ConstraintOptimal Control for Gyroscope

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ABSTRACT

In this paper, an optimal control is designed for a gyroscope system. The gyroscope is very much useful in various applications such as navigational, automotive, aeronautical and space engineering. Gyroscope control system dynamics is a very popular system which has been studied since many years. This paper describes modeling and application of optimal control specifically model predictive control on gyroscope plant. The model predictive control handle different external unmeasureable disturbances, uncertainties and it retains performance in the presence of measurement noise very well without violating the constrained of input and output. In order to test the robustness of the proposed scheme, different cases have been considered: like the system output with measurement noise, where measurement noise has zero mean value Gaussian noise is very commonly seen in practical situations. Additionally the performance of system is considered with different level of uncertainties and external disturbances.
Keywords:- Gyroscope, Model Predictive Control, Optimal Control.

1. Introduction

The controlled gyroscopes are mainly used for two applications. The role of controlled gyroscopes in field of control engineering has been limited to sensing devices such as vibration measurement. However there are many areas of applications, such as the automotive industry, where the use of a controlled gyroscope has many advantages and proven useful for these applications. On the other hand, linear, approximated by a low ordersystem,a stable and minimum-phase controller has the advantage of better and easier real time implementation, especially for multivariable systems [1], [2]. An optimal control is well established technique for many control engineering applications. Model predictive control is a special type of optimal control, where the controller is designed based on the model of the plant.Model predictive control (MPC) [3] is a proven technic forthe controlling of multivariable systems in the presence of the control input, the output and the states constraints and it is capable of tracking pre-scheduled reference signals. These attractive features make MPC widely accepted in the process industry. Many efforts have been made for fast dynamical systems.Basically MPC uses a model of the plant to predict its future behavior in order to compute appropriate control signals to control outputs/states of the plant. To do so, at each sample time MPC uses the current measurement of outputs/states and solves an optimization problem. The result of the optimization problem is a sequence of control inputs of which only the first move is applied to the plant and the procedure is repeated at the next sample time with new measurements [3] as shown in Fig.1. This approach is called receding horizon control.
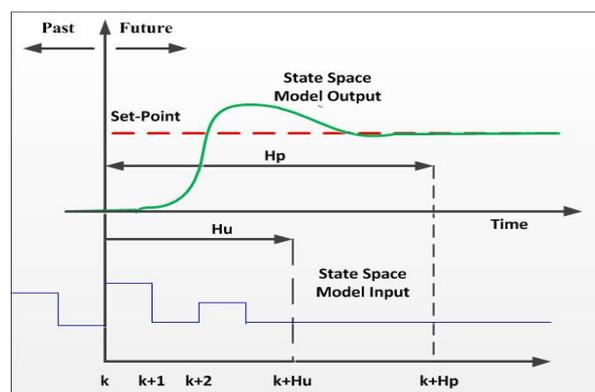

Fig. 1 Model Predictive Control Philosophy.

International Journal of Application or Innovation in Engineering & Management (IJAIEM)
Web Site: www.ijaiem.org Email: editor@ijaiem.org
Volume 3, Issue 10, October 2014   **ISSN 2319 - 4847**

Therefore basic elements of MPC are: a model of the plant to predict its future, a cost function which reflects control objectives, constraints on inputs and states/outputs, an optimization algorithm and the receding horizon principle. In this paper, we describe the development of a state space based model predictive controller and linear model of Gyroscope. The outline of the paper is as follows. In Section II, a constraint model predictive control formulation is described. In the section III, a mathematical model of Gyroscope is developed. Based on this model, Section III presents the development of the proposed controller and observer. The effect of the controller parameters such as the prediction and control horizons on the response and robustness of the drive is discussed in the Section IV using simulation results. The effectiveness of the proposed approach is evaluated experimentally in Section V b subjecting the motor to the inertia loads and the time varying loads. The results have shown that both the controller and observer perform well under unknown dynamic loading conditions.

