A fuzzy mixture two warehouse inventory model with Linear Demand

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ABSTRACT

This paper deals with fuzzy based two warehouse inventory model with linear demand. Deterioration rates in both warehouses are considered to be different due to change in environment. Shortages are not allowed. In the real life and global market situations some parameters like ordering cost, holding costs and deteriorating cost fluctuate with their actual values. So the parameters are not assumed to be constant. To deal with such type of uncertainty, consider a fuzzy model in which holding costs are assumed in warehouses, ordering cost and deteriorating cost in a fuzzy number which is represented by triangular numbers. We obtained the total inventory cost in crisp environment as well as fuzzy environment with the help of Signed distance method. A numerical example is provided to illustrate both the proposed crisp model and fuzzy model which shows the effects of fuzziness of the parameters on the optimal solution.

Keywords: Linear demand, deterioration, holding cost, Crisp model, Fuzzy model and Signed distance Method.

1. INTRODUCTION

In the last few decades, inventory models have been widely applied in business world. However, one of the weaknesses of current inventory models is the unrealistic assumption that all items produced are of good quality. In the crisp environment, all parameters in the total inventory cost such as holding cost, ordering cost, set-up cost, deterioration rate and demand rate etc. are known and have definite value without ambiguity. Some of the business situations fit such conditions, but in most of the situations and in the day-to-day changing market scenario the parameters and variables are highly uncertain or imprecise. The concept of soft computing techniques (fuzzy logic) was first introduced by Zadeh (1965). Bellman and Zadeh (1970) developed the difference between randomness and fuzziness by showing that the former deals with uncertainty regarding membership or non-membership of an element in a set while the later is concerned with the degree of uncertainty by which an element belongs to a set. Hartely (1976) considered an inventory model with two storage facilities. It is generally assumed that the holding cost in the RW is greater than holding cost in the OW. Hence, the items are stored first in the OW, and only excess of stock is stored in the RW. Further, the items of the RW are released earlier than the items of the OW. Zimmermann (1985) has given a review on fuzzy set theory and its applications.

There is a long history of gradual progress in the development of inventory models. Park (1987) and Vujosevic et al (1996) developed the inventory models in fuzzy sense in which ordering cost and holding cost are represented by fuzzy numbers. Park has represented costs as trapezoidal fuzzy numbers. While Vujosevic et al has discussed the ordering cost by triangular fuzzy number and holding cost by trapezoidal fuzzy number, Yao and Lee (1999) presented a fuzzy inventory model with and without backorder for fuzzy order quantity with trapezoidal fuzzy number. C. Kao and W. K. Hsu (2002) presented a lot size-reorder point inventory model with fuzzy demands. Hsieh (2002) considered two fuzzy production-inventory models: one for crisp production quantity with fuzzy parameters and the other one for fuzzy production quantity. Hsieh used the graded mean integration representation method for defuzzifying the fuzzy total inventory cost. J. S. Yao and J. Chiang (2003) developed an inventory without back order with fuzzy total cost and fuzzy storing cost deffuzified by centroid and singed distance. Chang et al. (2004) discussed a fuzzy mixture inventory model with variable lead-time based on probabilistic fuzzy set and triangular fuzzy number. Chang et al. (2006) presented a model in which they considered a lead-time demand as fuzzy random variable instead of a probabilistic fuzzy set. Dutta et al. (2007) considered a continuous review inventory system, where the annual average demand was treated as a fuzzy random variable. The lead-time demand was also assessed by a triangular fuzzy number. Yung et al. (2007) discussed procurement planning of time-variable demand in manufacturing system based on soft computing techniques. Singh and Singh (2008) considered the fuzzy inventory model for finite rate of replenishment using signed distance method. C. C. Chou (2009) discussed a fuzzy economic order quantity inventory model. Halim et al. (2010) addressed the lot sizing problem in an unreliable production system with stochastic machine breakdown and fuzzy repair time. They defuzzified the cost per unit time using the signed distance method. Yong et al. (2010) discussed an optimal production-inventory model for deteriorating items with multiple-market demand. Malik and Singh (2011) considered an inventory model for deteriorating items with soft computing techniques and variable demand. Liao et al

(2012) determined an economic order quantity for deteriorating items with two-storage facilities (one is an owned warehouse and the other in a rented warehouse) where trade credit is linked to order quantity. Malik, Singh and Gupta (2012) presented a fuzzy based two warehouses inventory model for deteriorating items. Hsieh and Dye (2013) discussed a production–inventory model for deteriorating items with time-varying demand and finite replenishment rate by allowing preservation technology cost as a decision variable in conjunction with production policy.

