

# System Stability of Synchronous Machine with Small Signal Representation including Rotor Circuit Dynamics

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## ABSTRACT

*In an interconnected power system, continuously growing in size and extending over vast geographical regions it becomes very difficult to maintain synchronism between various parts of power system. In this paper, the focus is made on small signal performance of a synchronous machine connected to a large system through transmission lines. The model detail is gradually increased by accounting for the effects of the dynamics of the field circuit. By using Eigen value, analysis has been made about the small signal performance.*

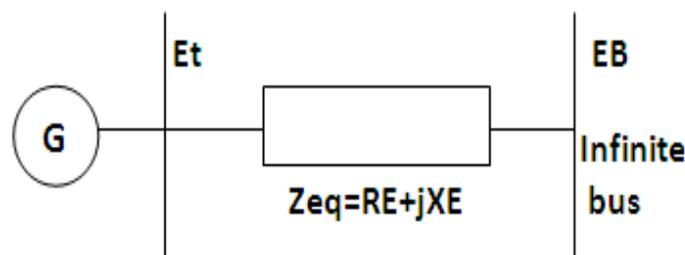
**Keywords:-** Stability, Field Circuit Dynamics, Synchronous Machine, State Matrix, Field Flux Linkage etc.

## 1. INTRODUCTION

Successful operation of a power system depends on the ability to provide reliable and uninterrupted service to the loads. The first requirement is to keep the synchronous generators running in parallel and with adequate capacity. Synchronous machines do not easily fall out of step under normal conditions. Conditions do arise, however, such as a fault on the network, failure in a piece of equipment, sudden application of a major load may not be adequate, and small impacts in the system may cause these machines to lose synchronism. A second requirement is to maintain the integrity of the power network. Interruptions in the transmission system may hinder the flow of power to the load. This usually requires a study of large geographical areas since almost all power systems are interconnected with neighboring systems.

## 2. SMALL SIGNAL STABILITY

Small signal stability is the ability of the power system to maintain synchronism when subjected to small disturbances. Here we study the small-signal performance of a machine connected to a large system through transmission lines. A equivalent system configuration is shown below in Fig.1 which is equivalent of the transmission network external to the machine and adjacent transmission.



**Figure 1:** Small Signal Studies of equivalent system

By gradually increasing the model detail by accounting for the effects of the dynamics of the field circuit and excitation systems, the development of the expressions for the elements of the state matrix as explicit functions of system parameters. It is useful in gaining a physical insight into the effects of field circuit dynamics and in establishing the basis for methods of enhancing stability through excitation control.

## 3. EFFECT OF FIELD CIRCUIT DYNAMICS

The system performance is considered including the effect of the field flux variations. The field voltage is assumed constant (manual excitation control). The state-space model of the system is developed by first reducing the

synchronous machine equations to an appropriate form and then combining them with the network equations. Time is expressed in seconds, angles in electrical radians and all other variables in per unit.

**3.1 Equations for Synchronous Machine**

The rotor angle  $\delta$  is the angle ( in electrical radian) by which the q-axis leads the reference EB. The equivalent circuits relating the machine flux linkages and current are as shown min Fig. 2.

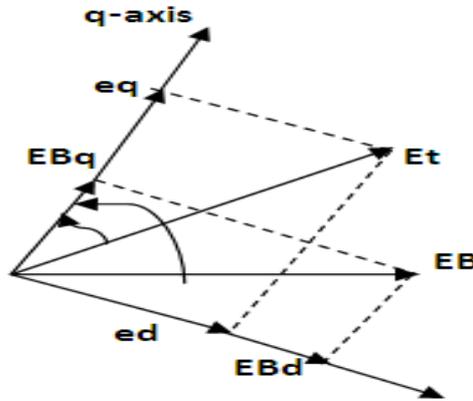


Figure 2: Rotor angle & EB representation

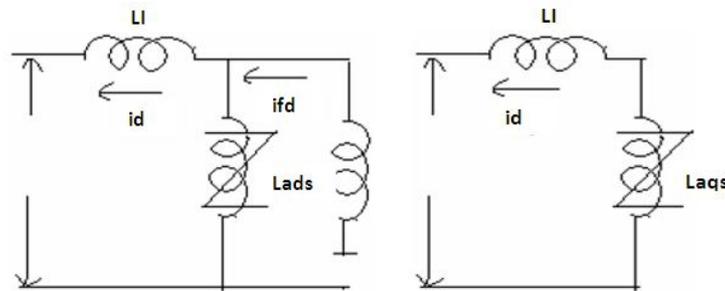


Figure 3: Equivalent circuit for Machine Flux Linkages & Current

The Stator and Rotor flux linkage are given by:

