreases with increase in the value of c_1 *and* R. The obtained results show that Q^* is highly sensitive to changes in s, c_1, R, I_e, M *and* δ. It is less sensitive to changes in H.

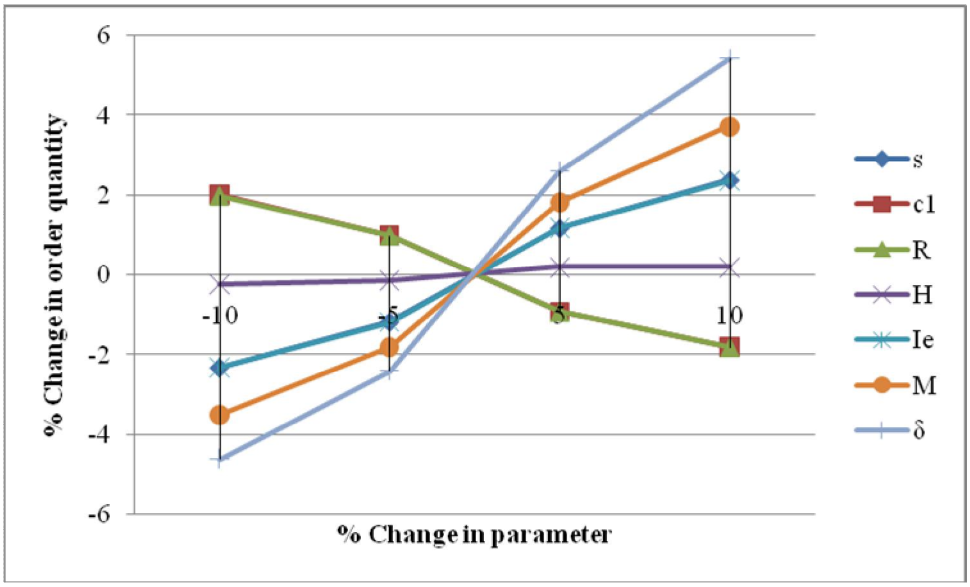

Fig 6. Behavior of optimal order quantity in Study 2

4. From Table 2 it is clear that Q^* decreases with decrease in the values of model parameters s, H, I_e, M and δ while Q^* increases with decrease in the value of c_1 *and* R. The obtained results show that Q^* is highly sensitive to changes in s, c_1, R, I_e, M *and* δ. It is less sensitive to changes in H.

Summary and Conclusions

In inventory control theory, practitioners assume that the retailer must pay for the items as soon as the items are received. However, in practice the supplier will offer the retailer a delay period, in paying for the amount of purchasing cost for attracting the retailer to buy more. Before the end of trade credit period, the retailer can sell the products and accumulate revenue and earn interest. A higher interest is charged if the payment is not settled by the end of trade credit period. Since, a large pile of stock in shelf attracts the customer to buy more; therefore, stock dependent demand is taken in this study. Holding cost rate per unit time is assumed as representing the fact that the rent or depreciation of warehouse is fix however handling charges may vary with time and this variation is directed by the deterioration of item in stock. The complete study has divided in two parts. First part gives the optimal schedule when delay period is less than that of the point at which inventory becomes zero and second part gives the optimal schedule when delay period is greater than the point from shortages starts to occur. Some but not all customers will wait for the backlogged items; for using this real fact of business, we have considered partial backlogging in the proposed study. From sensitivity analyses it is observed that the model is plenty stable.

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