In this paper we have presented an inventory model with two warehouses for deteriorating items. Here we have two warehouses for study-one own warehouse and the second rented warehouse. The holding cost of rented warehouse is higher than the holding cost of own warehouse. The holding cost and deteriorating costs for two warehouses are considered as fuzzy numbers. The triangular type of fuzzy number is used for representing the fuzzy parameters. The total inventory costs are obtained in crisp as well as fuzzy model with the help of a numerical example.

2. NOTATION AND ASSUMPTIONS

The inventory model is based on the following assumptions:

- A the ordering cost per order
- hr the inventory holding cost in RW per unit per time unit
- h_o the inventory holding cost in OW per unit per time unit
- C_d the deteriorating cost per unit per time unit
- t_1 the time at which the inventory level in RW reaches zero, $t_1 \geq 0$
- T = $(t_1 + t_2)$ the length of cycle time.
- W the maximum inventory level in OW during $[0, T]$.
- $I_R(t)$ the level of inventory in RW at time t, $0 \leq t \leq t_1$
- $I_{O1}(t)$ the level of inventory in OW at time t, $0 \leq t \leq t_1$
- $I_{O2}(t)$ the level of inventory in OW at time t, $t_1 \leq t \leq t_1 + t_2$
- TC the total cost per time unit.

{~ Sign represents the fuzziness of the parameters}

In addition, the following notations are used:

- The inventory system deals with single item.
- Shortages are not allowed.
- The demand rate is *a-bt*.
- The deterioration rate in RW is α and in OW is β.
- The lead – time is zero or negligible.
- ***Triangular Fuzzy Number:*** Let k = (k_1, k_2, k_3) is a triangular fuzzy number, where k_1=k-Δ_1, k_2 =k, k_3 = k+Δ_2. The membership function of k is

$$\mu_{\tilde{k}}(\tilde{k}) = \begin{cases} \dfrac{k - k_1}{k_2 - k_1} & k_1 \leq k \leq k_2 \\ \dfrac{k_3 - k}{k_3 - k_2} & k_2 \leq k \leq k_3 \\ 0 & otherwise \end{cases}$$

3. MATHEMATICAL MODEL (CRISP MODEL)

The inventory level at any instant of time during $[0, T]$ is described by following the differential equation:

$$\frac{dI_R(t)}{dt} + \alpha I_R(t) = -(a - bt) \qquad 0 \leq t \leq t_1 \qquad \text{.... (1)}$$

$$\frac{dI_{O1}(t)}{dt} + \beta I_{O1}(t) = 0 \qquad 0 \leq t \leq t_1 \qquad \text{.... (2)}$$

$$\frac{dI_{O2}(t)}{dt} + \beta I_{O2}(t) = -(a - bt) \qquad t_1 \leq t \leq T \qquad \text{.... (3)}$$

With boundary conditions IR(t1)=0, IO1(0)=W and IO2(T)=0.

Solutions of above equations are:

$$I_R(t) = -\frac{a - bt}{\alpha} - \frac{b}{\alpha^2} + \left(\frac{a - bt_1}{\alpha} + \frac{b}{\alpha^2}\right) e^{\alpha(t_1 - t)} \qquad \text{.... (4)}$$

$$I_{O1}(t) = W e^{-\beta t} \quad \text{.... (5)}$$

$$I_{O2}(t) = -\frac{a-bt}{\beta} - \frac{b}{\beta^2} + \left(\frac{a-bT}{\beta} + \frac{b}{\beta^2}\right)e^{\beta(T-t)} \quad \text{.... (6)}$$

According to given conditions at t=t_1, $I_{O1}(t_1) = I_{O2}(t_1)$

$$W = -\left(\frac{a-bt_1}{\beta} + \frac{b}{\beta^2}\right)e^{\beta t_1} + \left(\frac{a-bT}{\beta} + \frac{b}{\beta^2}\right)e^{\beta T} \quad \text{... (7)}$$