$$\Psi_d = -L_1 i_d + \Psi_{ad} \quad (1)$$

$$\Psi_q = -L_1 i_q + \Psi_{aq} \quad (2)$$

$$\Psi_{fd} = L_{fd} i_{fd} + \Psi_{ad} \quad (3)$$

In the above equations  $\Psi_{ad}$  and  $\Psi_{aq}$  are the air-gap (mutual) flux linkages and  $L_{fd}$  is the saturated value of mutual inductance.

The field current:

$$i_{fd} = (\Psi_{fd} - \Psi_{ad}) / L_{fd} \quad (4)$$

The d-axis mutual flux linkage in terms of  $\Psi_{fd}$  and  $i_d$  is,

$$\Psi_{ad} = L'_{ads} (-i_d + \Psi_{fd}) / L_{fd} \quad (5) \text{ Where}$$

$$L'_{ads} = 1 / (1/L_{ads} + 1/L_{fd}) \quad (6)$$

Since there is no rotor circuit considered in the q-axis, the mutual flux linkage is given by,

$$\Psi_{aq} = -L_{aqs} * i_q \quad (7)$$

The air-gap torque is,

$$T_e = \Psi_{ad} * i_q - \Psi_{aq} * i_d \quad (8)$$

With  $\dot{\Psi}$  terms and speed variations neglected the stator voltage equations become,

$$e_d = -R_a i_d + (L_1 i_q - \Psi_{aq}) \quad (9)$$

$$e_q = -R_a i_q + (L_1 i_d - \Psi_{ad}) \quad (10)$$

**3.2 Network Equations**

Since there is only one machine, the machine as well as network equations can be expressed in terms of one reference frame, ie. d-q reference frame of the machine. Referring to Figure, the machine terminal and infinite bus voltages in terms of the d and q components are:

$$E_t = e_d + je_q \tag{11}$$

$$E_B = E_{BD} + jE_{Bq} \tag{12}$$

$$E_t = E_B + (R_E + jX_E)I_t \tag{13}$$

$$e_d + je_q = (E_{BD} + jE_{Bq}) + (R_E + jX_E)(i_d + ji_q) \tag{14}$$

$$e_d = R_E i_d - X_E i_q + E_{Bd} \tag{15}$$

$$e_q = R_E i_d - X_E i_d + E_{Bq} \tag{16}$$

$$E_{Bd} = E_B \sin\delta \tag{17}$$

$$E_{Bq} = E_B \cos\delta \tag{18}$$

Where

Using equations and to eliminate  $e_d, e_q$  in equations and using the expressions for  $\Psi_{ad}$  and  $\Psi_{aq}$  given by equations and the following expressions are obtained for  $i_d$  and  $i_q$  in terms of state variables  $\Psi_{fd}$  and  $\delta$ .

$$i_d = (X_{Tq} [\Psi_{fd} \left( \frac{L_{ads}}{L_{ads} + L_{fd}} - E_B \cos\delta \right) - R_T E_B \sin\delta]) / D \tag{19}$$

$$i_q = (R_T [\Psi_{fd} \left( \frac{L_{ads}}{L_{ads} + L_{fd}} - E_B \cos\delta \right) - X_{Td} E_B \sin\delta]) / D \tag{20}$$

The reactance is saturated values and in per unit they are equal to the corresponding inductances.

**3.3 Linearized System Equations**

Expressing equations and in terms of perturbed values, we may write,

$$\Delta i_d = m_1 \Delta\delta + m_2 \Delta\Psi_{fd} \tag{21}$$

$$\Delta i_q = n_1 \Delta\delta + n_2 \Delta\Psi_{fd} \tag{22}$$

Where,

$$m_1 = E_B (X_{Tq} \sin\delta_0 - R_T \cos\delta_0) / D \tag{23}$$

$$n_1 = E_B (R_T \sin\delta_0 - X_{Td} \cos\delta_0) / D \tag{24}$$

$$m_2 = \left( \frac{X_{Tq}}{D} \right) * \frac{L_{ads}}{L_{ads} + L_{fd}} \tag{25}$$

$$n_2 = \left( \frac{R_T}{D} \right) * \frac{L_{ads}}{L_{ads} + L_{fd}} \tag{26}$$

