Grounded Wye – Grounded Zigzag Transformer Connection Modelling in Phase Coordinates for Steady-State Studies

Saraín Montero Corzo[1], Nayeli Ramón Lara [2] , Sergio Baruch Barragán Gómez [3]

[1]Instituto Politécnico Nacional, México.

[2]Instituto Politécnico Nacional, México.

[3]Instituto Politécnico Nacional, México.

ABSTRACT

In this paper, an electrical model of a three-phase transformer connected in grounded wye – grounded zigzag configuration is developed, based in the common core three-phase transformer model as in transformation Banks. The proposed model operates under the phase coordinates scheme in order to simulate operating scenarios with balanced and unbalanced load and perform electrical studies in steady-state in an electrical circuit. The theory basis for the three-phase Transformers electrical modelling is presented and after that, the considered criteria for the suggested model development are shown. Also two ways of characterize the obtained model are presented, one by using an equivalent electrical network and other trough current injection sources, the last one is used to simulate an electrical test circuit via an iterative method which calculates the secondary voltages for three different cases with load variation. The load scenarios analyzed are: Very Low Load (open circuit approach), and balanced and unbalanced load per phase.

Keywords: electrical analysis, modelling, Zigzag connection, leakage admittance, connection matrix, nodal admittance matrix.

1. INTRODUCTION

The use of Transformers in electrical systems is fundamental since they are necessary to change the voltage levels to suitable ones for the planned purpose, for example; to elevate the voltage for long distance energy transmission and then avoid major voltage drop, or to pursue the proper and safe operation of connected loads. There are several transformer connections and each one of them has a specific aim. The IEEE transformer connections guideline [1] indicates in a detailed way, the characteristics of every connection. For transmission and distribution systems, the wye and delta connections and their diverse configurations are commonly used but there are special cases in which a Zigzag connection is required. This special cases can be: To ground an isolated system to generate a low impedance path for the zero component currents; to provide the suitable current to make protection systems work in a ground fault scenario when they are not ground referenced (ungrounded systems); to catch triplen harmonics (which produce non-linear loads) by connecting the transformer close to the harmonic source thus avoiding their propagation throughout the electrical network; to prevent the transformer saturation when a CA-CD converter is used and last but not least, to create a voltage level with a required phase shift when it is combined with a wye or delta connection.

There is not much information about the Zigzag transformer connection modelling. For example, in [2] an electrical network that includes a Zigzag connected transformer is modeled to analyze several fault conditions as the use of this device helps in the ground fault detection and prevents over-voltages in the non-fault phases. In [3] the Zigzag connection is used to eliminate third order harmonics which generate neutral conductor overheating.

When Transformers are involved in power flow studies, the wye and delta connections are commonly used and the magnitude of the input and output voltages are provided and obtained in per unit values regarding the phase to neutral voltages in both sides of the transformer. The achieved voltages must be multiplied or divided by a $\sqrt{3}$ factor to reach the phase and line-to-line voltages, depending on the type of connection that is used. The Zigzag connection produces an output with an ideal magnitude of 0.8666 and a phase shift of -30°. Therefore, it is needed to have a proper model of this connection since although the purpose of its use is to solve a specific problem, it is often required to perform power flow studies for different load scenarios and then accomplish the electric protection coordination, power factor correction, load restoration studies achievement, etc., and the inclusion of a transformer connected in this way, impacts the output voltage in the secondary side. The submitted paper shows the development of the modelling of a three-phase transformer connected in grounded wye – grounded zigzag configuration, based in the common core three-phase transformer model as in transformation banks under the phase coordinates scheme in order to analyze its electrical behavior in steady-state with balanced and unbalanced load. In addition to this, the simulation results are obtained by proving the suggested model response using an algorithm implemented in the programming language FORTRAN.

2. THREE-PHASE TRANSFORMER MODELLING

2.1 SINGLE PHASE TRANSFORMER MODEL

The two winding single phase transformer model can be represented as a four terminal device as it is shown in figure 1.