The equation shows the relation between t_1 & t_2.
Next, the total relevant inventory cost per cycle consists of the following elements:

1. **Ordering cost per cycle is** OC = A. ..... (8)
2. Inventory **holding cost** per cycle in **RW** is given by

$$HC_r = h_r \int_0^{t_1} I_R(t)\, dt$$

$$= h_r \left\{ -\frac{a}{\alpha^2}(1+\alpha t_1) + \frac{b}{\alpha}\left(\frac{t_1^2}{2} - \frac{1}{\alpha^2}\right) + \left(\frac{a-bt_1}{\alpha^2} + \frac{b}{\alpha^3}\right)e^{\alpha t_1} \right\} \quad \text{.... (9)}$$

3. Inventor**y holding cost** per cycle in **OW** is given by

$$HC_o = h_o \left(\int_0^{t_1} I_{O1}(t)\, dt + \int_{t_1}^{t_1+t_2} I_{O2}(t)\, dt \right)$$

$$= h_o \left[\frac{W}{\beta}(1-e^{-\beta t_1}) - \frac{a}{\beta^2}(1+\beta t_2) + \frac{b}{\beta}\left(\frac{t_2^2}{2} + t_1 t_2 + \frac{t_1}{\beta} - \frac{1}{\beta^2}\right) + \left\{\frac{a}{\beta} - \frac{bT}{\beta} + \frac{b}{\beta^2}\right\} e^{\beta t_2} \right] \quad \text{.... (10)}$$

4. Deterioration cost per cycle in **RW** is given by

$$DC_r = C_d \int_O^{t_1} \alpha\, I_R(t)\, dt$$

$$= \alpha C_d \left\{ -\frac{a}{\alpha^2}(1+\alpha t_1) + \frac{b}{\alpha}\left(\frac{t_1^2}{2} - \frac{1}{\alpha^2}\right) + \left(\frac{a-bt_1}{\alpha^2} + \frac{b}{\alpha^3}\right)e^{\alpha t_1} \right\} \quad \text{.... (11)}$$

6. Deterioration cost per cycle in **OW** is given by

$$DC_{OW} = \beta C_d \left\{ \int_0^{t_1} \beta\, I_{O1}(t)\, dt + \int_{t_1}^{t_1+t_2} \beta\, I_{O2}(t)\, dt \right\}$$

$$= \beta C_d \left[\frac{W}{\beta}(1-e^{-\beta t_1}) - \frac{a}{\beta^2}(1+\beta t_2) + \frac{b}{\beta}\left(\frac{t_2^2}{2} + t_1 t_2 + \frac{t_1}{\beta} - \frac{1}{\beta^2}\right) + \left\{\frac{a}{\beta} - \frac{bT}{\beta} + \frac{b}{\beta^2}\right\} e^{\beta t_2} \right]$$

.... (12)

Therefore, the total inventory cost per unit time is given by

$$TC(t_1, t_2) = \frac{1}{T}\left[OC + HC_r + HC_o + DC_r + DC_o\right] \quad \text{..... (13)}$$

Substituting Equation (8)–(12) in the above equation (13), we get

$$TC(t_1,t_2) = \frac{1}{T}\left[A + (h_r + \alpha C_d)\left\{ -\frac{a}{\alpha^2}(1+\alpha t_1) + \frac{b}{\alpha}\left(\frac{t_1^2}{2} - \frac{1}{\alpha^2}\right) + \left(\frac{a-bt_1}{\alpha^2} + \frac{b}{\alpha^3}\right)e^{\alpha t_1} \right\} \right.$$

$$\left. + (h_o + \beta C_d)\left\{ \frac{W}{\beta}(1-e^{-\beta t_1}) - \frac{a}{\beta^2}(1+\beta t_2) + \frac{b}{\beta}\left(\frac{t_2^2}{2} + t_1 t_2 + \frac{t_1}{\beta} - \frac{1}{\beta^2}\right) + \left(\frac{a}{\beta} - \frac{bT}{\beta} + \frac{b}{\beta^2}\right)e^{\beta t_2} \right\} \right]$$

... (14)