By linearizing equations (5) & (7) and substituting them in the above expressions for  $\Delta i_d$  and  $\Delta i_q$  we get,

$$\Delta\Psi_{ad} = \left( \frac{1}{L_{fd}} - m_2 \right) L'_{ads} \Delta\Psi_{fd} - m_2 L'_{ads} \Delta\delta \tag{27}$$

$$\Delta\Psi_{aq} = -n_2 L_{ads} \Delta\Psi_{fd} - n_1 L_{ads} \Delta\delta \tag{28}$$

Linearizing equation (4) and substituting for  $\Delta\Psi_{ad}$  from equation (27) gives,

$$\Delta i_{fd} = \left( 1 - \frac{L'_{ads}}{L_{fd}} + m L'_{ads} \right) \Delta\Psi_{fd} / L_{fd} \tag{29}$$

The Linearized form of equation (8) is,

$$\Delta T_e = \Psi_{ad0} \Delta i_q + i_{q0} \Delta\Psi_{ad} - \Psi_{aq0} \Delta i_d + i_{d0} \Delta\Psi_{aq} \tag{30}$$

Substituting for  $\Delta i_d, \Delta i_q, \Delta\Psi_{ad}$  and  $\Psi_{aq}$  from equations (21) to (28), we obtain

$$\Delta T_e = K_1 \Delta\delta + K_2 \Delta\Psi_{fd} \tag{31}$$

Where,

$$K_1 = n_1 (\Psi_{ad0} + L_{aq0} i_{d0}) - m_1 (\Psi_{aq0} + L'_{ads} i_{q0}) \tag{32}$$

$$K_2 = n_2 (\Psi_{ad0} + L_{aq0} i_{d0}) - m_2 (\Psi_{aq0} + L'_{ads} i_{q0}) + L'_{ads} / L_{fd} i_{q0} \tag{33}$$

By using the expressions for  $\Delta i_{fd}$  and  $\Delta T_e$  given by equations (29) and (31), we obtain the system equations in the desired final form:

$$a_{11} = \frac{-K_D}{2H}, a_{12} = \frac{-K_1}{2H}, a_{13} = \frac{-K_2}{2H} \quad (34)$$

$$a_{21} = \omega_0 = 2\pi f_0, a_{32} = -(\omega_0 R_{fd}/L_{fd})m_1 L'_{ads} \quad (35)$$

$$a_{33} = -\omega_0 R_{fd}/L_{fd} [1 - \frac{L'_{ads}}{L_{fd}} + L_{fd} L'_{ads}] \quad (36)$$

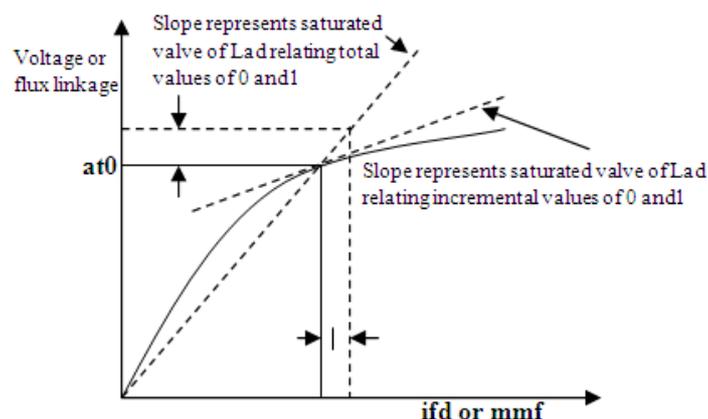
$$b_{11} = \frac{1}{2H}, b_{32} = \omega_0 R_{fd}/L_{ad0} \quad (37)$$

and  $\Delta T_m$  and  $\Delta E_{fd}$  depends on prime-mover and excitation controls. With constant mechanical input  $\Delta T_m = 0$ ; with constant exciter output voltage  $\Delta E_{fd} = 0$ . The mutual inductances in the above equations are saturated values.