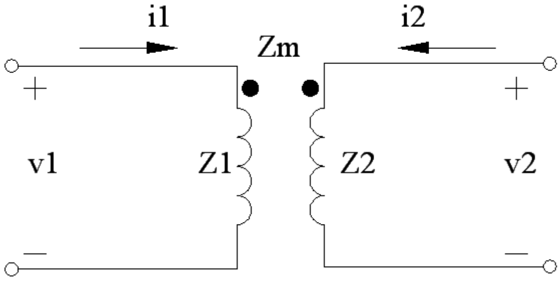

Figure 1 Four terminal single phase transformer representation

Based in this representation, both the primary and secondary currents can be determined as in (1)

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} y_1 & -y_m \\ -y_m & y_2 \end{bmatrix}^{-1} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \quad (1)$$

Where:

y_1 and y_2 Primary and secondary winding self-admittance

y_m Mutual admittance between primary and secondary winding

Due to $y_1 = y_2 \cong y_m = y$ as mentioned in [4], (1) stays represented in the way of (2)

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} y & -y \\ -y & y \end{bmatrix}^{-1} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \quad (2)$$

Where:

y Leakage Reactance (calculated from the percentage value of the short circuit impedance ($\%Z$), the ratio $\frac{X}{R}$ and the base impedance).

Another way to model the single phase transformer is through the equivalent π circuit. This model is used for the single phase representation but it is not a suitable base for the construction of the three-phase transformer model [4]. Figure 2 presents this model.

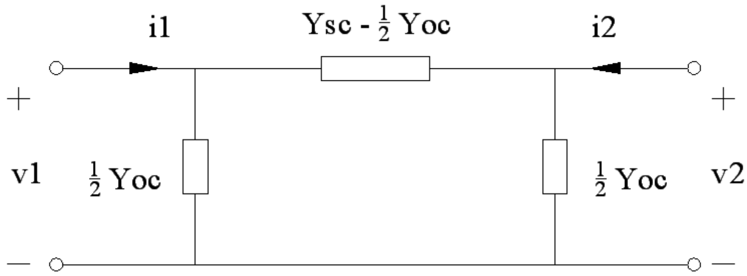

Figure 2 Single phase transformer representation using the π - equivalent circuit.

Associating Figure 2 with (1):

$$y_1 = y_2 \cong y_{sc} \quad (3)$$

And

$$y_m \cong y_{sc} - \tfrac{1}{2} y_{sc} \quad (4)$$

2.2 THREE-PHASE TRANSFORMER MODEL

The basis for the three-phase transformer model development is the construction of the primitive admittance matrix. This matrix is created from the single phase transformer model explained in (1). By using the figure 3 and the equation (5), (6) is obtained.

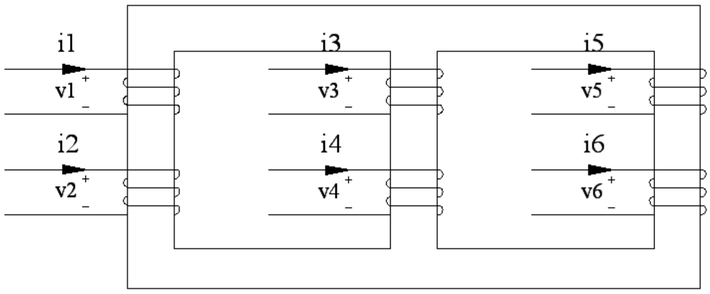

Figure 3 Two windings Three-phase transformer representation

$$[I]=\left[Y_{prim}\right][V] \qquad (5)$$

Where:

$[I]$ Nodal currents vector

$[V]$ Branch voltages vector

$\left[Y_{prim}\right]$ Primitive admittance matrix

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} & y_{13} & y_{14} & y_{15} & y_{16} \\ y_{21} & y_{22} & y_{23} & y_{24} & y_{25} & y_{26} \\ y_{31} & y_{32} & y_{33} & y_{34} & y_{35} & y_{36} \\ y_{41} & y_{42} & y_{43} & y_{44} & y_{45} & y_{46} \\ y_{51} & y_{52} & y_{53} & y_{54} & y_{55} & y_{56} \\ y_{61} & y_{62} & y_{63} & y_{64} & y_{65} & y_{66} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} \qquad (6)$$

Where:
y_{ki} Self-winding admittance, when $k=i$
y_{ki} Mutual winding admittance, when $k \neq i$

To obtain the primitive admittance matrix in (6) involves a high level of difficulty due to the need of make at least 21 short circuit tests to get the matrix elements. This task can be completed by energizing the coil i, and then bypassing the rest of the windings, the column i can be obtained with $y_{ki}={I_k}/{V_i}$ [5]. Consequently, if the magnetic flux in the transformer core is considered perfectly symmetric, the primitive matrix in (6) can be simplified as shown in (7).

