Numerical Modeling of One-Dimensional Hydrous Transfer in the Soil.

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ABSTRACT

We propose, in this study, a mathematical model, which describes the hydrous transfer in the soil-plant-atmosphere systems. Indeed this problem is resulted from the combination of two processes essential to know the water run-off in the studied medium and the transfer of heat and mass. This model are developed using an electric analogy, where hydrous transport in the ground and the plant are described in a coupled way using an electric equivalent diagram, and the quadruple method. The balance equation; written and then numerically solved in the Laplace space. The model is written in a FORTRAN program. For that, we present the study of periodic regime (day and year). The solution in the Laplace space leads to the flow and potential evolution for long times. The return to real space is done by the stehfest numerical method.

Keywords: Modeling, hydrous, Heat Transfer; Mass Transfer; Electric analogy, Quadripoles

1. INTRODUCTION

The heat and water exchanges between the atmosphere and the ground surface are important processes for environmental studies. Heat energy and water supplied to the ground surface as solar and long wave radiation and precipitation are redistributed to the ground and the atmosphere through the ground-surface processes, becoming the driving force of atmospheric phenomena. Thus, the processes of heat and water exchanges at the ground surface play an important role in determining the meteorological and climatologically conditions. Moreover, the processes of heat and water exchanges at the ground surface are essential components for pollutant movement. However, the behavior of heat and water in the soil–plant-atmosphere systems, which can be found everywhere on the globe, is very complicated and is still poorly understood. Many factors including meteorological, hydrological, and biological factors control the processes of heat and water exchanges in the system. The property of water that has solid, liquid, and vapor phases under the natural environment makes the processes more complicated because phase change of water causes heat exchange. The mechanisms of water transfer between soil and atmosphere, particularly as influenced by plants, have been major themes in soil physics (Hillel, 1980), and many mathematical models describing these phenomena have been proposed (Campbell, 1985).

2. WATER TRANSPORT MODEL

The ground water, the roots and the leaves are assumed to be at the average heights $Z_{gw}$, $Z_R$ and $Z_L$. Due to transpiration (resulting from a heat and mass balance between the leaves and the surroundings) the plant loses a water flux versus time $T(t)$ whereas due both to the rain and the evaporation, the soil surface a net water flux $\varphi(t)$.

2.1. The soil

2.1. 1. Assumptions:

The following hypothesis has been made:

- In a bare soil of depth $e$, transport is unidirectional in vertical (or $z$ direction).
- Soil water diffusivity $D$ and hydraulic conductivity $k$ vary markedly with water content. However, $D$ and $K$ are assumed to be constant.
- Mass transfer is only in the liquid phase and the leaf water potential is considered to be the potential of the substomatal cells.

2.2 Water transport equations

The equation of continuity if the water density $\rho$ is assumed constant combined with generalized Darcy's law for all isotropic medium, leads to:
\[ \frac{\partial \theta}{\partial t} = K \Delta H \] (1)

Where \( \theta \) the soil volumetric water of the soil, \( K \) is the soil hydraulic conductivity and \( H \) the piezometric water head or hydraulic head.

As indicated when developing the various assumptions made, the hydraulic conductivity \( K \) is supposed here to be independent of the volumetric soil water content \( \theta \) and hence of the space variable. For unsaturated soil, the capillary capacity \( C \) is introduced

\[ \frac{d\theta}{d\psi} = C_h \] (2)

If \( \psi \) denotes the capillary water pressure

\[ \psi = \frac{P_{atm} - P}{\rho g} \] (3)

Where: \( P \) is the water pressure, \( P_{atm} \) the atmospheric pressure, \( \rho \) the water density and \( g \) the gravity intensity. The water pressure head \( H \) is given by

\[ H = \psi + z \] (4)

The capillary transport of water is given by the Richard's equation [1]

\[ \frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} (K \frac{\partial H}{\partial z}) \] (5)

Assuming that the gas pressure is uniform in the soil and that \( \psi \) is only a function of \( \theta \), combining equations (4) and (5) leads to:

\[ \frac{d\theta}{d\psi} \frac{\partial H}{\partial \psi} = \frac{\partial}{\partial z} (K \frac{\partial H}{\partial z}) = C \frac{\partial H}{\partial t} \] (6)

The flow equation is then

\[ \frac{\partial H}{\partial t} = D \Delta H \] (7)

Where; \( D = \frac{K}{C} \) the soil water diffusivity. However, \( D \) and \( K \) are assumed to be constant in this study.