### 3.4 Representation of Saturation in Small Signal Studies

Total saturation is associated with total values of flux linkages and currents. Incremental saturation is associated with perturbed values of flux linkages and currents. Therefore, the incremental slope of the saturation curve is used in computing the incremental saturation as shown in figure. Denoting the incremental saturation factor  $K_{sd(incr)}$ , we have

$$L_{ads(incr)} = K_{sd(incr)} L_{adu} \quad (38)$$



**Figure 4:** Distinction between incremental & total saturation

### 3.5 Formulating the State Matrix

a) The following steady-state operating conditions, machine parameters and network parameters are given:

Pt Qt Et RE XE

Ld Lq Ll Ra Lfd Rfd Asat Bsat ΨT1

Alternatively  $E_B$  may be specified instead of  $Q_t$  or  $E_t$ .

b) The first step is to compute the initial steady-state values of system variables:

It, power factor angle  $\Phi$ , Total saturation factors  $K_{sd}$  and  $K_{sq}$

$X_{ds} = L_{ds} = K_{sd} L_{adu} + L_l$

$X_{qs} = L_{qs} = K_{sq} L_{aqu} + L_l$

$\delta i = \tan^{-1}((I_t X_{qs} \cos \Phi - I_t R_a \sin \Phi) / (E_t + I_t R_a \cos \Phi - I_t X_{qs} \sin \Phi))$

$ed0 = E_t \sin \delta i$

$eq0 = E_t \cos \delta i$

$id0 = I_t \sin(\delta i + \Phi)$

$iq0 = I_t \cos(\delta i + \Phi)$

$EBd0 = ed0 - REid0 + XEiq0$

$EBq0 = eq0 - REiq0 + XEid0$

$\delta 0 = \tan^{-1}(EBd0 / EBq0)$

$EB = (EBd0^2 + EBq0^2)^{1/2}$

$ifd0 = (eq0 + Raiq0 + Ldsido) / L_{ads}$

$Efd0 = L_{adu} ifd0$

$$\Psi_{ad0} = L_{ads}(-i_{d0} + i_{fd0})$$

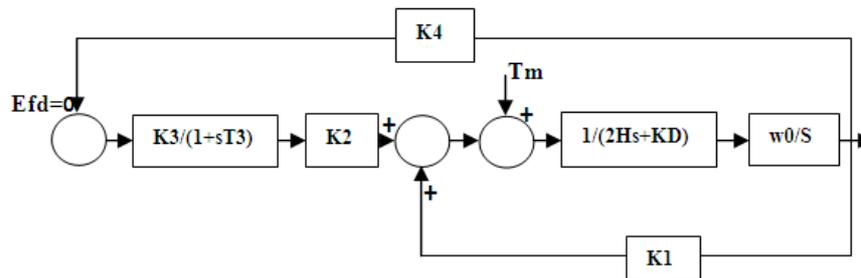
$$\Psi_{aq0} = -L_{aqs}i_{q0}$$

- c) The next step is to compute incremental saturation factors and the corresponding Saturated values of  $L_{ads}$ ,  $L_{aqs}$ ,  $L'_{ads}$  and then,  $R_T$ ,  $X_{Tq}$ ,  $X_{Td}$ ,  $D$ ,  $m_1$ ,  $m_2$ ,  $n_1$ ,  $n_2$ ,  $K_1$ ,  $K_2$

- d) Finally, compute the elements of matrix A from equation (34-37).

#### 4. BLOCK DIAGRAM REPRESENTATION

Figure.5 shows the block diagram representation of the small-signal performance of the system. In this representation, the dynamic characteristics of the system are expressed in terms of the so-called K constants. The basis for the block diagram and the expressions for the associated constants are developed below.



**Figure 5:** Block Diagram Representation of Small Signal Performance

From equation (32) we may express the change in air-gap torque as a function of  $\Delta\delta$  and  $\Delta\Psi_{fd}$  as follows:

$$\Delta T_e = K_1 \Delta\delta + K_2 \Delta\Psi_{fd}$$

Where  $K_1 = \Delta T_e / \Delta\delta$  with constant  $\Psi_{fd}$

$K_2 = \Delta T_e / \Delta\Psi_{fd}$  with constant rotor angle  $\delta$

The component of torque given by  $K_1 \Delta\delta$  is in phase with  $\Delta\delta$  and hence represents a synchronizing torque component. The component of torque resulting from variations in field flux linkage is given by  $K_2 \Delta\Psi_{fd}$ . The variation of  $\Psi_{fd}$  is determined by the field circuit dynamic equation.