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} = \begin{bmatrix} y_p & -y_m & y_m^{'} & y_m^{''} & y_m^{'} & y_m^{''} \\ -y_m & y_s & y_m^{''} & y_m^{'''} & y_m^{''} & y_m^{'''} \\ y_m^{'} & y_m^{''} & y_p & -y_m & y_m^{'} & y_m^{''} \\ y_m^{''} & y_m^{'''} & -y_m & y_s & y_m^{''} & y_m^{'''} \\ y_m^{'} & y_m^{''} & y_m^{'} & y_m^{''} & y_p & -y_m \\ y_m^{''} & y_m^{'''} & y_m^{''} & y_m^{'''} & -y_m & y_s \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} \qquad (7)$$

Where:
y_p Transformer primary winding self-admittance
y_s Transformer secondary winding self-admittance

y_m Same phase primary – secondary windings mutual admittance

$y_m^{'}$ Different phase primary windings mutual admittance

$y_m^{''}$ Different phase primary – secondary windings mutual admittance

$y_m^{'''}$ Different phase secondary windings mutual admittance

Usually the mutual admittance values with apostrophe are significantly smaller than the ones without, and then as these last are very similar between them, the matrix can be simplified from (7) to (8).

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} = \begin{bmatrix} y & -y & 0 & 0 & 0 & 0 \\ -y & y & 0 & 0 & 0 & 0 \\ 0 & 0 & y & -y & 0 & 0 \\ 0 & 0 & -y & y & 0 & 0 \\ 0 & 0 & 0 & 0 & y & -y \\ 0 & 0 & 0 & 0 & -y & y \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} \qquad (8)$$

With (8) the behavior of the three-phase transformer cannot still be simulated so it is necessary to include the way in which their windings are connected. In order to achieve this, the equations (9), (10) and (11) are used.

$$[Y_{node}] = [C^T][y_{prim}][C] \qquad (9)$$

$$[I_{node}] = [Y_{node}][V_{node}] \qquad (10)$$

$$[V] = [C][V_{node}] \qquad (11)$$

Where:

$[Y_{node}]$ Nodal Admittance Matrix

$[C]$ Winding Connection Matrix

$[C^T]$ Transposed Matrix of $[C]$

$[V]$ Branch Voltages Vector

$[V_{node}]$ Nodal Voltages Vector

$[I_{node}]$ Nodal currents vector

$$[V_{node}] = \begin{bmatrix} V_A \\ V_B \\ V_C \\ V_a \\ V_b \\ V_c \end{bmatrix}$$

$$[I_{node}] = \begin{bmatrix} I_A \\ I_B \\ I_C \\ I_a \\ I_b \\ I_c \end{bmatrix}$$

To consider changes in the turns ratio not equal to the nominal values (off-nominal tap ratio), the following must be done:

- Divide upper right and lower left Y_{node} submatrices by $\alpha\beta$
- Divide left upper submatrix Y_{node} by α^2
- Divide right lower submatrix Y_{node} by β^2

Where:
α Tapping on the primary side
β Tapping on the secondary side

When per unit values are used in (10), the previous concept must be applied in the case of delta connection so the output variation is affected by the $\sqrt{3}$ factor. Therefore, α and/or β would be $\sqrt{3}$. In [4], [5] and [6] the basis for building the Y_{node} matrix for several three-phase transformer connections is displayed.

3. ZIGZAG MODEL DEVELOPMENT

In Figure 4 the physical wiring on the secondary side grounded Zigzag connection is shown thus its phasor diagram. In this work, the grounded wye connection in the primary side is used for the modelling development.