2. Optimal Control: Constrained Model Predictive Control Problem

Model predictive control (MPC) is a control strategy that has been widely adopted in the industrial process control community and implemented successfully in many applications [7], [8]. However, the traditional MPC strategy demands a great amount of online computation, since an optimization problem (often a constrained quadratic program) is solved at each sampling time. This has limited the use of these controllers to processes with relatively slow dynamics [9], [10]. MPC defines two parameters: receding horizon and control horizon. The receding horizon shows the number of predicted steps and the control horizon is the number of steps that input variables can be changed to control the system. The control horizon must be less than or equal to supposed receding horizon. Consider a linear time invariant control system model in its discrete time state space form.

$$x(k+1) = Ax(k) + Bu(k) \qquad (1)$$
$$y(k) = Cx(k) \qquad (2)$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^l$ is the input vector and $y \in \mathbb{R}^m$ is the vector measured outputs which are to be controlled A, B and C are constant matrices with compatible dimensions. According to the above paragraph, a predictive controller operates as the following steps:

1. The measurement of the system output;
2. Prediction of the system future using proper model of the plant;
3. Solving an optimization problem such that $u(k)$ inputs minimize a cost function;
4. Applying these $u(k)$ inputs to the system;
5. Repeat the step again from 1 to 4.

In general, we can say the most time-consuming task of a model predictive controller is the cost function optimization problem, there are different approaches are reported in the literature to solve this problem. Amongst them quadratic programming technique has received considerable attention due to its ability to robustify the solution as compared to other techniques. The cost function is defined as follows as a function of reference tracking error and the rate of change of the control input as:

$$J = \frac{1}{2}\sum_{k=1}^{H_p} \|Y(k) - r(k)\|^2_Q + \frac{1}{2}\sum_{k=0}^{H_u-1} \|\Delta u(k)\|^2_R \qquad (3)$$

Subject to linear inequality constraints on system inputs, outputs and states. Here r the set-point, H_p and H_u are the prediction and control horizons respectively. It may be noted that the rate of change of the control input is limited in the real time system, a penalty function is introduced additionally in the cost function definition to limit the rate of change of the control input. There are many kinds of problem formulations for MPC are reported in the literature. However, it fundamentally is a QP problem is accepted for the control engineering problems. We follow a standard to convert the MPC problem to a QP problem [4],[5],[6]. We first define

$$Y = \Phi x_0 + \Gamma U \qquad (4)$$

where, Y, Γ, Φ, U are given below,
The constraint MPC problem reduces to stander QP problem discrete time.

$$Y = \begin{bmatrix} y(k+1) \\ y(k+2) \\ \vdots \\ y(k+H_p) \end{bmatrix}, \Phi = \begin{bmatrix} CA \\ CA^2 \\ \vdots \\ CA^{H_p} \end{bmatrix}, U = \begin{bmatrix} \Delta u(k) \\ \Delta u(k+1) \\ \vdots \\ \Delta u(k+H_u-1) \end{bmatrix}$$

$$\Gamma = \begin{bmatrix} CB & 0 & \cdots & 0 \\ CAB & CB & \cdots & 0 \\ \vdots & & \ddots & 0 \\ CA^{H_p-1}B & CA^{H_p-2}B & \cdots & CA^{H_p-H_u}B \end{bmatrix}$$

$$J = \frac{1}{2} U^T H U + F^T U \tag{5}$$

Subject to

$$u_{min} \leq u(k) \leq u_{max}$$
$$\Delta u_{min} \leq \Delta u(k) \leq \Delta u_{max}$$
$$y_{min} \leq y(k) \leq y_{max}$$

In the above equation $H = 2\Gamma^T \Gamma$ is an(H_u) positive definite matrix, and $F = 2\Gamma^T (\Phi x(k))$ is a H_i vector.