3. Laplace transforms

In the steady state, the resistive analogy is generally introduced in order to express the linear relation between the potential variation and the rate of flow. In the transient case, the extension of this notion is a quadripole; a quadripole is built with three impedances, which physically mean the simultaneity of resistive and capacitive effects in the medium. When the time tends to infinity (phenomena approach the steady state) the limit of the quadripole is represented in figure (1) with \( R_1 \) and \( R_2 \) that expresses the resistance of the soil to the movement of water and \( C \) the capacity of storage of the soil. Mathematically when a Laplace transformation \( (f(s) = \int f(t) \exp(-st)dt) \) is applied, the partial differential equation of diffusion (7) becomes a differential equation of second order which solution is exactly known; using the boundary conditions one can write a linear relation between potential flux at the input and the same at the output. The quadripole expresses this relation. Let us point out that the impedance is only significant in Laplace space and the inverse transformation in temporal space can only be made for the final solution (potential and/or rate of flow).
We assume for simplicity that at t = 0, the soil is at hydrostatic equilibrium (that is to say $H = H_i$ is constant elsewhere). If $\overline{H} = L(H - H^1)$ et $\overline{q} = L(q)$ designs the Laplace transforms of the pressure head and the volumetric water flux ($q = -kS \frac{\partial H}{\partial z}$), the linear relation between $\overline{H}_1$, $\overline{q}_1$ et $\overline{H}_2$, $\overline{q}_2$ at two different sections $z = z_1$ and $z = z_2$ of the soil is written with $e = z_1 - z_2$ [2]:

$$
\begin{bmatrix}
\overline{H}_1 \\
\overline{q}_1
\end{bmatrix} = \begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix} \begin{bmatrix}
\overline{H}_2 \\
\overline{q}_2
\end{bmatrix}
$$

(8)

With: $A_{11} = A_{22} = ch(le)$; $A_{12} = \frac{sh(le)}{kl}$; $A_{21} = (kl) sh(le)$.

Where: $l = \sqrt{\frac{p}{D}}$ and p is the Laplace variable.

Thus, in the unsteady state a soil layer of thickness e can be associated with a quadripole. As this quadripole is passive, it can be represented by a T - association of impedances (see figure 2b) with [3]:

$Z_1 = Z_2 = \frac{A_{11} - 1}{A_{21}}$, $Z_3 = \frac{1}{A_{21}}$

It is interesting to find the behaviours for these impedances when $t \to \infty$ ($\frac{Dl}{e^2} \gg 1$): $\frac{Dl}{e^2}$ Fourier number in relation to mass transfer in Laplace space at $\frac{Pe^2}{D} << 1$

In this case the quadripole approximation is given by: $Z_i = Z_i = \frac{e}{2k}$ (Equivalent to a resistance).

$Z_3 = \frac{1}{eC_kp}$ (Equivalent to a capacity)

4. PLANT

According to Katerji’s model [4] there are several resistance mechanisms to water transport inside a plant. But the most important ones are the resistance between the soil and the roots $R_r$ and the stomatic resistance of the leaves $R_f$. More the plant is able to accumulate water [4, 5].