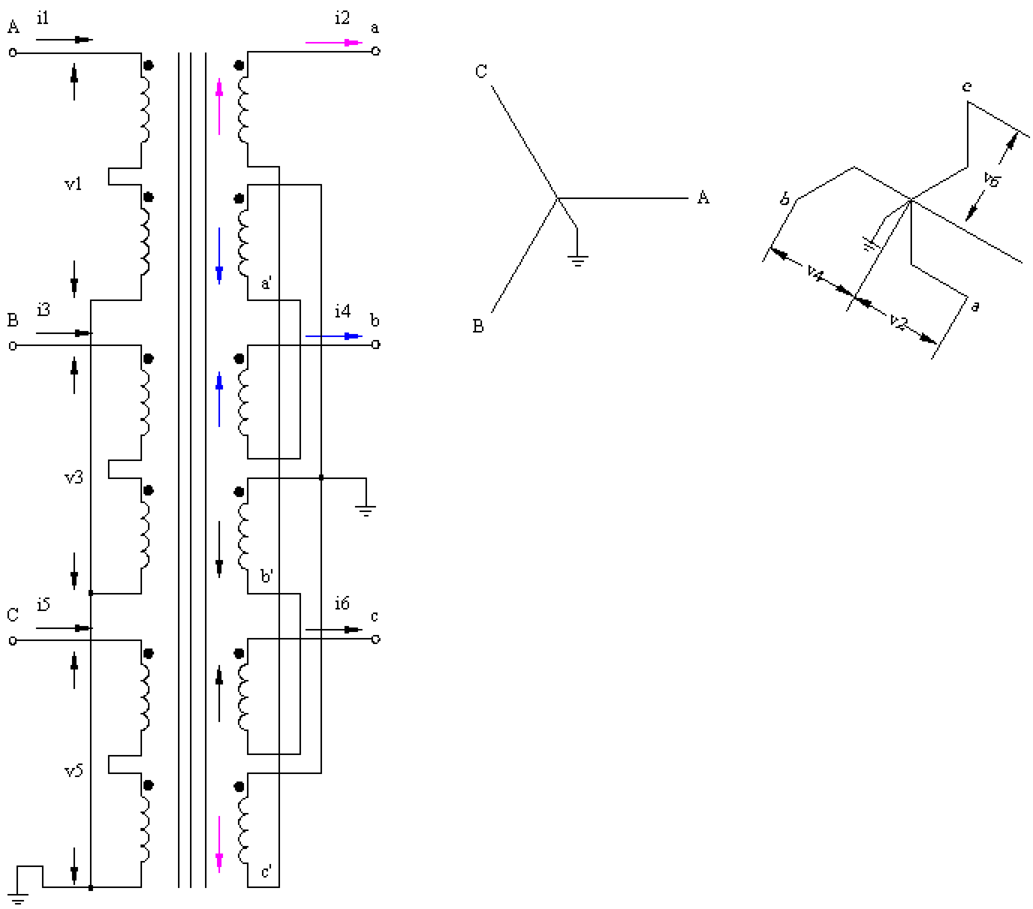

Figure 4 Electrical and phase diagrams of the suggested connection

In order to determinate the output voltage in the Zigzag connection, KVL (Kirchhoff's Voltage Law) is applied for each phase, therefore, according to the single secondary winding voltage as the new formed secondary phase voltage and per unit value, it has (12), (13) and (14) as follows:

$$\begin{aligned} V_a &= \sqrt{3}E\angle{-30^\circ} \\ V_b &= \sqrt{3}E\angle{-150^\circ} \\ V_c &= \sqrt{3}E\angle{90^\circ} \end{aligned} \quad (12)$$

$$E = \frac{V_s}{2} \quad (13)$$

$$V_s = V_p \frac{1}{a} \quad (14)$$

$$a = \frac{N_p}{N_s} \quad (15)$$

Replacing (13) in (12), the phase voltages are obtained as:

$$\begin{aligned} V_a &= \frac{\sqrt{3}}{2} V_s \angle -30^\circ = 0.866 V_s \angle -30^\circ \\ V_b &= \frac{\sqrt{3}}{2} V_s \angle -150^\circ = 0.866 V_s \angle -150^\circ \\ V_c &= \frac{\sqrt{3}}{2} V_s \angle 90^\circ = 0.866 V_s \angle 90^\circ \end{aligned} \qquad (16)$$

If $V_s = 1$

$$\begin{aligned} V_a &= \frac{\sqrt{3}}{2} V_s \angle -30^\circ = 0.866 \angle -30^\circ \\ V_b &= \frac{\sqrt{3}}{2} V_s \angle -150^\circ = 0.866 \angle -150^\circ \\ V_c &= \frac{\sqrt{3}}{2} V_s \angle 90^\circ = 0.866 \angle 90^\circ \end{aligned} \qquad (17)$$

