The Resolution in the Electron Microscopy

Mohammed Jawad Yaseen

Department of Physics, College of Education, the University of Mustansiriyah, Baghdad, Iraq

ABSTRACT

Benefit from the group's equations, especially the resolution limits in the transmission electron microscope (TEM) and scanning transmission electron microscope (STEM) calculated the resolution limits in the objective lens of these microscopes using a magnetic field resulting from mathematical models. Magnetic fields of these models have been limited region relatively on the optical axis and vanish before the end of the lens limited. The resolution limits calculated when the imagining formation is coherent time and incoherent another time. Also, depending on the optimum aperture was calculated depth of field and depth of focus of the objective lens in electron microscopes. Present work proved more convergence results of the resolution limits for both cases.

Keywords: Electron Optics, Electron Microscopy, Magnetic Lenses, Charged Particle, Distortion.

1. INTRODUCTION

The scanning transmission electron microscopy (STEM) is an invaluable tool for the characterization of nanostructures, providing a range of different imaging modes with the ability to provide information on elemental composition and electronic structure at the ultimate sensitivity, that of a single atom. The STEM works on the same principle as the normal scanning electron microscopy (SEM), by forming a focused beam of electrons that is scanned over the sample while some desired signal is collected to form an image [1]. The difference with SEM is that thin specimens are used so that transmission modes of imaging are also available. Although the need to thin bulk materials down to electron transparency can be a major task, it is often unnecessary for nanostructured materials, with sample preparation requiring nothing more than simply sprinkling or distributing the nanostructures onto a commercially available thin holy carbon support film. No long and involved grinding, polishing, or ion milling is required, making the STEM a rapid means for nanostructure characterization.

As in the SEM, secondary or backscattered electrons can be used for imaging in STEM; but higher signal levels and better spatial resolution are available by detecting transmitted electrons. A bright field (BF) detector includes the transmitted beam and so the holes appear dark. Each detector provides a different and complementary view of the specimen. It is one of key advantages of the STEM to have multiple detectors operating simultaneously to collect the maximum possible information from each scan. Although transmitted electron detectors are usefully fitted to conventional SEM instruments working at relatively low voltages, there are major advantages in increasing the accelerating voltage. Increased specimen penetration means that thicker specimens can be tolerated; but more importantly, the decreasing electron wavelength leads to higher spatial resolution and the ability to see the actual atomic configurations within the nanostructure.

Thus the STEM can take many forms: a simple add-on detector to a standard low-voltage SEM; a dedicated, easy-to-use, intermediate voltage STEM with rapid throughput; or an instrument more comparable to a high-resolution transmission electron microscope (TEM), which is able to provide the ultimate spatial resolution and analytical sensitivity. All have important and complementary roles in nanostructure characterization. Rapid feedback is critical to synthesis, and commercially available SEMs with subnanometer resolution at 30kV have the ability to image tens of samples within a few hours. Similar throughput is also available with dedicated STEMs giving at best around 0.2nm resolution at 200kV accelerating voltage. Such instruments can be used to guide the synthesis on a day-to-day basis and represent an invaluable first step in characterization.

2. FUNDAMENTAL PROPERTIES OF ELECTRONS

It is widely known that electrons show both particle and wavelike characteristics, both of which are demonstrated within an TEM/STEM. For example, a microscope operating at about 100keV will have a beam of electrons traveling about half the speed of light c. This corresponds to a distance between electrons of 0.16cm [2], thus there is never more than one electron in the sample at any one time. Electrons will still however, undergo diffraction and interference, both of which are properties of waves. The wavelength \( \lambda \) can be related to the particle momentum \( p \) through the Plank's constant, \( h \). Shown in equation 1.

\[
\lambda = \frac{h}{p}
\] (1)
Within the instrument, electrons are accelerated through a potential \( V \), giving each electron a kinetic energy \( E_k = eV \). This potential equal to the kinetic energy, thus we can equate the momentum to the electron mass, \( m \) and velocity, \( v \). This step can be seen in equation 2.

\[
eV = \frac{1}{2}mv^2
\]

(2)

\[
p = mv = \sqrt{2meV}
\]

(3)

Thus substituting 3 into 1, the electron wavelength is represented in terms of the accelerated voltage \( V \), in the instrument.

\[
\lambda = \frac{h}{\sqrt{2meV}}
\]

(4)

From equation 4 it can be seen that by increasing the accelerating voltage it is possible to decrease the wavelength of the electrons. The treatment of this principle given here is effective for explanation but neglects relativistic effects. For microscopes operating at 100kV and above the velocity of electrons is \( >0.5c \), thus relativistic effects cannot be ignored. Equation 5 is a modified version of 4 including relativistic effects. At larger accelerating voltages, larger relativistic effects will be seen [2].

\[
\lambda = \frac{h}{\sqrt{2meV_r}(1 + \frac{eV_r}{2mc^2})}
\]

(5)

The advantage of using electrons rather than photons to study materials is clear due to the increase in spatial resolution at high energies. The use of electrons however presents challenges in understanding their interaction with material. For instance electrons can be scattered by gas molecules, thus the environment in which the electron beam is created must be evacuated [3]. Table 1 shows several electron wavelengths at some acceleration voltages used in TEM [4].

