AN OPTIMAL PRODUCTION CYCLE FOR NON-INSTANTANEOUS DETERIORATING ITEMS IN WHICH HOLDING COST VARIES QUADRATIC IN TIME

Ashendra Kumar Saxena¹, Dr. Ravish Kumar Yadav²

¹Reader, Teertanker Mahaveer University, Moradabad
²Reader, Hindu College, Moradabad

ABSTRACT

In this paper we considered that the Economic Production Quantity (EPQ) model for non-instantaneous deteriorating item in which production and demand rate are constant and the holding cost varies with quadratic in time. We also considered the concept of price discount. Almost complete deteriorated units/items are discarded and partially deteriorated units/items are allowed to carry discount. Shortages are not allowed in this paper. We derived a formula for the optimal cycle time and the results are applied to numerical problems using different values of deterioration and price discount.

Keywords: EPQ, Non-instantaneous deterioration, Price discount, time dependent holding cost.

1. INTRODUCTION

An inventory model has a general assumption is that products or goods generated for indefinitely long lives. Generally, almost all items deteriorate over time. Often the rate of deterioration is low and there is some requirement to consider the deterioration in the determination of economic lot size. Therefore, so many products exist in the real world that is related to a significant rate of deterioration. Hence, the impact of product deterioration should not be neglected in the decision process of production lot size. Deterioration can be classified as age dependent on-going deterioration and age-independent on-going deterioration. Blood, fish and strawberries are some examples of commodities facing age-dependent on-going deterioration. Volatile liquids such as alcohol and gasoline, radioactive chemicals, and grain products are examples of age-independent on-going deteriorating items. Legally these products do not have an expiry date; they can be stored indefinitely, though they suffer natural attrition while being held in inventory. In general, deterioration can be explained as the damage, decline, corrosion, dryness, spoilage, etc., that result in the decrease of usefulness of the commodity the decrease or loss of utility due to decay is usually a function of the on-hand inventory. It is reasonable to note that a product may be understood to have a life time which ends when utility reaches zero.

An economical production quantity (EPQ) model is an inventory control model that determines the amount of product to be produced on a single facility so as to meet a deterministic demand over an infinite planning horizon. Misra (1975), first studied the EPQ model for deteriorating items with the varying and constant rate of deterioration. Choi et. al. (1986), developed a model determining the production rate for deteriorating items to minimize the total cost function over a finite planning period. Raafat (1985) extended the model, given in Park (1983), to deal with a case in which the finished product is also subject to a constant rate of decay. Yang et. al. (2003), considered a multi-lot-size production-inventory system for deteriorating items with constant production and demand rates. Aggarwal et. al. (1991), studied a model assuming that items are deteriorating at a constant rate, the demand rate is known and decreases negative exponentially, no shortages are allowed, and the production rate is known, but can vary from one period to another over a finite planning period. Pakkal et. al. (1992), considered a production-inventory model of deteriorating items with two storage facilities and a constant demand rate. Shortages are allowed. Gary et al. (2006), obtained common production cycle time for an ELSP with deteriorating items. Sugapriya et. al. (2008), considered EPQ model for non-instantaneous deteriorating item in which holding cost varies with time. Sugapriya et. al. (2008), considered a common production cycle time for an EIOQ model with non-instantaneous deteriorating items in which price discount is allowed with permissible delay in payments.

In this paper, an EPQ model for single product subject to non-instantaneous deterioration under a production-inventory policy in which holding cost varies with quadratic time is considered. Each unit of the item is provided with price discount decayed units. In the next section, assumptions and notations that are employed for the development of the model are given. The optimal cycle time is derived.
2. NOTATION AND ASSUMPTIONS

The following notation and assumptions are used throughout the paper.

A  Cost of Set up of a production run for the product.

D  Actual demand of the product given in the number of units per unit time.

K  Production rate per unit time, (K>D).

h= a +bt +ct² Inventory carrying cost per unit time. Where a and b are positive constants.

θ  A constant deterioration rate (unit time).

Cpro Production cost per unit of the product.

r  Price discount per unit cost.

T  Optimal cycle time.

T1 Production period.

T2 Time during there is no production of the product i.e. T1 = T - T2.

I1(t) The inventory level at time t (0 ≤ t ≤ T1) in which the product has no deterioration.

I2(t) The inventory level at time t (T1 ≤ t ≤ T2) in which there is no production.

I0 Maximum inventory level of the product.

Z(T) The total cost per unit time.

In addition, the inventory model is developed under the following assumptions:

(1) A single non-instantaneous deteriorating item is modeled and the demand rate for the product is known and finite.