$$p\Delta\Psi_{fd} = a_{32}\Delta\delta + a_{33}\Delta\Psi_{fd} + b_{32}\Delta E_{fd}$$

By grouping terms involving  $\Delta\Psi_{fd}$  and rearranging, we get

$$\Delta\Psi_{fd} = K_3(\Delta E_{fd} - K_4\Delta\delta) / (1 + pT_3)$$

##### 4.1 K-constants in the expanded form

Expression for K constants in terms of the elements of matrix A is made. In the literature, they are usually expressed explicitly in terms of the various system parameters. The constants can be computed as :

$$K_1, K_2, K_3, K_4, a_{32}, a_{33} \& T_3$$

If the elements of matrix A are available, the K constants may be computed directly from them. The expanded forms are derived here to illustrate the form of expressions used in the literature. An advantage of these expanded forms is that the dependence of the K constants on the various system parameters is more readily apparent. A disadvantage, however, is that some inconsistencies appear in representing saturation effects.

##### 4.2 Effect of field flux linkage variation on system stability

It was seen from the block diagram of figures that, with constant field voltage ( $\Delta E_{fd} = 0$ ), the field flux variations are caused only by feedback of  $\Delta\delta$  through the coefficient  $K_4$ . This represents the demagnetizing effect of the armature reaction. The change in air-gap torque due to field flux variations caused by rotor angle changes is given by:

$$(\Delta T_e / \Delta\delta) \text{ due to } \Delta\Psi_{fd} = -K_2 K_3 K_4 / (1 + sT_3)$$

The constants  $K_2$ ,  $K_3$  and  $K_4$  are usually positive. The condition of  $\Delta\Psi_{fd}$  to synchronizing and damping torque components depends on the oscillating frequency as discussed below.

- (a) In the steady state and at very low oscillating frequencies ( $s = j\omega \rightarrow 0$ ):

$$\Delta T_e \text{ due to } \Delta\Psi_{fd} = -K_2 K_3 K_4 \Delta\delta$$

The field flux variation due to  $\Delta\delta$  feedback (i.e., due to armature reaction) introduces a negative synchronizing torque component. The system becomes monotonically unstable when this exceeds  $K_1 \Delta\delta$ . The steady state stability limit is reached when

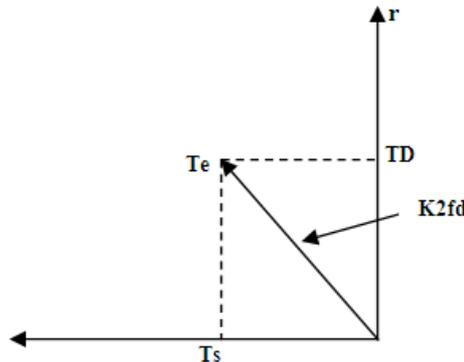
$$K2K3K4 = K1$$

(b) At oscillating frequencies much higher than  $1/T3$

$$\begin{aligned} \Delta T_e &\approx -K2K3K4 \Delta\delta / j\omega T3 \\ &= K2K3K4 j \Delta\delta / \omega T3 \end{aligned}$$

Thus, the component of air-gap torque due to  $\Delta\Psi_{fd}$  is  $90^\circ$  ahead of  $\Delta\delta$  or in phase with  $\Delta\omega$ . Hence,  $\Delta\Psi_{fd}$  results in a positive damping torque component.

(c) At typical machine oscillating frequencies of about 1 Hz (2 rad/s),  $\Delta\Psi_{fd}$  results in a positive damping torque component and a negative synchronizing torque component. The net effect is to reduce slightly the synchronizing torque component and increase the damping torque component.



