APPLICATIONS OF GRAPH THEORY IN VARIOUS FIELDS

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ABSTRACT
Mathematics plays vital role in various fields. One of the most important areas of Mathematics is graph theory which is used for structural model. This paper aims to focus the applications of graph theory in various fields. Graph theory has significant role in our day to day life. Graphs give a convenient way to represent various types of Mathematical objects. There are many uses of graphs in providing problem solving techniques.
Keywords: Graph, Path, connected graph, Hamiltonian graph, Euler graph

1. INTRODUCTION

There are many papers on Graph Theory and its applications. In [2] BhagyaJyoti discussed applications of Graph Theory in different branches of science. In [11] Veena Rani discussed the uses of Graph Theory in day to day life.In [10] S.G. Srinivas and others discussed the concept of Graph Theory in Computer Science. Recently, in [3] BadwaikJyoti described the recent advances in Graph theory and its applications. Now in 2020 PranabKalita [8] gives uses graph theory in DNA sequencing and its uniqueness issue, Global Positioning System (GPS), the latest technological development. For this they use shortest path algorithms. He also discussed about the fingerprint recognition and its related areas like image processing, computer vision, sensor network, cryptography and other areas.

Graph theory is the most useful subject in all branches of Mathematics and widely applied in subjects like Computer Technology, Communication Science, Electrical Engineering, Physics, Architecture, Operation Research, Economics, Sociology, Genetics etc. Graph Theory is the study of graphs which are mathematical structures used to model pairwise relation between objects. It is a bridge connecting mathematics with different branches of Science.

It is a branch of discrete Mathematics. The graph is a way to express the information in picture form.

Graph theory is increasing area as it is applied to areas of Mathematics, Science and Technology. From the last centuary, many researchers attracted towards Graph theory. Graph theory provided healthy atmosphere for research of provable technique in discrete Mathematics for researchers. Many applications are studied by Graph Theory in the computing, Industrial, natural and social sciences. Graph theory is a vast area with many applications to real life which helps the researchers to get more ideas to manage the problems in the real life situation. It has tremendous application in modern science and engineering. In this paper we study how problem in almost every conceivable discipline can be solved using graph models.

Before we can understand application of graphs we need to know some basic definitions that are part of graphs theory.The undefined terms are from Harary [3], D.B. West [12].

Definition: 1.1A graph G is a triple consisting of vertex set V, edge set E and the relation that associated with each edge two vertices called its end points.

Definition1.2:A graph G is called a simple graph if it has no loop and multiple edges.

Definition 1.3: A simple graph in which every pair of vertices are adjacent with each other is called a complete graph.

Definition 1.4: A bipartite graph or bigraph is a graph whose vertex set can be divided into two disjoint and independent sets V1 and V2 such that every edge connect a vertex V1 to one in V2.

Definition 1.5:A bipartite graph G is called as complete bipartite graph if there exits an edge between every vertex in V1 with every vertex in V2.

Definition1.6:A walk in a graph G is defined as it is alternating sequence of vertices and edges with finite length.

Definition 1.7: A trail is a walk in which no repetition of edges.

Definition 1.8:A open walk in which no vertex appears more than once is called a path.

Definition 1.9: A closed walk in which no vertex appear more than once is called a cycle.

Definition 1.10:A simple circuit which contains every edge of graph G is called as Euler circuit.

Definition 1.11 An Euler path in a graph G is a simple path containing every edge of G.

Definition1.12: A graph G is called as Eulerian graph if it contains either Euler circuit or Euler path.

Definition 1.13: A simple path in a graph G which passes through every vertex exactly once is called a Hamiltonian path.

Definition 1.14: A simple circuit in a graph G which passes through every vertex exactly once is called a Hamiltonian path.

Definition 1.15: A graph which contains either Hamiltonian circuit or Hamiltonian path is called as Hamiltonian graph.

Definition 1.16: A directed graph is graph that is a set of objects (called vertices or nodes) that are connected together, where all the edges are directed from one vertex to another.

1.1 Applications of Graph Theory in Technology

The concepts and ideas of graph theory are widely used in various branches of science. Now without knowledge of graph we use graph theory concepts in our daily life. For example when we have to go to a place which is connecting with our starting point by different ways then we use the shortest road to arrive the destination earlier. Here if we consider this problem in terms of graph theory the two places can be considered as vertices and roads are as edges. If we also assume the direction of travel, then the graph is directed. In similar way we can use concepts of graph theory in various situations.

A] Chemistry: For mathematical modelling of chemical phenomena graph theory is used. For this we represent vertices as atom and edges as bond. To solve the molecular problems chemical graph theory is used.

For example: A hydrocarbon molecule consists of a carbon and hydrogen atom which is connected by chemical bonds. We can represent a hydrocarbon by a graph in which the vertices are the atoms forming the molecules and the edges are the bonds connecting two atom results in the formation of connected graph.In model molecule structures for computer processing graphs are used. Atoms are considered as vertices of a graph and the bonds that connect them are edges between them. Based on the properties of compounds these structures are created and then taken for analysis and processing. It is used to study the structure of molecules and to check similarity level between molecules.

