Simulation of Tie Lines in Interconnected Power Systems

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ABSTRACT

Modern day tradition is to interconnect systems operated by different power companies through tie lines and this increases the reliability of electric energy supply. It is therefore important to introduce the concepts for modeling interconnected networks for addition or removal of the tie lines for contingency analysis. This paper validates the modeling of the power networks, interconnected through the tie line. A piecewise method of interconnection has been presented. The method is based on the basic assumption that each network is simulated by a linear model for its analysis by the individual operating authority.

Keywords: simulation, incidence matrix, stand-alone, tie lines etc.

1. INTRODUCTION

Most if not all of the world’s electric power supply systems are widely interconnected involving the connections inside utilities’ own territories which extend to inter-utility interconnections and then to inter-regional and international connections. This is done for economic reasons, to reduce the cost of electricity and to improve reliability of power supply [4]. The purpose of the transmission network is to pool power plants and load centers in order to minimize the total power generation capacity and fuel cost. These interconnections enable taking advantage of diversity of loads, availability of sources, and fuel price in order to supply electricity to the loads at minimum cost with a required reliability. In general, if the power delivery system was made up of radial lines from the individual local generators without being part of a grid system, many more generation resources would be needed to serve the load with the same reliability, and the cost of electricity would be much higher. Less transmission capability means more generation resources would be required regardless of whether the system is made up of large or small power plants. In fact small distributed generation become more economically viable if there is a backbone of a transmission grid [7]. One can-not be really sure about what is optimum balance is between generation and transmission unless the system planners use advanced methods of analysis which integrate transmission planning into an integrated based transmission/generation planning scenario.

Simulation Method

Figure 1 shows a four bus power system A interconnected through two tie lines, of impedances $Z_a$ & $Z_b$ , with another three bus power network B. Both the power systems are independent except for the interconnections and are earthed thereby providing a common reference point. Initially it is assumed that both the systems operate in stand alone mode, and use the $Z_{bus}$ of their own networks for analyzing their systems. These results can be later modified, by each system, to take advantage of the interconnections. between two networks. It may be noted that buses 3 and 4 of the system A are connected to buses 5 and 6 of system B. The current injections and voltage at each bus, shown in figure, are assumed known and represent the networks in the stand-alone mode. Mathematically, the relationship between the bus voltages and injected currents with the tie lines open may be written as [1-3].
\[ V = \begin{bmatrix} V_1 \\ \vdots \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} = \begin{bmatrix} Z_{A_{bus}} & 0 \\ 0 & Z_{B_{bus}} \end{bmatrix} \begin{bmatrix} I_4 \\ \vdots \\ I_7 \end{bmatrix} \] (1)

where \( Z_{A_{bus}} \) is the 4 x 4 bus impedance matrix of system A. Similarly \( Z_{B_{bus}} \) is the 3 x 3 bus impedance matrix of system B. The task is to determine the new injected bus currents and the resulting changed bus voltages, in the two power systems, when they are interconnected via two tie lines. Assume that the impedances of the two tie lines connected between buses 3-5 and 4-6 are \( Z_a \) and \( Z_b \) respectively, while the currents flowing through them are \( I_a \) and \( I_b \) respectively.

Assume all the changed voltages are represented by \( V'_{1}, V'_{2}, V'_{3} \) and \( V'_{4} \) in system A and by \( V'_{5}, V'_{6}, V'_{7} \) in power system B due to the injected currents \( I_a \) and \( I_b \). The effect of the tie line currents can be simulated by tearing the interconnected network shown in figure 1 into two pieces at the tie lines and using equation.

\[ \begin{bmatrix} a & b \\ b & a \end{bmatrix} \begin{bmatrix} \epsilon_{ij} - Z_{ij} & \epsilon_{ij} - Z_{ij} \\ \epsilon_{ji} - Z_{ji} & \epsilon_{ji} - Z_{ji} \end{bmatrix} \begin{bmatrix} \epsilon_{pq} - Z_{pq} \\ \epsilon_{qp} - Z_{qp} \end{bmatrix} = \begin{bmatrix} \epsilon_{ij} - Z_{ij} & \epsilon_{ij} - Z_{ij} \\ \epsilon_{ji} - Z_{ji} & \epsilon_{ji} - Z_{ji} \end{bmatrix} \begin{bmatrix} \epsilon_{pq} - Z_{pq} \\ \epsilon_{qp} - Z_{qp} \end{bmatrix} \] (2)

Figure 1: Two Power System Networks Connected Through Tie Lines

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Figure 2 shows the two equivalent power systems along with the injected currents.

The impedance matrix $Z$ can be computed by substituting $i = 3, j = 5, p = 4,$ and $q = 6$ in equation 2. Thus

$$Z = \frac{a}{b} \begin{bmatrix} Z_{33}^A + Z_{55}^B + Z_a & Z_{34}^A + Z_{56}^B \\ Z_{43}^A + Z_{65}^B & Z_{44}^A + Z_{66}^B + Z_b \end{bmatrix}$$

For the individual system A, the bus voltages $V_3$ and $V_4$ are unknown. Similarly for the system B, the bus voltages $V_5$ and $V_6$ are also known. By using following equation, the tie line currents are computed.

$$\begin{bmatrix} I_a \\ I_b \end{bmatrix} = Z^{-1} \begin{bmatrix} V_i - V_j \\ V_p - V_q \end{bmatrix}$$

From the above equation, the Thevenin’s equivalent circuits for the power system A and B can be perceived directly.
It may be observed that matrix A shows the incidence of tie lines on the buses of system A and B.