Where:
E Secondary winding RMS voltage per coil
V_s Same phase interconnected secondary total voltage
V_p Primary voltage
a Turns ratio
N_p Primary winding turns ratio
N_s Secondary winding turns ratio

In Figure 4 can be observed that the secondary winding is divided in two parts and then one single secondary winding current generates a magnetic flux in opposite direction of the other. This can be seen for example that one single-phase current circulating throughout the b phase (i_4) is actually circulating through the second part of the phase a winding. This feature is considered to create the primitive admittance matrix as shown in (18). The $2y$ term is due to the primary winding leakage reactance doubles the value of one single winding.

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} = \begin{bmatrix} 2y & -y & 0 & y & 0 & 0 \\ -y & 2y & 0 & 0 & y & 0 \\ 0 & 0 & 2y & -y & 0 & y \\ y & 0 & -y & 2y & 0 & 0 \\ 0 & y & 0 & 0 & 2y & -y \\ 0 & 0 & y & 0 & -y & 2y \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} \qquad (18)$$

Applying (11) it has:

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_A \\ V_B \\ V_C \\ V_a \\ V_b \\ V_c \end{bmatrix} \qquad (19)$$

By using (9), the nodal admittance matrix is achieved

$$y_{node} = \begin{bmatrix} 2y & 0 & 0 & -y & y & 0 \\ 0 & 2y & 0 & 0 & -y & y \\ 0 & 0 & 2y & y & 0 & -y \\ -y & 0 & y & 2y & 0 & 0 \\ y & -y & 0 & 0 & 2y & 0 \\ 0 & y & -y & 0 & 0 & 2y \end{bmatrix} \quad (20)$$

By making use of (9) and (10) the following procedure is made for the phase A

$$I_1 = 2yV_A - yV_a + yV_b$$

Grouping terms

$$I_1 = 2yV_A + y(V_A - V_a) - y(V_A - V_b)$$

For the secondary phase a

$$I_2 = -yV_A + yV_C + 2yV_a$$

Grouping terms

$$I_2 = 2yV_a + y(V_a - V_A) - y(V_a - V_C)$$

Performing the same method for getting I_3, I_4, I_5 and I_6, the electrical representation in (20) is get as it is displayed in Figure 5.

$$\begin{aligned} I_1 = I_A &= 2yV_A + y(V_A - V_a) - y(V_A - V_b) \\ I_2 = I_a &= 2yV_a + y(V_a - V_A) - y(V_a - V_C) \\ I_3 = I_B &= 2yV_B + y(V_B - V_b) - y(V_B - V_c) \\ I_4 = I_b &= 2yV_b + y(V_b - V_B) - y(V_b - V_A) \\ I_5 = I_C &= 2yV_C + y(V_C - V_c) - y(V_C - V_a) \\ I_6 = I_c &= 2yV_c + y(V_c - V_C) - y(V_c - V_B) \end{aligned} \quad (21)$$