B] Physics: The concept of directed graph is used in physics. To show the flow of current in circuit we use directed graph. Also the voltage, resistance on a circuit can be shown by using graph theory concepts.

C] Biology: To study the structure of DNA and RNA we use graph theory concepts. Also if we want to study the food chain of various animals in a ecological system then we use graph theory concept such as animals are vertices and they connect with each other by an edge if they are depend on each other for food.

For biological networks analysis graph theory concept is also used. Graph theory is widely used in following biological analysis; Protein-protein interaction (PPI) networks, Regulatory networks (GRNs), Signal transduction networks, and Metabolic and biochemical networks. When we analysis these different components then it will be generated the structure network which is similar to one of the graph component in graph theory. To solve the problem of DNA fragment assembly Eulerian circuit concept from graph theory is used.

D] Mathematics:

i) Operation Research: The concepts of graph theory are very useful tool in operation research. Operation research problems can be solved by using graphs. In transportation problem, to minimize the cost or to maximize the profit, we need the graph theoretical concepts.

In different assignment problem such as assigning different peoples to different jobs, manage time table of school, colleges, universities and office etc. Graph theory is dynamic tool in combinatorial operation research. The most popular and successful applications of network in OR is the planning and scheduling of large complicated project. Game theory is applied to the problems in engineering, economics and war science to find optimal way to perform certain tasks in competitive environment. The method of finite game is represented by bigraph. Many important OR problems are solved using graphs. A Graph is used to model the transportation of commodity from one destination to another. A network called transport network. The main objective is to maximize and minimize the cost within the prescribed flow.

ii) Lattice Theory: There are many papers which interlink graph theory and lattice theory. In [6] Survase and Wasadikar defined the zero-divisor graphs of lattices and in [7] they defined incomparability graphs of lattices. By using graph theoretical concepts they got many interesting results.

iii) Group Theory: We known that the subgroup diagram of groups can be drawn with the help of graphs. Also lattice diagram of groups can be drawn with the help of graphs. Here subgroups obtained from given groups can be considered as atoms and covering relations are considered as edges between them.

E] Computer Science: Graph theory plays important role in computer science. To develop the algorithm of different programmes graph theory concepts is used. By using these algorithm and programme we can solve many theoretical problem. In computer science, graphs are used to represent communication network, data mining, the flow of computation,

etc. The link structure of a website can be represented by a directed graph, in which the vertices represent web pages and directed edges represent links from one page to another.

Vertex coloring is very important concepts in graph theory. It is used in many real time applications in computer science. Different methods of coloring are available and can be used on requirement basis. The minimum number of colors required to label the vertices of graphs such that no two vertices have same color. So that number is called as chromatic number. Chromatic number concepts are widely used in network analysis.

Software Engineering: Graphs have many applications in software engineering. For example: between the requirements specification, data flow diagrams are used where vertices represent transformation and edges indicate data flow. In the design phase, graphical design is used to describe the relationships between the modules. In Software process management uses graph algorithms for network diagram.

F] Geography:-
Consider a map of India. Let us consider each country as vertex and they connect with each other by an edge if those countries share a border. This problem remains unsolved form many years and that problem is four color problems. We know that any map can be colored with at most 4 colors in such a way that no two adjacent vertices (countries) have the same color. This problem motivated a lot of the development of graph theory and was finally proved with the help of a computer in 1976.

1.2 Applications in Daily life
i) GPS or Google Map: To find the shortest distance from one destination to another we use GPS map. Here destination is vertices and their connections are edges. The optimal route is obtained by this software. School and colleges are used this technique to pickup students from their stop to school. Each stop is considered as vertex and the route is considered as an edge. The concept of Hamiltonian path is used.

ii) Social Network: We connect with friends and family via social media or video get viral. Here user is a vertex and other connected users are edge. Therefore video get viral when reached to certain connection.

iii) Traffic lights: For turning Green/red lights and timing between them the vertex coloring technique is used to solve conflicts of time and space by identifying the chromatic number for the number of cycles required.

iv) Google Search: By using google we search the web pages. Pages on the internet are linked by the hyperlink. Here page is a vertex and the link between two pages is an edge. Here connected graph concept is used. The problem is solved by using connectivity algorithm.

v) To clear road blockage: In city when roads are blocked due to ice. For this we have to plane to put salt on the road. To solve this problem Euler path or circuit is used to traverse the road in significant way.

vi) GSM mobile phone networks and Map coloring: All mobile phones connect to GSM network by searching for cells in the neighbors. For this concept of coloring from graph theory is used. For this process only four colors may be used to color the cellular regions.

vii) Measuring performance and Progress report: Graph theory is widely used in an organization/ Institutes to measure the performance of anemployee for certain period. The graph can be shown the progress or degrading of an employee or region. It will help an employee to motivate and work with more dedication.

CONCLUSION: This paper is helpful for students and researchers to get overview of graph theory and its applications in different fields. Graph theory is a vast area with more applications to real life which helps the researchers to get more ideas to manage the problems in the real life situation. We hope that this paper would attract many new researchers towards graph theory. The main purpose of this paper is to give some important graph theoretical applications in different fields so that the readers or researchers and students may create some interest or ideas for future research.

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