(2) An infinite planning horizon is assumed.

(3) Shortages are not allowed.

(4) Inventory holding cost is charged only to the amount of undecayed stock.

(5) The production rate of each product is finite. The machine has large enough capacity to produce all the items to meet the demand of all products.

(6) Once a unit of a product is produced, it is available to meet the demand.

(7) Once the production is terminated, the product starts deterioration and the price discount is considered.

(8) The production rate of the product is independent of the production lot size.

(9) There is no replacement or repair for a deteriorated item.

3. MODEL FORMULATION

In this paper, at start t = 0, the inventory level is zero. The production and supply start simultaneously and the production stops at t = T1, at which the maximum inventory I(T1) is reached. The inventory built up at a rate is K - D in the interval [0, T1] and there is no deterioration. After the time T1, the maximum inventory is reached, production is terminated and the deterioration starts. Production will be resumed when all on hand inventories are depleted at time T. Then an identical production run will begin. Since an exponential deterioration process is assumed, the inventory level of the system for the product at time t over period [0, T] can be the represented by the differential equations: when the inventory reduces to zero level and production run begins.

\[ \frac{dI_1(t)}{dt} = K - D, \quad \text{for} \quad 0 \leq t \leq T_1 \quad \text{……(1)} \]

\[ \frac{dI_2(t)}{dt} + \theta I_2(t) = -D, \quad \text{for} \quad 0 \leq t \leq T_2 \quad \text{……(2)} \]

Using the boundary conditions \( I_1(0) = 0 \) and \( I_2(T_2) = 0 \).

Solving equation (1) with respect to time t,

\[ I_1(t) = (K - D)t + C_1 \]

Where \( C_1 \) is the constant of integration.

Using the boundary condition \( I_1(0) = 0 \) then \( C_1 = 0 \)

\[ I_1(t) = (K - D)t, \quad \text{for} \quad 0 \leq t \leq T_1 \quad \text{……(3)} \]

From equation (2)

\[ \frac{dI_2(t)}{dt} + \theta I_2(t) = -D \]

\[ I.F. = e^{\int \theta dt} = e^{\theta t} \]

Hence the solution of the differential equation (2)

\[ e^{\theta t} I_2(t) = \frac{(D)e^{\theta t}}{\theta} + C_2 \]
Apply the boundary condition $I_1(T_2) = 0$.

$$I_2(t) = \frac{D}{\theta} \left( e^{\theta T_2} - 1 \right), \quad \text{for} \quad 0 \leq t \leq T_2 \quad \ldots \quad (4)$$

(1) **Holding Cost:** The holding cost per unit time is given by

$$HC = \frac{1}{T} \int_0^T (a + bt + ct^2) I_1(t) \, dt + \frac{1}{T} \int_0^T (a + bt + ct^2) I_2(t) \, dt$$

$$= \frac{1}{T} \int_0^T \left( (a + bt + ct^2) (K - D) t + \int_0^T (a + bt + ct^2) (T_2 - t) \, dt \right)$$

Assuming $t \theta < 1$, an approximate value is got by neglecting those items of degree greater than or equal to 2.

$$HC = \frac{a(K - D)}{2T} + b\frac{(K - D) T_2^2}{2T} + c(T_2 - t) \left( \frac{T_2}{4T} \right) \quad \ldots \quad (5)$$

(2) **Production Cost:** The production cost per unit time is given by

$$PC = KC_{pro} \frac{T_1}{T} \quad \ldots \quad (6)$$

(3) **Setup Cost:** The setup cost per unit time is given by

$$SC = \frac{A}{T} \quad \ldots \quad (7)$$

(4) **Deterioration Cost:** The number of units that deteriorate in a cycle is the difference between the maximum inventory and the number of units used to meet the demand. Hence the deterioration cost per unit time is given as

$$DC = \frac{C_{mu}}{T} \left[ I_2(0) - \int_0^T D \, dt \right]$$

$$DC = \frac{C_{mu}}{T} \left( e^{\theta T_2} - 1 \right) - \int_0^T D \, dt$$

Assuming $t \theta < 1$, an approximate value is got by neglecting those items of degree greater than or equal to 2.

$$DC = \frac{C_{mu}}{T} \left( \frac{D}{\theta} \left[ 1 + \theta T_1 + \left( \frac{\theta T_1}{2} \right)^2 + \ldots \right] \right) - DT_2 \quad \ldots \quad (8)$$

