FALSE DYNAMIC EIV MODEL IDENTIFICATION IN THE PRESENCE OF NON-PARAMETRIC DYNAMIC UNCERTAINTY

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Abstract

For the past decade noise corrupted output measurements have been a fundamental research problem to be investigated. On the other hand, the estimation of the parameters for linear dynamic systems when also the input is affected by noise is recognized as more difficult problem which only recently has received increasing attention. Representations where errors or measurement noises/disturbances are present on both the inputs and outputs are usually called errors-in-variables (EIV) models. In these systems the input is corrupted by noise during measurement only but there is possibility that the noise affects the input and can also affect the output measurements, consequently this gives rise to a false EIV problem [17]. Parameter estimation of this false EIV problem with additive uncertainty effects, with the possibility of noise bearing different assumptions, will be considered in this paper using equation error and output error schemes. The comparative study of these two schemes has also been carried out.

Keywords: Errors-in-variable(EIV), False EIV, additive uncertainty, equation error, output error, Gaussian noise.

1.1 Introduction

In the basic dynamic EIV models the input is corrupted by noise during measurement only but there may be possibility that noise gets added in the input and affects the performance of the system, which can be termed as false EIV problem [10, 17]. With reference to these systems, the assumptions (prejudices) which lie behind the identification procedure have been thoroughly analyzed in Kalman with particular attention to the Frisch scheme [7, 9, 10]. This scheme assumes that each variable is affected by an unknown amount of additive noise and each noise component is independent of every other noise component and of every variable. Some classical work on EIV include the work by Adcock, Koopmans (4a), Reiersøl and others whose extensive analysis is given in Anderson [1, 2, 3, 13]. Other works deal with ‘EIV filtering’. This refers to filtering problems, where both input and output measurements are contaminated by noise which has been treated by Guidorzi, Diversi, and Soverini and Markovsky, Willems, and De Moor [8, 12]. Scaled prediction variances in the errors-in-variables models are investigated and their performance is compared with those in classical model of response surface designs [21]. A number of common methods for errors-in-variables methods can be put into a general framework, resulting into a Generalized Instrumental Variable Estimator (GIVE). Söderström presented various computational aspects of GIVE and the asymptotic distribution of the parameter estimates [18].

This paper will be concentrated on the estimation of parameters of false EIV model, with and without additive uncertainty using equation error (EQ) and output error (OP) identification methods [15, 16, 19, 20].

1.2 False dynamic errors-in-variables problem

Dealing with false dynamic EIV models the output measurement noise \( \tilde{y}(n) \) is considered as a form of process disturbance and the total effect of \( \tilde{u}(n) \) and output measurement noise \( \tilde{y}(n) \) can be modeled as a single auto-correlated disturbance on the output side as shown in fig. 1.1.

Fig. 1.1 False Errors-In-Variables Problem

Here \( u(n) \) is the designed input but \( \tilde{u}(n) \) is added (due to distortions or other unavoidable reasons) before the input reach the system as \( u(n) \) which is the noise free input in EIV model. Assumptions on input and output noises can be taken as:

Assumption 1 (A1): When both input and output noises are ARMA (auto regressive moving average) models.
Assumption 2 (A2): When input noise is white noise sequence and output noise is ARMA model.
Assumption 3 (A3): When both input and output noises are white noise sequence.

Apart from above assumptions another possibility can be there regarding the noise which may be called as assumption A4.
Assumption 4 (A4): When input noise is ARMA model and output noise is white noise sequence.

### 1.3 False v/s basic dynamic EIV model

- The dynamics between \( u(n) \) and \( y(n) \) is the same as between \( u(n) \) and \( y(n) \) as shown in Fig. 1.1. Hence the presence of \( \theta(n) \) is not so problematic here as was in basic dynamic EIV model [17].
- \( \theta(n) \) affects also the output measurements \( y(n) \) in case of false dynamic EIV problem.
- For the situation as shown in fig. 1.1, it is appropriate to regard \( \theta(n) \) as a form of process disturbance. The total effect of \( u(n) \) and \( \theta(n) \) can be modeled as a single auto correlated disturbance on the output side.

From the above considered facts it has been realized that false EIV problem is not actually an EIV problem. Therefore the false EIV problem may lie in the following two categories:

- Category-1 When both input-output observations are corrupted by noise.
- Category-2 When only the output observation is corrupted by noise.