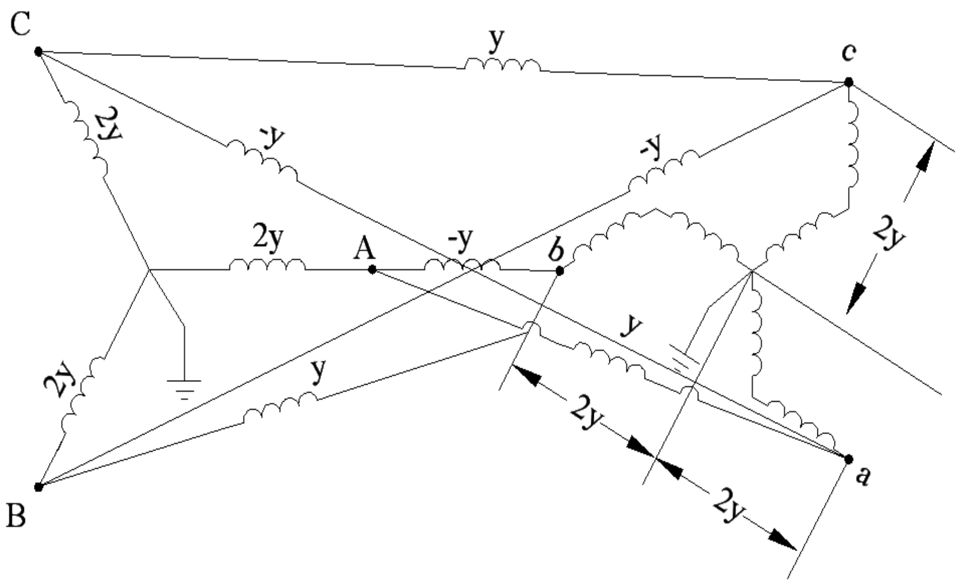

Figure 5 Electrical Representation of the Nodal Admittance Matrix y_{node}

Another way of represent the circuit exhibited in Figure 5, is employing six current sources, three for the primary and three for the secondary (one for each phase). This methodology is proposed in [6]. By applying this, the following nodal currents are got and after that, the final circuit is illustrated in Figure 6.

$$\begin{aligned} I_p^A &= -2yV_A + y\left(V_A - V_b\right) \\ I_p^B &= -2yV_B + y\left(V_B - V_c\right) \\ I_p^C &= -2yV_C + y\left(V_C - V_a\right) \\ I_s^a &= -2yV_a + y\left(V_a - V_C\right) \\ I_s^b &= -2yV_b + y\left(V_b - V_A\right) \\ I_s^c &= -2yV_c + y\left(V_c - V_B\right) \end{aligned} \quad (22)$$

$y_3 = y$

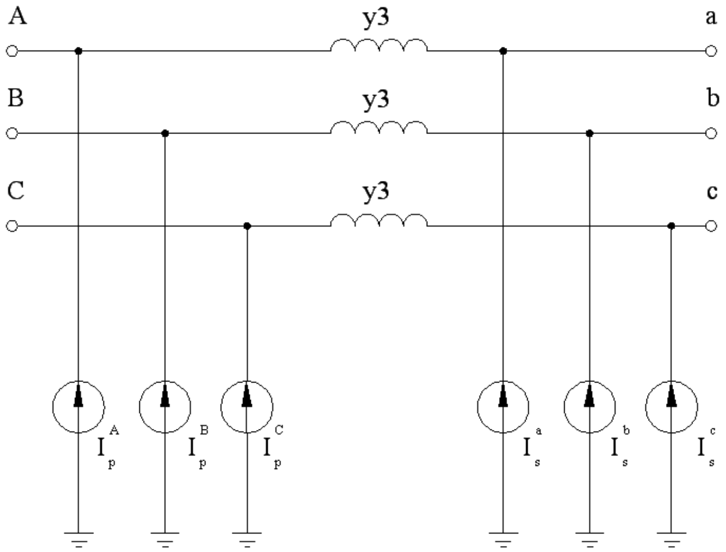

Figure 6 Grounded Wye – Grounded Zigzag Transformer Model Representation using current injection

4. RESULTS

The Test System showed in Figure 7 was used to prove the Grounded Wye – Grounded Zigzag Connection performance. The simulation model employed was the current injection one mentioned among a computational algorithm which determinates the voltages in the secondary side via a convergence iterative method. The algorithm was developed using the programming language FORTRAN and implemented at three different test cases. The three test cases analyzed are: 1MΩ pure resistive load (open circuit approach), 400 + 300j kVA balanced load, and unbalanced load at an unbalance rate as follows: Phase a=50%, Phase b=30% and Phase c=20%.

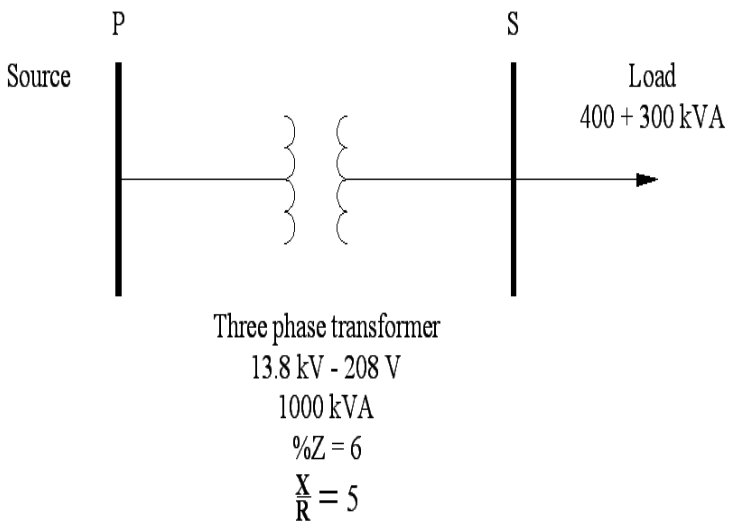

Figure 7 Test System

The results obtained in the simulation procedure for the three suggested scenarios are displayed in tables I, II and III. The maximum amount of executed iterations is indicated when a precision greater than or equal to 1E-4 is achieved.