(5) **Price discount:** Price discount is offered as a fraction of production cost for the units in the period $[0, T_2]$

$$PD = \frac{C_{mu} r}{T} \int_0^T D \, dt = \frac{C_{mu} r DT_1}{T} \quad \ldots \quad (9)$$

The total average cost per unit time can be formulated as

$$Z(T) = HC + PC + SC + DC + PD$$

$$Z(T) = \left[ \frac{a(K - D) T_2^2}{2T} + b(K - D) \frac{T_2^3}{3T} + c(K - D) \frac{T_2^4}{4T} \right] + \frac{a D T^2}{2T}$$

$$+ \frac{b D T^3}{6T} + \frac{c D T^4}{12T} + K_{pro} T \frac{A C_{mu} r D T^2}{T} + \frac{C_{mu} r D T^2}{T}$$

To minimize the total cost per unit time $Z(T)$ to express in terms of $T$ in equation (10) so that there is only one variable $T$ in the equation. At the

$$I_1(T_1) = I_2(0)$$

$$P - D \quad T_1 = \frac{D}{\theta} \left[ e^{\theta T_1} - 1 \right]$$
\[ (P-D) T = \frac{D}{2} \left( \frac{\theta_1}{2} + \frac{\theta_1}{2} \right) \]

\[ (P-D) T = \frac{D}{2} \left( \frac{\theta_1}{2} \right) \]

\[ T_1 = \frac{TD}{P-D} \]

Now \[ T = T_1 + T_2 \]

\[ T_2 = \frac{(P-D)T}{P} \]

\[ T_i = \frac{DT}{P} \]

Substitute the value of \( T_1 \) and \( T_2 \) in equation (10)

\[ Z(T) = \left[ \frac{a(K-D)}{2K} + b(D) + cKD + \frac{d(D)(K-D)}{3K} \right] + \frac{\alpha}{T} \left[ \frac{a(K-D)}{2K} + b(D) + cKD + \frac{d(D)(K-D)}{3K} \right] \]

The objective of this paper is to determine the optimal ordering policy for minimizing the total variable cost per unit time.

\[ \frac{\partial Z(T)}{\partial T} = 0 \]

For this, the first derivative of \( Z(T) \) with respect to \( T \) is given by:

\[ \frac{\partial Z(T)}{\partial T} \]

Using the equation (14), we will get the optimum value of \( T \).

\[ \frac{\partial^2 Z(T)}{\partial T^2} > 0 \]

<table>
<thead>
<tr>
<th>( r )</th>
<th>( \theta )</th>
<th>( T )</th>
<th>( T_1 )</th>
<th>( T_2 )</th>
<th>Holding Cost (HC)</th>
<th>Total Cost Z(T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>.01</td>
<td>.02</td>
<td>3.9721</td>
<td>1.58848</td>
<td>2.38326</td>
<td>175.4041</td>
<td>2604.426</td>
</tr>
<tr>
<td>.04</td>
<td>.02</td>
<td>3.7784</td>
<td>1.51136</td>
<td>2.26704</td>
<td>162.1322</td>
<td>2605.151</td>
</tr>
<tr>
<td>.06</td>
<td>.02</td>
<td>3.6115</td>
<td>1.4446</td>
<td>2.1669</td>
<td>151.0940</td>
<td>2606.972</td>
</tr>
<tr>
<td>.08</td>
<td>.02</td>
<td>3.4610</td>
<td>1.3844</td>
<td>2.0766</td>
<td>141.4216</td>
<td>2609.691</td>
</tr>
<tr>
<td>.10</td>
<td>.02</td>
<td>3.326</td>
<td>1.3304</td>
<td>1.9956</td>
<td>133.0615</td>
<td>2613.108</td>
</tr>
<tr>
<td>.02</td>
<td>.02</td>
<td>3.9721</td>
<td>1.58848</td>
<td>2.38326</td>
<td>175.4041</td>
<td>2724.426</td>
</tr>
<tr>
<td>.04</td>
<td>.02</td>
<td>3.7784</td>
<td>1.51136</td>
<td>2.26704</td>
<td>162.1322</td>
<td>2725.151</td>
</tr>
<tr>
<td>.06</td>
<td>.02</td>
<td>3.6115</td>
<td>1.4446</td>
<td>2.1669</td>
<td>151.0940</td>
<td>2726.972</td>
</tr>
<tr>
<td>.08</td>
<td>.02</td>
<td>3.4610</td>
<td>1.3844</td>
<td>2.0766</td>
<td>141.4216</td>
<td>2729.691</td>
</tr>
<tr>
<td>.10</td>
<td>.02</td>
<td>3.326</td>
<td>1.3304</td>
<td>1.9956</td>
<td>133.0615</td>
<td>2733.108</td>
</tr>
<tr>
<td>.03</td>
<td>.02</td>
<td>3.9721</td>
<td>1.58848</td>
<td>2.38326</td>
<td>175.4041</td>
<td>2844.426</td>
</tr>
<tr>
<td>.04</td>
<td>.02</td>
<td>3.7784</td>
<td>1.51136</td>
<td>2.26704</td>
<td>162.1322</td>
<td>2845.151</td>
</tr>
<tr>
<td>.06</td>
<td>.02</td>
<td>3.6115</td>
<td>1.4446</td>
<td>2.1669</td>
<td>151.0940</td>
<td>2846.972</td>
</tr>
<tr>
<td>.08</td>
<td>.02</td>
<td>3.4610</td>
<td>1.3844</td>
<td>2.0766</td>
<td>141.4216</td>
<td>2849.691</td>
</tr>
<tr>
<td>.10</td>
<td>.02</td>
<td>3.326</td>
<td>1.3304</td>
<td>1.9956</td>
<td>133.0615</td>
<td>2853.108</td>
</tr>
</tbody>
</table>
### 4. Numerical Example