In the paper, the false EIV problem with and without additive uncertainty for the category-1 will be identified using EQ and OP formulation for all the assumptions pertaining to input-output noise [4].

### 1.4 Mathematical formulation for category-1 false EIV model

In this formulation both input-output observations are corrupted by noise and the available signals take the form as:

\[
\begin{align*}
\hat{u}'(n) &= \hat{u}(n) + \hat{\epsilon}(n) \\
\hat{y}'(n) &= \hat{y}(n) + \hat{\eta}(n)
\end{align*}
\]

(1.1)

The ARMAX system is given by

\[
\begin{align*}
(1 - A(n, q))\hat{y}'(n) &= B(n, q)\hat{u}'(n) \\
&= B(n, q)u'(n) + \eta(n) + \hat{\eta}(n)
\end{align*}
\]

(1.2)

where

\[
A(n, q) = \sum_{i=1}^{I} a_i q^{-i}
\]

\[
B(n, q) = \sum_{i=1}^{P} b_i q^{-i}
\]

\[
y'(n) = y'(n) + \eta(n)
\]

(1.3)

Prediction of the output Eq. 1.3 using Eq. 1.2 is given by:

\[
\hat{y}'(n) = \sum_{i=1}^{P} \hat{h}_i y'(n-i) + \sum_{i=1}^{I} \hat{h}_i u'(n-i)
\]

(1.4)

or

\[
\hat{y}'(n) = \phi'(n) B(n-1)
\]

(1.5)

\[\phi' = (\phi(n-1), \ldots, \phi(n-M), u(n-M), \ldots, u(n-M+1))^{T}\]

(1.6)

The parameter vector.

The identification of parameters of above described false EIV model will now be carried out considering category-1 with and without uncertainty for all the four assumptions using EQ and OP formulations [14].

### 1.5 Case study-I (without uncertainty)

False dynamic errors-in-variables problem is now implemented on the ARX model \( H_2(z) \) given as [14]:

\[
H_2(z) = \frac{\hat{B}_2(z)}{\hat{A}_2(z)} = \frac{\sum_{i=1}^{P} b_i z^{-i}}{1 - \sum_{i=1}^{I} a_i z^{-i}}
\]

(1.7)

where \( \hat{B}_2(z) = \hat{b}_0 + \hat{b}_1 z^{-1} + \ldots + \hat{b}_P z^{-P} \) and \( \hat{A}_2(z) = 1 - \hat{a}_I z^{-1} - \hat{a}_2 z^{-2} - \ldots - \hat{a}_I z^{-I} \).

The output \( u(n) \) in periodic form is taken as:

\[
u(n) = \sin(\pi (1.42n) + \sin(1.42n)) + \sin(2.142n)
\]

(1.8)

This false EIV model given by Eq. 1.7 is subjected to input-output noise form \( \nu(n) \) taken as:

\[
\nu(n) = \begin{cases} v_1(n), & \text{if } \gamma \geq 0.0; \\ v_2(n), & \text{otherwise} \end{cases}
\]

(1.9)

where \( v_1(n) \) is the white Gaussian noise with variance 0.01, and \( v_2(n) \) is also the white noise with variance 1. \( \gamma \) is a random variable and its distribution is uniform within (0, 1).

The transfer function of the input-output noise is given by:

\[
\frac{v_1(n)}{\nu(n)} = H_{I1}(z)
\]

(1.10)

\[
\frac{v_2(n)}{\nu(n)} = H_{I2}(z)
\]

(1.11)

In difference equation form:

\[
\hat{a}(\gamma) = 0.68 \hat{a}(\gamma) + 0.68 \hat{x}(n-1)
\]

(1.12)

\[
\hat{y}'(n) = 0.2 \hat{y}'(n) + 0.8 \hat{y}'(n-1)
\]

(1.13)
The parameters of false EIV model without uncertainty are estimated using EQ and OP formulations [19] considering all the assumptions A1 to A4.

- **EQ and OP formulation of false EIV model without uncertainty**
The estimated parameters using all A1 to A4 assumptions are shown in fig. 1.2 to 1.9 for EQ and OP techniques.