If we admit a linear relation between the accumulated water volume $V$ in the plant and water pressure of the plant $\Psi_p$, then:

$$
V = V_{max} \left(1 - \frac{\Psi_p}{\Psi_M}\right)
$$

(9)
Where $V_{\text{max}}$ is the accumulated water volume when the plant is fully wet ($\psi_p = 0$) and $\psi_m < 0$ the pressure corresponding to definitive withering of the plant (figure 2). Therefore, the water flux $q$ from the plant is given by:

$$q = -\frac{dV}{dt} = \frac{V_{\text{max}}}{-\psi_m} \left( \frac{d\psi_p}{dt} \right) = C \left( \frac{d\psi_p}{dt} \right)$$ (10)

The plant acts as a capacitance $C$ which is supposed to be connected to the main water flux by a resistance $r$ (figure 2). Laplace transformation in the time $t$ from equation (10) gives

$$\bar{q} = C(\psi_p^i - p \bar{\psi}_p) = -Cp\bar{H}_p$$ (11)

Where $\psi_p^i$ is the initial value of $\psi_p$. For simplicity we will assume here that the system is initially at equilibrium.

5. The Atmosphere
The main factors of atmosphere influencing the behaviour of water flow towards plant are: the net solar radiation, the rains, the humidity and temperature of air, and the wind speed.

6. Balance equation

6.1. Radiation Balance
The main energy source on earth surface is from solar origin. The net radiation $R_N$ is the balance of radiative energy received by the covered vegetal.

$$R_N = (1 - a)R_G + \varepsilon_s R_C - \varepsilon_s \sigma T_F^4$$ (12)

Where $R_C$ is the atmospheric radiation and $R_G$ the total radiation, which is compound of the direct and diffuse radiations [6].

6.2 Energetic Balance
A part of net radiation serves to vaporize the water available on the surface of the covered vegetal and give a flux of latent heat ($\text{LE}$). The rest is dissipated by convection as heat in the air (sensitive heat $H_C$) or stored in the soil (as a conductive heat flux S). If the part of net radiation $R_N$ used for photosynthesis by the vegetation and the conductive flux in the soil $S$ are neglected, the equation of the energy balance can be written:

$$R_N = \text{LE} + H_C$$ (13)

The electrical equivalent schema soil-plant-atmosphere system represented in Laplace space is compound of three quadripoles figure (3).

![Figure 3: schematic diagram of soil-plant- atmosphere system.](image)

7. Results and Discussion
The system of the established equation from this electrical representation see (figure 3) has been given as follows:
\[
\mathcal{H}_f = - \left( \frac{abc}{abc} \mathcal{E} + \frac{ab}{abc} \mathcal{V}_1 + \frac{ab}{abc} \mathcal{V}_2 \right)
\]
\[
\mathcal{H}_p = \mathcal{H}_f + q \mathcal{V}
\]
\[
q = - c_p (\mathcal{H}_p) = - \frac{c_p \mathcal{H}_f}{1 + r c_p}
\]
\[
q_a = c_{11} \mathcal{E} + c_{12} \mathcal{H}_f + v_1
\]
\[
\mathcal{H}_0 = \frac{a_{21} b_{11} \left( c_{11} \mathcal{E} + c_{12} \mathcal{H}_f + v_1 \right)}{1 - \left( a_{21} b_{11} + a_{22} b_{11} \right)}
\]
\[
\mathcal{H}_r = a_{11} \mathcal{H}_0
\]

Where:

\[
ab_{11} = c_h (\beta e_1); \quad ab_{12} = K \beta (c_h (\beta e_1) th (\beta e_2) + sh (\beta e_1)); \quad ab_{21} = \frac{sh (\beta e_1)}{K \beta}; \quad ab_{22} = sh (\beta e_1) th (\beta e_2) + c_h (\beta e_1);
\]
\[
abc_{11} = c_h (\beta e_1) + K \beta (c_h (\beta e_1) th (\beta e_2) + sh (\beta e_1)) R_p; \quad abc_{12} = \frac{c_h (\beta e_1) C_p}{(1 + r C_p)} + K \beta (c_h (\beta e_1) th (\beta e_2) + sh (\beta e_1)) R_p;
\]
\[
abc_{21} = \frac{sh (\beta e_1)}{K \beta} + (c_h (\beta e_1) th (\beta e_2) + sh (\beta e_1)) \left( \frac{1 + C_p (r + R_p)}{(1 + r C_p)} \right); \quad abc_{22} = \left( \frac{sh (\beta e_1)}{K \beta} \right) + (c_h (\beta e_1) th (\beta e_2) + sh (\beta e_1)) \left( \frac{1 + C_p (r + R_p)}{(1 + r C_p)} \right).
\]