The total relevant inventory cost per unit time is minimum if

$$\frac{\partial TC}{\partial t_1} = 0,\ \frac{\partial TC}{\partial t_2} = 0,\ \left(\frac{\partial^2 TC}{\partial t_1^2}\right)\left(\frac{\partial^2 TC}{\partial t_2^2}\right) - \left(\frac{\partial^2 TC}{\partial t_1 t_2}\right)^2 > 0 \text{ and } \frac{\partial^2 TC}{\partial t_1^2} > 0.$$

4. FUZZY MODEL

In the above model we develop a crisp model in which assumed all the parameters are fixed or as considered the rate of deterioration in both RW and OW assumed is constant, but in real life and global market situations they fluctuate with own actual values. So the parameters are not assumed to be constant. To deal with such type of uncertainty consider a fuzzy model in which holding costs are assumed in both warehouses, ordering cost and deteriorating cost in a fuzzy number which is represented by triangular numbers. We discussed this fuzzy model used by signed distance method.

5. SIGNED DISTANCE METHOD

This method we used triangular fuzzy number for holding costs, ordering cost and deteriorating cost. Suppose the following fuzzy numbers:

(1) $h_r \in [h_r-\Delta_1, h_r+\Delta_2]$, where $0<\Delta_1<h_r$ and $0<\Delta_1\Delta_2$

(2) $h_o \in [h_o-\Delta_3, h_o+\Delta_4]$, where $0<\Delta_3<h_o$ and $0<\Delta_3\Delta_4$

(3) $A \in [A-\Delta_5, A+\Delta_6]$, where $0<\Delta_5<A$ and $0<\Delta_5\Delta_6$

(4) $C_d \in [Cd -\Delta_7, C_d +\Delta_8]$, where $0<\Delta_7< C_d$ and $0<\Delta_7\Delta_8$.

The signed distance of the above fuzzy numbers is

(1) $d\left(\tilde{h}_r, 0\right) = h_r + \frac{1}{4}(\Delta_2 - \Delta_1)$ (2) $d\left(\tilde{h}_0, 0\right) = h_o + \frac{1}{4}(\Delta_4 - \Delta_3)$

(3) $d\left(\tilde{A}, 0\right) = A + \frac{1}{4}(\Delta_6 - \Delta_5)$ (4) $d\left(\tilde{C}_d, 0\right) = C_d + \frac{1}{4}(\Delta_8 - \Delta_7)$

Using equation (14), we have

$$T\tilde{C} = (TC_1, TC_2, TC_3)$$

$$TC_1(t_1,t_2) = \frac{1}{T}\Bigg[(A-\Delta_5) + \{(h_r-\Delta_1)+\alpha(C_d-\Delta_7)\}\left\{-\frac{a}{\alpha^2}(1+\alpha t_1)+\frac{b}{\alpha}\left(\frac{t_1^2}{2}-\frac{1}{\alpha^2}\right)+\left(\frac{a-bt_1}{\alpha^2}+\frac{b}{\alpha^3}\right)e^{\alpha t_1}\right\}$$

$$+\{(h_0-\Delta_3)+\beta(C_d-\Delta_7)\}\left\{\frac{W}{\beta}(1-e^{-\beta t_1})-\frac{a}{\beta^2}(1+\beta t_2)+\frac{b}{\beta}\left(\frac{t_2^2}{2}+t_1t_2+\frac{t_1}{\beta}-\frac{1}{\beta^2}\right)+\left(\frac{a}{\beta}-\frac{bT}{\beta}+\frac{b}{\beta^2}\right)e^{\beta t_2}\right\}\Bigg]$$

… (15)

$$TC_2(t_1,t_2) = TC(t_1,t_2) \quad \text{… (16)}$$

$$TC_3(t_1,t_2) = \frac{1}{T}\Bigg[(A+\Delta_6) + \{(h_r+\Delta_2)+\alpha(C_d+\Delta_8)\}\left\{-\frac{a}{\alpha^2}(1+\alpha t_1)+\frac{b}{\alpha}\left(\frac{t_1^2}{2}-\frac{1}{\alpha^2}\right)+\left(\frac{a-bt_1}{\alpha^2}+\frac{b}{\alpha^3}\right)e^{\alpha t_1}\right\}$$

$$+\{(h_0+\Delta_4)+\beta(C_d+\Delta_8)\}\left\{\frac{W}{\beta}(1-e^{-\beta t_1})-\frac{a}{\beta^2}(1+\beta t_2)+\frac{b}{\beta}\left(\frac{t_2^2}{2}+t_1t_2+\frac{t_1}{\beta}-\frac{1}{\beta^2}\right)+\left(\frac{a}{\beta}-\frac{bT}{\beta}+\frac{b}{\beta^2}\right)e^{\beta t_2}\right\}\Bigg]$$