3. Mathematical Modeling of Gyroscope

In this section mathematical modeling of Gyroscope is derived. The basic feedback control scheme for implementation of MPC is descried in Fig.2. The motion of the Gyroscope plant is described by [10], [11];

$$J_y \ddot{\theta} + h\dot{\gamma} = M_y \tag{6}$$
$$-h\theta + J_z \ddot{\gamma} = 0 \tag{7}$$

The open loop transfer function G(s) for the gyroscope derived by using (6) and (7). Laplace transforms of (6) and (6) is,

$$J_y \theta s^2 + h\gamma s = M_y \tag{8}$$
$$-h\theta s + J_y \gamma s^2 = 0 \tag{9}$$

Re-arranging (9) to solve for θ we get:

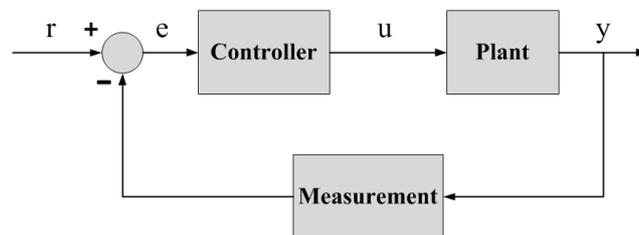$$\theta = \frac{J_z \gamma s}{h} \tag{10}$$

Fig. 2 Block diagram of feedback control system.

Substituting (10) into (8) and rearranging, open loop transfer function G(s) for the gyroscope plant is obtained as,

$$\frac{\varphi}{M_y} = \frac{h}{J_y J_z s^3 + h^2 s} \tag{11}$$

State-space form of a control system is given by

$$\begin{aligned} x &= Ax + bu \\ y &= Cx + Du \end{aligned} \tag{12}$$

Using equation (6) and (7) and the state vector given below, obtain the complete state-space representation for the gyroscope plant.

$$x = \begin{pmatrix} \theta \\ \gamma \\ \dot{\gamma} \end{pmatrix} \tag{13}$$

Re-arranging (6),

$$\ddot{\theta} = \frac{-h}{J_y} \dot{\varphi} + \frac{1}{J_y} M_y \tag{14}$$

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Volume 3, Issue 10, October 2014   **ISSN 2319 - 4847**

Rearranging (7),

$$\dot{\gamma} = \frac{h}{J_z}\theta \tag{15}$$

Combining (13), (14) and (15),

$$x = \begin{pmatrix} \theta \\ \gamma \\ \dot{\gamma} \end{pmatrix} = \begin{pmatrix} 0 & 0 & \frac{-h}{J_y} \\ 0 & 0 & 1 \\ \frac{h}{J_z} & 0 & 0 \end{pmatrix} \begin{pmatrix} \theta \\ \gamma \\ \dot{\gamma} \end{pmatrix} + \begin{pmatrix} \frac{1}{J_y} \\ 0 \\ 0 \end{pmatrix} M_y \tag{16}$$

Table 1: Nomenclature of Gyroscope plant [13]

Parameter	Unit	Numerical Value
h	$Kg.m^2/s$	1.0891
Jy	$Kg.m^2$	0.0026
Jz	$Kg.m^2$	0.0342

The output of the gyroscope plant is the variable γ, the angle about the z-axis. Therefore the second equation given in (12) takes the following form,

$$y = \gamma = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{pmatrix} \theta \\ \gamma \\ \dot{\gamma} \end{pmatrix} \tag{17}$$

Substituting the parameter valuesfrom Table 1, the state-space equation is represented as,

$$A = \begin{pmatrix} 0 & 0 & -418.88 \\ 0 & 0 & 1 \\ 31.84 & 0 & 0 \end{pmatrix} \tag{18}$$

$$B = \begin{pmatrix} 384.62 \\ 0 \\ 0 \end{pmatrix} \tag{19}$$

For a calculation nominal plant parameter value is taken from Table 1.