### Table 1: shows several electron wavelengths at some acceleration voltages used in TEM

<table>
<thead>
<tr>
<th>Accelerating Voltage (kv)</th>
<th>Nonrelativistic Wavelength (nm)</th>
<th>Relativistic Wavelength (nm)</th>
<th>Mass ((x\ m_o))</th>
<th>Velocity ((x10^8\ m/s))</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0.00386</td>
<td>0.00370</td>
<td>1.196</td>
<td>1.644</td>
</tr>
<tr>
<td>120</td>
<td>0.00352</td>
<td>0.00335</td>
<td>1.235</td>
<td>1.759</td>
</tr>
<tr>
<td>200</td>
<td>0.00273</td>
<td>0.00251</td>
<td>1.391</td>
<td>2.086</td>
</tr>
<tr>
<td>300</td>
<td>0.00223</td>
<td>0.00197</td>
<td>1.587</td>
<td>2.330</td>
</tr>
<tr>
<td>400</td>
<td>0.00193</td>
<td>0.00164</td>
<td>1.783</td>
<td>2.484</td>
</tr>
<tr>
<td>1000</td>
<td>0.00122</td>
<td>0.00087</td>
<td>2.957</td>
<td>2.823</td>
</tr>
</tbody>
</table>

The resolution \( \delta \) of a microscope is defined as the distance between two details just separable from one another. It can be calculated using the Abbe's theory of images formation for optic systems. For incoherent light or electron beam, in fact, resolution distance \( \delta \), or what sometimes called by critical distance, may be expressed by the following relation according to Rayleigh criterion, and called lateral resolution [5].

\[
\delta = \frac{0.61\lambda}{nsina}
\]

(6)

Where \( \lambda \) is the wavelength of the light \( n \) is the medium refractive index, and \( \alpha \) is the maximum angle between incident and deflected beam in the limit of the lens aberrations, and also is called the axial resolution [6].

\[
\delta = \frac{2n\lambda}{(nsina)^2}
\]

(7)

For optical microscopy, the resolution is therefore limited by the wavelength of light (410-660 nm). The X or \( \gamma \) rays have lower wavelength, but unfortunately, high-performance lenses necessary to focus the beam to form an image do not exist yet (however, X-rays can reveal structural information of materials by diffraction techniques). A first image with TEM was obtained by Ernst Ruska and Max Knoll. In a TEM, the electrons are accelerated at light voltage (100-1000 kV) to a velocity approaching the speed of light (0.6-0.9c); they must therefore be considered as relativistic particles. The associated wavelength is five orders of magnitude smaller than the light wavelength (0.04-0.008 \( \lambda \)). Nevertheless, the magnetic lens aberrations limit the convergence angle of the electron beam to 0.5°(instead of 70° for the glass lens used in optics), and reduce the TEM resolution to the 5th order. This resolution enables material imaging and structure determination at the atomic level. They are somewhat different form the coherent conditions, as seen in Table 2 [7].
For the incoherent image formation, the two points are easily resolved (minimum). In the case of the coherent image formation, both points can no longer be separated [8].

For a lens with aperture angle \( \alpha \), the minimum blur is given by [9]-[10].

\[
\delta_{\text{min}} = \frac{C_s \alpha^2}{2}
\]

(8)

For a rough estimate of the optimum aperture size, convolve blurring terms, if the point spreads were Gaussian, we could add in quadrature [9].

\[
\delta_f^2 \approx \delta_{\text{min}}^2 + \delta^2 = \left( \frac{C_s \alpha^2}{2} \right)^2 + \left( \frac{0.61 \lambda}{\alpha} \right)^2
\]

(9)

### 3. DEPTH OF FIELD AND DEPTH OF FOCUS

The object points \( O_1 \) and \( O_2 \) objects are separated by the resolution limit \( \delta \) of the lens. Rays from these points cross the axis at A and B equally. Hence, points between A and B will look equally sharp, and AB is the depth of field \( D_o \) of the lens for a semi-angular \( \alpha \), and depth of field varies with magnification, see figure 1 [9].

\[
D_o = \frac{\delta}{\tan \alpha}
\]

(10)

We also need to consider the depth of focus (vertical resolution). This is the ability to produce a sharp image from a non-flat surface.

\[
D_{\text{of}} = \frac{\lambda}{n \sin \alpha}
\]

(11)

Depth of focus is increased by inserting the objective aperture (just in iris that cuts down on light entering the objective lens). However, this decreases resolution.