… (17)

The total inventory fuzzy cost $T\tilde{C}$ per unit time by signed distance method is

$$d\left(T\tilde{C}\right) = TC(t_1,t_2)$$

$$+\frac{1}{T}\Bigg[\frac{1}{4}(\Delta_6-\Delta_5)+\frac{1}{4}\{(\Delta_2-\Delta_1)+\alpha(\Delta_8-\Delta_7)\}\left\{-\frac{a}{\alpha^2}(1+\alpha t_1)+\frac{b}{\alpha}\left(\frac{t_1^2}{2}-\frac{1}{\alpha^2}\right)+\left(\frac{a-bt_1}{\alpha^2}+\frac{b}{\alpha^3}\right)e^{\alpha t_1}\right\}$$

$$+\{(\Delta_4-\Delta_3)+\beta(\Delta_8-\Delta_7)\}\left\{\frac{W}{\beta}(1-e^{-\beta t_1})-\frac{a}{\beta^2}(1+\beta t_2)+\frac{b}{\beta}\left(\frac{t_2^2}{2}+t_1t_2+\frac{t_1}{\beta}-\frac{1}{\beta^2}\right)+\left(\frac{a}{\beta}-\frac{bT}{\beta}+\frac{b}{\beta^2}\right)e^{\beta t_2}\right\}\Bigg]$$

… (19)

Hence find the optimum solution for any particular situation with the help of the above equations.

6. NUMERICAL EXAMPLE

In order to illustrate the above solution procedure, consider an inventory system with the following data:

1. Crisp Model

A = 1500, $h_r = 0.75$, $h_o = 0.4$, $C_d = 0.5$, a = 10,000, b = 100, α = 0.06, β=.07, t_1=74.84241164, t_2=22.57170445, W=11434856, TC=1461584.

As can be observed in the above illustrations the total cost TC is very sensitive to changes in demand. With increase in the demand part *a*, the optimum time t_1 for inventory in RW, time t_2 for inventory in OW and total cost TC increases. But the demand part *b* is very sensitive, if *b* increases then the total cost TC decreases very fast. Increases in the rate of deterioration in RW, the optimum time t_1 and the total cost TC are increase and t_2 and W are decrease but increase in the rate of deterioration in OW, the optimum time t_1 and the total cost TC are decrease and; t_2 and W increase. When the deteriorating cost, holding costs of RW and OW, increases then the total Cost TC is increases. Increase in the ordering cost does not produce significant changes in the optimal solution.

Changes in		t_1	t_2	W	TC
a	10000	74.842412	22.571704	11434856.092149	1461583.654803
	10100	74.888327	23.564404	12751859.916943	1548475.488495
	10200	74.937297	24.552778	14182424.287745	1640834.300027
b	100	74.842412	22.571704	11434856.092149	1461583.654803
	105	74.666632	17.782286	6821829.842718	1168718.193180
	110	74.575389	13.327105	3876449.783894	959468.556664
α	0.060	74.842412	22.571704	11434856.092149	1461583.654803
	0.061	80.847849	16.489522	8328917.833036	1499439.734488
	0.062	88.707093	8.511129	3829934.237699	1555651.704281
β	0.068	89.378133	7.639075	2827109.678035	1390576.718942
	0.069	81.166470	16.079157	7442118.232486	1416538.967013
	0.070	74.842412	22.571704	11434856.092149	1461583.654803
C_d	0.50	74.842412	22.571704	11434856.092149	1461583.654803
	0.55	74.475588	22.944741	11609290.097202	1470253.967240
	0.60	74.115252	23.311082	11778719.989910	1478968.961514
A	1500	74.842412	22.571704	11434856.092149	1461583.654803
	2000	74.842411	22.571695	11434853.549409	1461588.787529
	2500	74.842411	22.571683	11434851.006654	1461593.920257
h_r	0.75	74.842412	22.571704	11434856.092149	1461583.654803
	0.76	75.974749	21.419540	10884130.542041	1471301.668884
	0.77	77.110807	20.262558	10313414.472063	1481462.145467
h_o	0.38	79.112416	18.221392	9266047.670584	1431454.341059
	0.39	76.923520	20.453368	10408725.138278	1445740.693678
	0.40	74.842412	22.571704	11434856.092149	1461583.654803