4. Numerical simulations of the gyroscope with Model Predictive Controller

In this section, the results of simulation of the SISO model of Gyroscope are presented and discussed.The model has the torque as its input, and the rotor position (rad) as output. The torque is limited to ±5 Nm. These are limits imposed by the equipment design and cannot be exceeded. A MPC controller was designed with a sampling interval of 1s, H_p= 10, H_u= 8. The following constraints,

$$|u| \leq \pm 5, \ |y| \leq 1.5$$

were also included. This translates to a standard QP problem the MPC has to compute on-line at every sampling instant. The gyroscope model and predictive control was simulated using MATLAB.Figure3 shows the response of controlled output (rotor position) for set-point tracking.
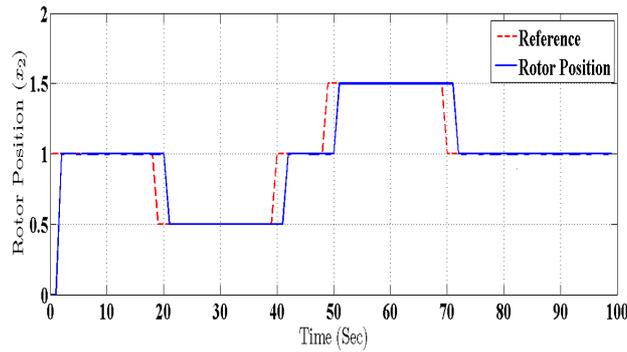
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Volume 3, Issue 10, October 2014 **ISSN 2319 - 4847**

Fig. 3 Response of output to set-point tracking.

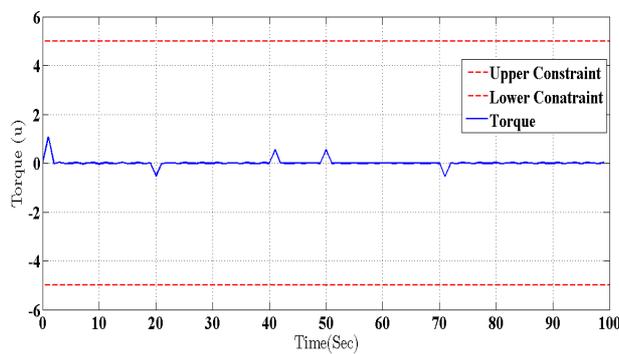Figure4 shows the response of the control input (torque) during   set-point tracking of the system.

Fig. 4 Response of input to set-point tracking.

Fig. 5, a disturbance was introduced at time (20-30) and (60-70)seconds, onthe controlled output (the rotor position).
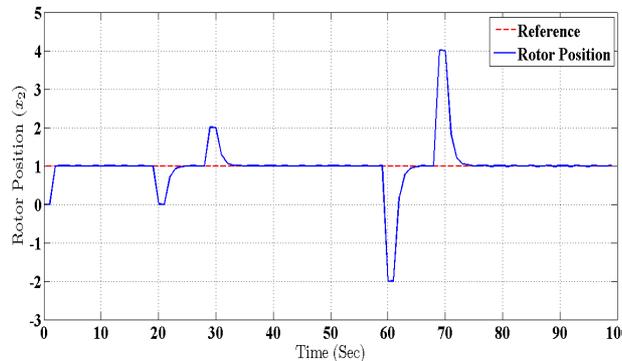
Fig. 5 Output plant response using MPC during disturbance.

Figure 6 shows the input response of input to applied disturbance. The figure showed that this implementation ofMPC is able to handle such disturbance.
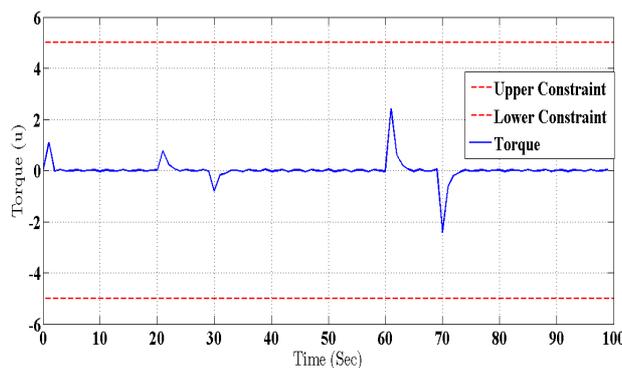

Fig. 6 Input response using MPC during disturbance.