Based on the results from fig. 1.2 to 1.9 the estimated values of the parameters are quantitatively shown using all the four assumptions in table 1.1 for EQ and OP schemes whereas the errors in the parameters and system output are graphically shown by fig.1.10 for these schemes and it has been observed that these errors are very small. Proceeding to this, the assumption wise comparisons for errors in parameters and system output for EQ and OP schemes are given.

<table>
<thead>
<tr>
<th>True value</th>
<th>EQ (BAR)</th>
<th>OP (BAR)</th>
<th>EQ (IWOAR)</th>
<th>OP (IWOAR)</th>
<th>EQ (BWN)</th>
<th>OP (BWN)</th>
<th>EQ (IAROW)</th>
<th>OP (IAROW)</th>
</tr>
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<tbody>
<tr>
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<td>1.1314</td>
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<td>1.1371</td>
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<td>-0.2584</td>
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<td>0.0587</td>
<td>0.0569</td>
<td>0.0570</td>
<td>0.0586</td>
<td>0.0609</td>
<td>0.0580</td>
<td>0.0520</td>
</tr>
<tr>
<td>b1</td>
<td>-0.4</td>
<td>-0.4065</td>
<td>-0.4099</td>
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<td>-0.3901</td>
<td>-0.3955</td>
<td>-0.3907</td>
<td>-0.3989</td>
</tr>
</tbody>
</table>
Comparison of EQ and OP schemes without uncertainty for errors in the parameters and output

For both input and output noise as ARMA models
- a₁* is less for EQ as compared to the a₁* of OP scheme.
- a₂* is less for EQ as compared to the a₂* of OP scheme.
- b₀* is less for OP as compared to the b₀* of EQ scheme.
- b₁* is less for EQ scheme as compared to the b₁* of OP scheme.
- e₁ is less for EQ as compared to the e₁ of the OP scheme.

For input noise as white noise sequence and output noise as ARMA model
- a₁* is less for EQ as compared to the a₁* of OP scheme.
- a₂* is less for OP as compared to the a₂* of EQ scheme.
- b₀* is less for EQ as compared to the b₀* of OP scheme.
- b₁* is less for EQ scheme as compared to the b₁* of OP scheme.
- e₁ is less for EQ as compared to the e₁ of the OP schemes.

For both input and output noise as white noise sequences
- a₁* is less for EQ as compared to the a₁* of OP scheme.
- a₂* is less for OP as compared to the a₂* of EQ scheme.
- b₀* is less for OP as compared to the b₀* of EQ scheme.
- b₁* is less for EQ scheme as compared to the b₁* of OP scheme.
- e₁ is less for EQ as compared to the e₁ of the OP schemes.

For input noise as ARMA model and output noise as white noise sequence
- a₁* is less for EQ as compared to the a₁* of OP scheme.
- a₂* is less for OP as compared to the a₂* of EQ scheme.
- b₀* is less for EQ as compared to the b₀* of OP scheme.
- b₁* is less for EQ scheme as compared to the b₁* of OP scheme.
- e₁ is less for EQ as compared to the e₁ of the OP schemes.

After analyzing the results obtained above it is concluded that when false EIV model without uncertainty is identified by using equation error and output error formulation considering the category-1 (both input-output observations corrupted by measurement noise) the equation error formulation is giving less error in almost all the parameters and hence output as compared to output error scheme in all the possibilities of noise entering into the system.

1.6 Case study-II (with uncertainty)
The parameters of the system given by Eq. 1.7 are estimated considering all the four assumptions A1-A4 pertaining to input-output noise using EQ and OP methods [19]. The ARMA model of the input-output noise is given by Eq. 1.12 and 1.13 respectively whose general form is given by Eq. 1.9. The input to the system is periodic given by Eq. 1.8. The system is perturbed by additive uncertainty given by Eq. 1.14 and 1.15 respectively [15]:

\[ H_2(z) = \frac{b(z)E(z)}{1-a(z)} \]  
(1.14)

where \( H_2(z) \) is the perturbed plant.

\[ B(z) = \frac{a(z) - e(z)}{1-a(z)} \]  
(1.15)

where \( B(z) \) is the perturbation bound based on a priori physical information.