2. Fuzzy Model (Signed Distance Method)

A = 1500, $h_r = 0.75, h_o = 0.4$, $C_d = 0.5$, *a* = 10,000, b = 100, α = 0.06, β=.07, t_1^*=75.086558, t_2^*=22.323367, W^*=11317673, dTC=1454645.

Changes in		t_1^*	t_2^*	W^*	dTC
a	10000	75.086558	22.323367	11317672.955929	1454645.211292
	10100	75.130980	23.317813	12630607.618613	1541035.957349
	10200	75.178420	24.307954	14057139.334063	1632859.951235
b	100	75.086558	22.323367	11317672.955929	1454645.211292
	105	74.917475	17.525825	6719625.764435	1163514.209827
	110	74.832155	13.063047	3789881.955575	955478.181312
α	0.060	75.086558	22.323367	11317672.955929	1454645.211292
	0.061	81.135722	16.195947	8168921.916691	1492910.703694
	0.062	89.046291	8.163966	3638009.156865	1549715.124352
β	0.068	89.719043	7.289295	2668005.879413	1385362.666483
	0.069	81.454456	15.785112	7293671.696896	1410428.237132
	0.070	75.086558	22.323367	11317672.955929	1454645.211292
C_d	0.50	75.086558	22.323367	11317672.955929	1454645.211292
	0.55	74.715104	22.701180	11495616.622916	1463285.409470
	0.60	74.350212	23.072217	11668458.071541	1471971.329490
A	1500	75.086558	22.323367	11317672.955929	1454645.211292
	2000	75.086557	22.323356	11317670.391372	1454650.344239
	2500	75.086557	22.323345	11317667.826800	1454655.477187

h_r	0.75	75.086558	22.323367	11317672.955929	1454645.211292
	0.76	76.226898	21.162838	10759008.736275	1464459.192662
	0.77	77.370551	19.997879	10180445.496867	1474720.844893
h_o	0.38	79.397764	17.930111	9112662.023876	1424903.563853
	0.39	77.188163	20.183738	10273908.943095	1438982.654897
	0.40	75.086558	22.323367	11317672.955929	1454645.211292

As can be observed in the above illustrations the total cost dTC is very sensitive to changes in demand. Increase in the demand part *a*, the optimum time t_1^* for inventory in RW, time t_2^* for inventory in OW and total cost dTC increases. But the demand part *b* is very sensitive, if *b* increases then the total cost dTC decreases very fast. Increases in the rate of deterioration in RW, the optimum time t_1^* and the total cost dTC are increase and t_2^* and W^* are decrease but increase in the rate of deterioration in OW, the optimum time t_1^* is decrease and; t_2^*, W^* and the total cost dTC increase. Increase in the ordering cost does not produce significant changes in the optimal solution. When the deteriorating cost, holding costs of RW and OW increases then the total Cost dTC is increases.

7. CONCLUSIONS

This paper deals a fuzzy based inventory model for deteriorating items with linear demand and two warehouse facilities. The proposed model can be used in inventory control of deteriorating items such as fashionable items, medicines, food items, electronic components such as mobile, machines, circuit, toys and fashionable commodities etc. Here we use a numerical example to illustrate both the proposed crisp model and fuzzy model which shows the effects of fuzziness of the parameters on the optimal solution. In comparison the crisp model, the fuzzy model is giving the best optimal solution. In the future study, it is hoped to further incorporate the proposed model into more realistic assumptions, such as probabilistic demand, deterioration, shortages and holding cost like as stock dependent, production dependent, exponential, ramp type, quadratic, weibull deterioration, partial backlogging,.

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