1. **Example 1**
   \[ A = 1000 \text{$/set up}$, \( K = 100 \text{ units/unit time}$, \( D = 40 \text{ units/unit time}$, \( a = 2 \), \( b = 0.02 \), \( c = 0.01 \), \( C_{pro} = 50 \text{$/unit time}$, \( \Theta = 0.06 \), \( r = 0.02 \text{/unit}$, \( T = 3.6115 \), \( Z(T) = 2726.972 \text{ and $HC = 151.094$} \)

2. **Example 2**
   \[ A = 1000 \text{$/set up}$, \( K = 1000 \text{ units/unit time}$, \( D = 40 \text{ units/unit time}$, \( a = 2 \), \( b = 0.02 \), \( c = 0.01 \), \( C_{pro} = 50 \text{$/unit time}$, \( \Theta = 0.08 \), \( r = 0.02 \text{/unit}$, \( T = 3.4610 \), \( Z(T) = 2844.426 \text{ and $HC = 175.4041$} \)

3. **Example 3**
   \[ A = 1000 \text{$/set up}$, \( K = 100 \text{ units/unit time}$, \( D = 40 \text{ units/unit time}$, \( a = 2 \), \( b = 0.02 \), \( c = 0.01 \), \( C_{pro} = 50 \text{$/unit time}$, \( \Theta = 0.02 \), \( r = 0.03 \text{/unit}$, \( T = 3.9721 \), \( Z(T) = 2973.108 \text{ and $HC = 3089.691$} \)

4. **Example 4**
   \[ A = 1000 \text{$/set up}$, \( K = 100 \text{ units/unit time}$, \( D = 40 \text{ units/unit time}$, \( a = 2 \), \( b = 0.02 \), \( c = 0.01 \), \( C_{pro} = 50 \text{$/unit time}$, \( \Theta = 0.04 \), \( r = 0.04 \text{/unit}$, \( T = 3.7784 \), \( Z(T) = 2930.691 \text{ and $HC = 141.4216$} \)

Now, we computed Total cost, Production cycle time, Production run time, and Holding cost for different sets of deterioration rates and price discount. After that, we compared the results. The following tables and graphs show the total costs, the production cycle time, and the production run time with reference to deterioration rate and price discount.

**For \( r = 0.01 \)**

![Figure 2](image2.png)
**Figure 2**: Shows the graph of total cost \( Z(T) \) VS \( T_2 \). It appears that \( T_2 \) decreases as total cost increases.

![Figure 3](image3.png)
**Figure 3**: Shows the graph of total cost \( Z(T) \) VS \( T_1 \). It appears that \( T_1 \) decreases as total costs increases.

![Figure 4](image4.png)
**Figure 4**: Shows the graph of total cost \( Z(T) \) versus Holding Cost. It shows total costs increase as the holding cost decreases.
Figure 5: Shows the graph of total cost $Z(T)$ versus deterioration rate. It shows total costs increase as the deterioration rates increases.

5. CONCLUSION
In this paper, an EPQ model for a single machine producing single item which under non-instantaneous deterioration and holding cost varies with quadratic time has been developed. Almost complete deteriorated units/items are discarded and partially deteriorated units/items are allowed to carry discount which maintains the demand which is more realistic. The production cycle time is formulated and is found be a relatively simple expression. This paper helps to reduce the total cost for non-instantaneous deterioration. We considered the concept of price discount and shortages are not allowed.

REFERENCES