**Figure 6:** Positive damping torque and negative synchronizing torque due to  $K2\Delta\Psi_{fd}$

### 4.3 Special situations with K4 negative

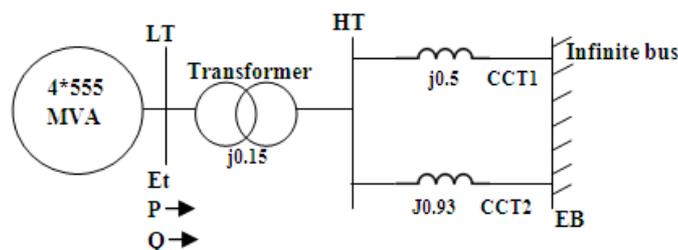
The coefficient  $K4$  is normally positive. As long as it is positive, the effect of field flux variation due to armature reaction ( $\Delta\Psi_{fd}$  with constant  $E_{fd}$ ) is to introduce a positive torque component. However, there can be situations where  $K4$  is negative. From the expression given by Equation:

$$K4 \text{ is negative when } (X_E + X_Q) \sin \delta_0 - (R_a + R_E) \cos \delta_0 \text{ is negative.}$$

This is the situation when a hydraulic generator without damper windings is operated at light load and is connected by a line of relatively high resistance to reactance ratio to a large system. Also  $K4$  can be negative when a machine is connected to a large local load, supplied partly by the generator and partly by the remote large system. Under such conditions, the torques produced by induced currents in the field due to armature reaction have components out of phase with  $\Delta\omega$ , and produce negative damping.

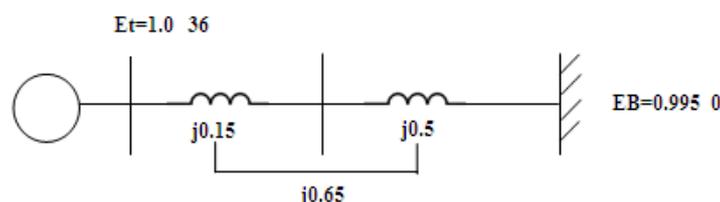
## 5. STEPS TAKEN FOR SIMULATION

The analysis of small-signal stability of the system can be carried out for the figure.7, Thermal Generating Station. Including the effects of the generator field circuit dynamics. The parameters of each of the four generators of the plant in pu on its rating.



**Figure 7:** Thermal generating station

The elements of the state matrix  $A$  representing the small signal performance of the system. The constants  $K1$  to  $K4$  and  $T3$  associated with the block diagram representation of figure. Eigen values of  $A$  and the corresponding Eigen vectors and participation matrix; frequency and damping ratio of the oscillatory mode. Steady state synchronizing torque coefficient, damping and synchronizing torque coefficients at the rotor oscillating frequency.



**Figure 8:** Equivalent Circuit model of the System

**6.RESULT ANALYSIS**

State matrix A

$$\mathbf{A} = \begin{bmatrix} 0 & -0.1094 & -0.1236 \\ 376.9911 & 0 & 0 \\ 0 & -0.1942 & -0.4229 \end{bmatrix}$$

Eigen vectors matrix

$$\mathbf{V} = \begin{bmatrix} -0.0003 + 0.0170i & -0.0003 - 0.0170i & 0.0004 \\ 0.9994 & 0.9994 & -0.7485 \\ -0.0015 + 0.0302i & -0.0015 - 0.0302i & 0.6631 \end{bmatrix}$$

Participation matrix

$$\mathbf{P} = \begin{bmatrix} 0.5005 - 0.0085i & 0.5005 + 0.0085i & -0.0011 - 0.0000i \\ 0.5005 - 0.0085i & 0.5005 + 0.0085i & -0.0011 - 0.0000i \\ -0.0011 + 0.0171i & -0.0011 - 0.0171i & 1.0022 + 0.0000i \end{bmatrix}$$

Steady-state synchronizing torque coefficient  $K_s = 0.368$ Synchronizing torque coefficient at rotor oscillating frequency  $K_{srf} = 0.764$ Damping coefficient at rotor oscillating frequency  $K_{drf} = 1.531$ Undamped natural frequency of the oscillatory mode  $\omega_n = 6.413$ Damping ratio of the oscillatory mode  $\zeta_p = 0.017$ **7. CONCLUSIONS**

For a given clearing angle, as the maximum power limit of the various power angles is raised, it adds to the transient stability limit of the system. The maximum steady power of a system can be increased by raising the voltage profile of a system and by reducing the transfer reactance. Thus it was observed that by considering the effect of rotor circuit dynamics, the study of model in greater details. The expressions were developed for the elements of the state matrix as explicit functions of system parameters. In addition to the state-space representation, block diagram representation was taken into consideration to analyze the system stability characteristics. While this approach is not suited for a detailed study of large systems, it is useful in gaining a physical insight into the effects of field circuit dynamics and in establishing the basis for methods of enhancing stability through excitation control.

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