The resolution of the system of equations obtained in the Laplace space is numerically given; have allowed us to know the evolution of all physical magnitudes (flow and potential) intervening in process of exchange at different levels of plant or soil. The return to real space is done by the Stehfest numerical method [6]. A part of results obtained has been illustrated in figures 4 and 5; they represent the evolution curves of the potential and density flow of water with respect to time for different parameters characterizing the soil-plant-atmosphere. The physical properties of the soil; (relative or balance humidity, hydraulic diffusivity and conductivity of soil), the depth of ground water and the water flux arriving on earth surface from the rain or the artificial irrigation. Different magnitudes in relation with the plant ( stomatal resistance, plant-reservoir capacity...) proposed in specialized literature have been tested too. The results obtained are in good agreement with the experimental results obtained by various authors. For a mean flow density \(q_0 = 0.82\text{mm.day}^{-1}\) at the soil surface, a mean transpiration \(T = 8, 47\text{ mm.day}^{-1}\), a volumetric water content of soil \(\theta = 35\%\) and a water at 0.5m depth (where the soil water being at atmospheric pressure) for a diurnal cycle (\(T_p = 24\text{h}\)). Curves relating to different potentials are presented in fig. 4a, the leaves potentials and the tank-plant potentials decrease very fast during the ten first hours. This is confirmed by the density flux evolution shown in fig. 4b. This brutal drop of tank-plant potentials shows that the tank-plant will be solicited first. It becomes rapidly empty. The roots flow issuing from the humid surrounding soil will feed the plants and will become the main water spring satisfying the climatic request, because the response times of soil are much longer than those of plant. The two curves are confused along the day; a low difference between them, explains the low value of flow coming from the tank-plant. A potentials (\(H_p\)) and that of the tank-plant (\(H_0\)): can follow a similar variation. This brutal drop of tank-plant potentials shows that the tank-plant will be solicited first. The Flow coming from the tank-plant is definitely more important, its contribution to satisfy the climatic request is considerable. Because of the diurnal cycle of the climatic demand inducing water uptake,
the model reproduces greater dryness around the roots during high demand (13 h) than during low demand (9 h). The same calculations were done for a ground water located at 0.5m depth. Then we compare the evolution of the curves relatively to potentials fig. 5a and to density flow fig. 5b for the two volumetric water content of soil \( \theta = 35\% \) and 25\%. The fig. 6(a, b), shows a variations of flow and potential for a period of one year (Tp = 12 month) at depth \( z = 0.5m \); with the same soil moisture \( \theta = 35\% \).

**Figure 4** (a): Potential variation for \( \theta = 35\% \) at 0.5m depth  
**Figure 4** (b): Flow density variation for \( \theta = 35\% \) at 0.5m depth

**Figure 5** (a, b): Comparison of potential and density flow evolution at the 0.5m depth for \( \theta = 35\%, 25\% \).
8. CONCLUSION
We developed, in this study a simplified mathematical model to describe the transfer in the soil-plant-atmosphere continuum system (SPAC). In this global approach of complicated and multifarious problems, the assumptions considered previously have allowed to obtain a simple and quick solution. These results have allowed us to prove the importance of taking into account the plant-reservoir, which appears during a water stress period. On the other hand, for short times case, the foliar resistance becomes a significant parameter and its influence on flux and the potentials is important in the first hours. We were interested thereafter, in the influence of a few numbers of parameters such: water content of the ground, depth of the ground, foliar resistance on the conditions of appearance of the hydrous stress. This study allows the possibility to analyze the sensitivity of each parameter on the evolutions of the potentials and flows at various levels of the ground and the plant.

References