**Figure 1** The depth of field

### 4. THE MATHEMATICAL MODELS

To calculate the axial magnetic field distribution \( B_z \), two different mathematical models have been adopted to be an objective functions for this work, which are respectively,


\[
B_z = \frac{B_{\text{max}}}{\left[ 1 + \left( \frac{2z}{w} \right)^4 \right]^4}
\]

(12)

2) The Grivet -Lenz Model [12].

\[
B_z(z) = B_{\text{max, sectant}} \left( \frac{2.634z}{w} \right)
\]

(13)
It is seen that the two above expressions are formulated such that $B_z$ distributions may assign in terms of a same optimization parameters. That are the maximum flux density $B_{\text{max}}$, field half-width $w$ and lens length $L=Z_2-Z_1$, where $Z_1$ and $Z_2$ are the axial field terminal coordinates respectively.

In the present work to calculate all requirement results we have been written in Fortran power station 90 language program. Therefore, figure 2 representing a block diagram for this work [13].

![Block Diagram](image)

**Figure 2** The block diagram for the present work

### 5. RESULTS AND DISCUSSION

The magnetic flux density distributions for the considered the two models are shown in figures 3. These $B_z$ distributions have been computed for $B_{\text{max}}=0.2\text{Tesla}$, the half-width of the field $w=1\text{ millimeter}$ and lens length $L=20\text{ millimeter}$. It is seen that each of these models has its own characteristic extend along the optical axis and vanishes after the lens end.

![Graph](image)

**Figure 3** Axial flux density distributions $B_z$ for the two models at $B_{\text{max}}=0.2\text{T}, w=1\text{mm}$ and $L=20\text{ mm}$
In order to reveal the influence of varying the half-width of an imaging formation on its resolution at coherent and incoherent imaging formation all relations in Table 2 have been used respectively. Therefore, five values of w have been chooses namely (1, 2, 3, 4 and 5 ) in unit of millimeter. However, at fixed values of (NI/Vr)1/2 = 20, Bmax=0.2T and L=20mm the resolution limit δ for each considered model are plotted as a function of w in figure 4, for coherent case, and figure 5a,b, for incoherent case. It is seen that the resolution get enhanced as along as the field half-width increases. Furthermore, the field of a wide extension along the optical axis has a better value of δ.

\[ \delta = 0.66(C_s \lambda^3)^{1/4} \]

**Figure 4** The Variation of the resolution limit \( \delta \) as a function of the half-width \( w \) for the two models for coherent case

Such a result, however, can be understood easily by plotting each of \( V_r, \lambda \), and \( C_s \) as a function of the chosen values of \( w \) as shown in figures 6, 7 and 8 respectively. Figure 6 shows clearly that increases of lens half-width leads to increases of accelerating potential required to obtain \( NI/V_r^{1/2} \) = 20. So, the associated electron wavelength must be decreases consequently as indicated in figure 7. Now, one may easy realize that the values of spherical aberration \( C_s \) increasing linearly with increasing the half–width \( w \), as shown in figure 8.
Figure 6 The variation of the relativistically accelerating voltage $V_r$ as a function of the half-width $w$ for the two models.

Figure 7 The variation of the associated electron wavelength $\lambda$ with the half-width $w$ for the two models.

Figure 8 The variation of spherical aberration $C_s$ versus the half-width $w$ for the two models.
According to equation 9, figure 9a,b, illustrates the relationship between the square resolution limit $\delta^2$, as a function of the half-width $w$ for the two models for coherent and incoherent case at optimum aperture. It can be noted that $\delta^2$ increases with increasing the half-width $w$, while according to equations 10 and 11, figure 10a,b represents the depth of field $D_o$ at optimum aperture, figure 10a, and figure 10b, for the coherent, and incoherent cases respectively, it will be seen that the depth of field increasing in two cases when the half-width increasing as well as the depth of focus at optimum aperture see figure 11a,b.

![Figure 9](image-url)

**Figure 9** The Variation of square resolution limit $\delta^2$ as a function of the half-width $w$ for the two models

![Figure 10](image-url)

**Figure 10** The Variation of the depth of field $D_o$ as a function of the half-width $w$ for the two models

![Figure 11](image-url)

**Figure 11** The Variation of the depth of focus $D_{of}$ as a function of the half-width $w$ for the two models
6. CONCLUSIONS
Through the results extracted can be seen some important results of the resolution limits for both proposed lenses, we find more convergence in the results at high values of the half-width, while, depth of field and depth of focus are increasing.

References