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Volume 3, Issue 10, October 2014   **ISSN 2319 - 4847**

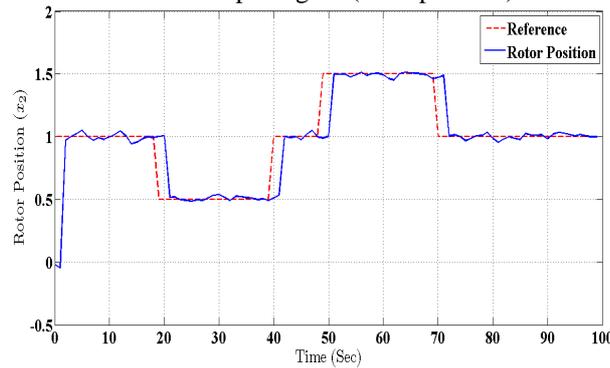Fig. 7 a Gaussian noisewas introduced in the output signal (rotor position).

Fig. 7 Output response during noise.

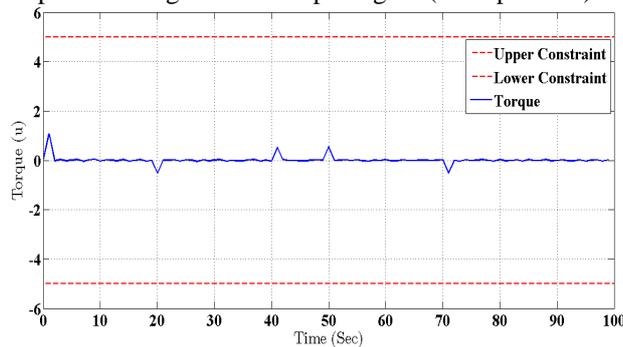Figure 8 depicts the input response during noise in output signal (rotor position).

Fig. 8 Inputresponses during noise.

From the figures it is clear that MPC can track set-point in noise. Next to see the effect of weight matrix on performance of controller, the value of R is taken as 0.0001 and Q is 0.0001. Figure 9 depicts the response of MPC and Fig.10 shows input response to new Q and Rvalues.
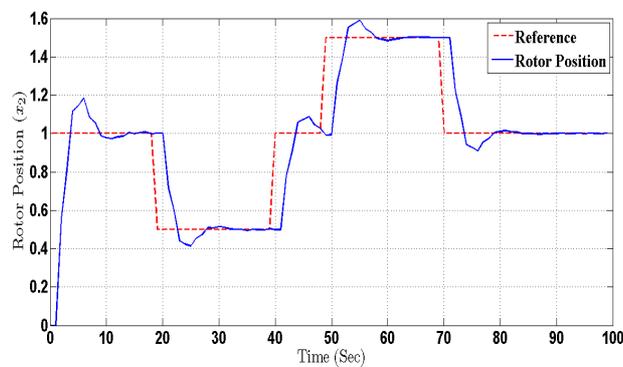

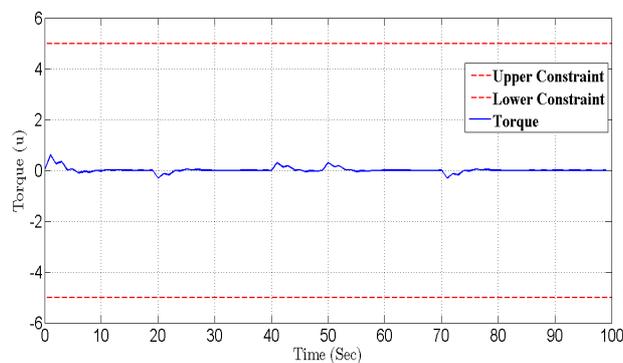**Fig. 9** Output responses when R=0.0001.

Fig. 10 Input responses when R=0.0001.

4. Conclusions

In this paper the mathematical modeling of the controlled gyroscopeand implementation of Model Predictive Controllerfor Gyroscope arepresented. The system uses the rotor axis position as the controlled output and torque as the controlinput. By selecting appropriate parametersof the proposed controller, the presented MPC control system canachieve satisfactory performance for set-point tracking. The MPC is shown to be robust against unknown external disturbance and in the presence of measurement noise while maintaining an actable set-point tracking performance. Overall performance indicated the feasibility of MPC for the controlled gyroscope plant implementation.

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