Assume that due to injected tie line currents $I_a$ and $I_b$, the changed bus voltages in system A are designated by $V_1', V_2', V_3'$ and $V_4'$, similarly the new bus voltages in system B are represented by $V_5', V_6', \text{ and } V_7'$. By using:

$$
\Delta V = V' - V = -Z_{\text{bus}} A^t \begin{bmatrix} I_a \\ I_b \end{bmatrix} \quad \text{Or} \quad V' = V - Z_{\text{bus}} A^t \begin{bmatrix} I_a \\ I_b \end{bmatrix}
$$

The new bus voltages are computed as follows:

$$
\begin{bmatrix} V_1' \\ V_2' \\ V_3' \\ V_4' \\ V_5' \\ V_6' \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} - Z_{\text{bus}} A^t \begin{bmatrix} a & 0 & 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & Z_{\text{bus}}^B & b & 0 & 0 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \end{bmatrix}
$$

Alternatively, the piece-wise method may also be applied to obtain the new system voltages as follows:

$$
\begin{bmatrix} V_1' \\ V_2' \\ V_3' \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} - Z_{\text{bus}}^A \begin{bmatrix} a & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} -I_a \\ -I_b \end{bmatrix}
$$

$$
\begin{bmatrix} V_5' \\ V_6' \end{bmatrix} = \begin{bmatrix} V_5 \\ V_6 \\ V_7 \end{bmatrix} - Z_{\text{bus}}^B \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \end{bmatrix}
$$

**Figure 3. Thevenin's equivalent circuits of the interconnected systems showing the paths of the tie line currents $I_a$ and $I_b$ through the reference bus**
In computing the new bus voltages, when the two networks are interconnected through tie lines, the major task is the formation of loop impedance matrix \( Z \). The computation of the inverse of the loop impedance matrix \( Z \), however, is simple since its order is dependent on the number of tie lines and is small.

**Formation of Loop Impedance Matrix \( Z \)**

Mathematically, the loop impedance matrix \( Z \) may be formed by using the piecewise method and using the following equation.

\[
Z = \begin{bmatrix}
    a & b
    
    c & d
\end{bmatrix}
\]

Equation 3 shows that the loop impedance matrix \( Z \) is made of the tie-line impedances and the respective sub-matrices from the bus impedance matrices \( Z_{bus}^A \) and \( Z_{bus}^B \). Hence, for the interconnected network shown in figure 1, the sub-matrices of interest would be,

\[
\begin{bmatrix}
    Z_{33} & Z_{34} \\
    Z_{43} & Z_{44}
\end{bmatrix}
\text{ from } Z_{bus}^A \text{ and } \begin{bmatrix}
    Z_{55} & Z_{56} \\
    Z_{65} & Z_{66}
\end{bmatrix}
\text{ from } Z_{bus}^B
\]

The branch to bus incidence matrix \( A^A \) showing the incidence of the tie lines on the boundary buses of system A is given by

\[
A^A = \begin{bmatrix}
    a & 1 & 0 \\
    b & 0 & 1
\end{bmatrix}
\]

\[
Z^A = A^A \begin{bmatrix}
    Z_{33} & Z_{34} \\
    Z_{43} & Z_{44}
\end{bmatrix} A^{A'}
\]

Similarly from system B

\[
A^B = \begin{bmatrix}
    a & -1 & 0 \\
    b & 0 & -1
\end{bmatrix}
\]

\[
Z^B = A^B \begin{bmatrix}
    Z_{55} & Z_{56} \\
    Z_{65} & Z_{66}
\end{bmatrix} A^{B'}
\]

The loop incidence matrix \( Z \), therefore is computed as.

\[
Z = Z^A + \begin{bmatrix}
    a & 0 \\
    0 & Z_b
\end{bmatrix} + Z^B
\]
In a generalized form, the sub-matrices, removed from the original $Z^A_{bus}$ and $Z^B_{bus}$ system impedance matrices may be expressed as:

$$
\begin{bmatrix}
  i & j & k \\
  i & Z_{ii} & Z_{ij} & Z_{ik} \\
  j & Z_{ji} & Z_{jj} & Z_{jk} \\
  k & Z_{ki} & Z_{kj} & Z_{kk}
\end{bmatrix}
$$

Taken out from $Z^A_{bus}$ matrix of system A

$$
\begin{bmatrix}
  p & q & r & s \\
  pp & Z_{pq} & Z_{pr} & Z_{ps} \\
  qp & Z_{qq} & Z_{qr} & Z_{qs} \\
  Z_{qp} & Z_{qr} & Z_{rr} & Z_{rs} \\
  Z_{qp} & Z_{qr} & Z_{sr} & Z_{ss}
\end{bmatrix}
$$

Taken out from $Z^B_{bus}$ matrix of system B

In the two sub-matrices, $i$, $j$, and $k$ are the boundary buses of system A and $p$, $q$, $r$, and $s$ are the boundary buses of system B. The two systems are interconnected, via tie lines at the boundary buses [8, 9]. Inverse of the above sub-matrices leads to the bus admittance matrices. These bus admittance matrices symbolize the system A and B connected via tie lines at the boundary buses. These equivalent admittance sub matrices are called ward equivalents.

**Conclusion**

The diagonal elements of the loop impedance matrix $Z$ are the sum of the impedances in the loop or the path traversed by the tie line currents starting from the reference bus of the system A to the reference bus of the system B. The off diagonal elements of $Z$ are represented by those impedances through which tie line currents are flowing and are the cause for producing new system voltages when the systems are interconnected.

**Reference:**