Here the two cases for the additive uncertainty have been considered [5, 6, 20]:

- \( \Xi(z) = 1 \)
- \( \Xi(z) = 0.968 \)
The parameters of false EIV model with additive uncertainty \( \hat{\theta}(z) = 1 \) and \( \hat{\theta}(z) = 0.66 \) are estimated using EQ and OP formulations considering all the assumptions A1 to A4.

The estimated parameters using all the assumptions are shown in fig. 1.11 to 1.18 for EQ and OP formulations with \( \hat{\theta}(z) = 1 \).

- The estimated parameters using all the assumptions are shown in fig. 1.11 to 1.18 for EQ and OP formulations with \( \hat{\theta}(z) = 1 \).

### Table 1.2 Estimated values of the parameters for false dynamic EIV model using EQ and OP formulation with \( \hat{\theta}(z) = 1 \).

<table>
<thead>
<tr>
<th>True value</th>
<th>EQ (BAR)</th>
<th>OP (BAR)</th>
<th>EQ (IWOAR)</th>
<th>OP (IWOAR)</th>
<th>EQ (BWN)</th>
<th>OP (BWN)</th>
<th>EQ (IAROW)</th>
<th>OP (IAROW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a1</td>
<td>1.1314</td>
<td>1.1516</td>
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<td>1.1375</td>
<td>1.1334</td>
<td>1.1414</td>
<td>1.1339</td>
<td>1.269</td>
</tr>
<tr>
<td>a2</td>
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<td>-0.2580</td>
<td>-0.2519</td>
<td>-0.2575</td>
<td>-0.2518</td>
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<td>-0.2522</td>
<td>-0.2456</td>
</tr>
<tr>
<td>b0</td>
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<td>0.0588</td>
<td>0.0526</td>
<td>0.0594</td>
<td>0.0519</td>
<td>0.0600</td>
<td>0.0526</td>
<td>0.0563</td>
</tr>
<tr>
<td>b1</td>
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<td>-0.4132</td>
<td>-0.4007</td>
<td>-0.4126</td>
<td>-0.4008</td>
<td>-0.4110</td>
<td>-0.3919</td>
<td>-0.4004</td>
</tr>
</tbody>
</table>
- **Comparison of EQ and OP schemes with** $\beta(z) = 1$ **for errors in the parameters and output**

- The error in the parameters and system output is minimum for output error algorithm as compared to equation error for all the assumptions subjected to input-output noise.

- Now the parameters of false EIV model with additive uncertainty $\beta(z) = 0.998$ are estimated using EQ and OP formulations with all assumptions described above which are shown in fig. 1.20 to 1.27
Based on the results from fig. 1.20 to 1.27 the estimated values of the parameters are quantitatively shown using all the four assumptions in table 1.3 for EQ and OP schemes whereas the errors in the parameters and system output are graphically shown in fig.1.28 for these schemes and it has been observed that these errors are very small.

**Table 1.3 Estimated values of the parameters for false dynamic EIV model using EQ and OP formulation**

<table>
<thead>
<tr>
<th>True value</th>
<th>EQ (BAR)</th>
<th>OP (BAR)</th>
<th>EQ (IWOAR)</th>
<th>OP (IWOAR)</th>
<th>EQ (BWN)</th>
<th>OP (BWN)</th>
<th>EQ (IAROW)</th>
<th>OP (IAROW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a1</td>
<td>1.1314</td>
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<tr>
<td>a2</td>
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<tr>
<td>b0</td>
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<td>0.0561</td>
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</tr>
<tr>
<td>b1</td>
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<td>-0.4088</td>
<td>-0.4071</td>
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<td>-0.4075</td>
<td>-0.3943</td>
<td>-0.4064</td>
<td>-0.3968</td>
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</tbody>
</table>

From the above table it is clear that the error in parameters and hence output is less using equation error formulation as compared to output error algorithm for all the assumptions subjected to input-output noise.

1.7 Conclusion

From the analysis of the results obtained above it is concluded that when false EIV model without uncertainty is identified by using equation error and output error formulation considering both input-output observations corrupted by measurement noise, the equation error formulation is giving less error in almost all the parameters as compared to output error scheme for all the assumptions subjected to input-output noise. When the same model is identified for using equation error and output error algorithms considering then it is found that the error in all the parameters and output is minimum for output error as compared to equation error. On the other hand when this case is considered for it is found that the error in parameters and hence output is less using equation error formulation as compared to output error algorithm.

**REFERENCES**


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