Table I Phase Voltages for the open circuit approach scenario

No. ite.	Va	angle	Vb	Angle	Vc	angle
0	1.0000	-60.0041	1.0000	179.9959	1.0000	59.9959
1	0.8660	-30.0003	0.8660	-150.0003	0.8660	89.9997
2	0.8660	-30.0014	0.8660	-150.0014	0.8660	89.9986

Table II Phase Voltages for the 400 + 300j kVA balanced load scenario

No. ite.	Va	angle	Vb	angle	Vc	angle
0	1.0066	-61.6756	1.0066	178.3244	1.0066	58.3244
1	0.8444	-30.5794	0.8444	-150.5794	0.8444	89.4206
2	0.8527	-30.7879	0.8527	-150.7879	0.8527	89.2121
3	0.8528	-30.7772	0.8528	-150.7772	0.8528	89.2228

Table III Phase Voltages for the unbalanced load scenario (a=50%, b=30% and c=20%)

No.ite.	Va	angle	Vb	angle	Vc	angle
0	1.0102	-62.5029	1.0059	178.4905	1.0039	58.9904
1	0.8332	-30.8639	0.8466	-150.5221	0.8531	89.6507
2	0.8458	-31.1995	0.8541	-150.7071	0.8581	89.5323
3	0.8460	-31.1743	0.8542	-150.6985	0.8582	89.5360

5. CONCLUSIONS

The use of three-phase transformers connected in the Zigzag configuration is commonly applied just for special cases in electrical systems but its acknowledgement is necessary to be considered in conventional electrical studies due to the changes in the observed variables into the electrical system performance. Thereby, knowing the models that provide a proper response under different steady-state operation scenarios is primary important. In this work, a procedure to generate a three-phase transformer model connected in the grounded wye - grounded zigzag topology was presented by using standard connection three-phase transformer models as starting point. The obtained results using the suggested model were satisfactory, in the open circuit approach load, the values in Table I are very similar to the calculated ones in (17). In the balanced load study performance results displayed in Table II can be noticed that the output voltage decrease the same in each one of the phases and the shift angles begin to deviate from the non-load scenario. If the calculation to determine the phase shift between lines is made, it can be proved the 120° shift between them. In the unbalanced load case, it can be appreciated in Table III that the phase voltages are a bit different and there is no longer the 120° phase shift. According to the simulation executed, the Zigzag connection responses better under unbalanced load scenarios, as in [6] where another simulation for the same unbalanced load conditions was run but for a Delta – Grounded wye Connection and the voltage deviation of the strong loaded phase respect to the ideal expected voltage (1 p.u.) was 3.52%, while in the Zigzag connection was around 2.06%.

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AUTHORS

Saraín Montero Corzo. M.Sc. in Electrical Engineering from SEPI-ESIME-IPN, México in 2006. Electrical Engineer graduated from Instituto Tecnológico de Tuxtla Gutiérrez, México in 1998. Nowadays is associate professor at ESIME-Zacatenco. The interest areas for him are Electrical System analysis for Power and Distribution, Power Electronics and Energy Quality.

Nayeli Ramón Lara. Is associate professor at the ESIME-IPN. M.Sc. in Electrical Engineering at SEPI-ESIME-IPN in 2006. Electronics and Communications engineer graduated from Instituto Tecnológico y de Estudios Superiores de Monterrey en 2002. The interest areas for her are, Electrical Machines Control, Power Electronics and Education.

Sergio Baruch Barragán Gómez. Associate professor from ESIME-IPN. M.Sc. in Electrical Engineering from SEPI-ESIME-IPN in 2004. Electrical Engineering from ESIME-IPN en 2001. The Interested areas for him are free Software, Analysis and Economic Operation of Electrical Systems.