Approximate solution to korteweg _ de vries (Kdv) equation by initial condition of Adomian decomposition method

A.M. Ayal , Ahmed Ismail Mohammed

Department of Mathematics, college of Basic Education, Misan University, Misan, Iraq.

ABSTRACT

Recently, Adomian decomposition has become an important method of interest. This paper suggests robust solution of korteweg-de vries (Kdv) equation with initial condition suitableby using Adomian decomposition. We primarily evaluate advantage of this method to avoid simplifications and restrictions which change the non-linear problem to mathematically tractable one. Our work primarily depends on robust solution but it is not consistent with physical solution; substantial changes to those methods achieve results that are competitive with prior state-of-the-art methods. The exact solution obtains as a special case according to the initial condition.

Keywords: Adomian Decomposition method, Kinematic wave equation.

1.INTRODUCTION

The Korteweg-de Vries (Kdv) equation is a non-linear Partial Differential Equation PDE[1] which describe weakly nonlinear shallow water waves.The solution to this equation leads to solitary waves solutions. In this article,our goal to obtain an analytic solution of a nonlinear problem in the form of a series by utilizing the Adomian decomposition method. Consequently, we consider the advantages of some prior existing methods and propose a unified deep methods[1],[2],[3]. We comprehensively study the method to provide solution technique thattackles anymathematical and physical problem directly. In addition, it is relatively easy to obtain an accurate and rapidly convergent series solution which is based on the Taylor series except that Adomian decomposition method expands the solution for a function, instead of reliable technique that requires less work if compared with the traditional techniques. Furthermore, this method does not require unjustified assumptions, linearization, or perturbation.

2.FORMULATION OF ADOMIAN DECOMPOSITION METHOD

Set We consider the equation

$$F\big(u(x)\big) = g(x) \qquad (1)$$

Where F represent a general nonlinear ordinary or partial differential operator including both linear and nonlinear terms, and g is a given function. The linear terms in $F\big(u(x)\big)$ are decomposed into $Lu + Ru,$ where L is an easily invertible operator (usually the highest order derivative), and R is the remainder of the linear operator. Thus, equation (1) can be written as

$$Lu + Ru + Nu = g \qquad (2)$$

Where, Nu indicates the nonlinear terms. By solving this equation (2) for $Lu,$ since L is invertible, and applying the inverse operator L^{-1} on both sides yields

$$u = A + L^{-1}(g) - L^{-1}(Ru) - L^{-1}(Nu), \qquad (3)$$

Where A can be found from the boundary or initial conditions.
Adomian method assumes the solution u can be expanded into infinite series as,

$$u = \sum_{n=0}^{\infty} u_n \qquad (4)$$

Also, the nonlinear term Nu can be written as infinite series in terms of the Adomian polynomials A_n of the form

$$Nu = \sum_{n=0}^{\infty} A_n \qquad (5)$$

Where the Adomian polynomials A_n, of Nu are evaluated using the formula

$$A_n(u_0, u_1, \dots, u_n) = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N\left(\sum_{i=0}^{\infty} u_i \lambda_i\right)\right]_{\lambda=0} \qquad n = 0,1,2,\dots \qquad (6)$$

Many computational algorithms are available to compute adomain polynomial [4], [5],[6},[7] that given a suitable and simpler one.So, we have adopted this algorithm and as follows for calculating $A_0, A_1, \dots A_n$

step1: Input nonlinear term N(u) and n, the number of Adomian polynomial needed.

step2: Set $A_0 = N(u_0)$

step3: For $k = 0$ to $n - 1$ do:

$$A_k(u_0, u_1, \dots u_k) := A_k(u_o + u_1\lambda, \dots, u_k + \qquad (k+1)u_{k+1}\lambda)$$

$$\{in\ A_k : u_i \to u_i + (i+1)u_{i+1}\lambda \qquad for\ i \quad 0,1,\dots,k\ \}$$

Step1: By taking the first order derivative of A_k, with

respect to λ, and then let

$$\lambda = 0: \frac{d}{d\lambda} A_k|_{\lambda=0} = (k+1)A_{k+1}$$

End do

Step2: Output $A_0, A_1, \dots A_n$.

According to the above Algorithm, Adomian polynomials will be computed as follows:

$$A_0 = N(u_0)$$

$$A_1 = \frac{d}{d\lambda} N(u_0 + u_1\lambda)|_{\lambda=0} = u_1 \acute{N}(u_0),$$

$$A_2 = \frac{1}{2}\frac{d}{d\lambda}\Big((u_1 + 2u_2\lambda)\acute{F}(u_0 + u_1\lambda)\Big)\Big|_{\lambda=0} = u_2\acute{N}(u_0) + \frac{u_1^2}{2!}\acute{\acute{N}}(u_0), \qquad (7)$$

And so on. The components of $u_n,\ n \geq 1.$

3.EXAMPLE: THE KORTEWEG–DE VRIES (KDV) EQUATION IS:

$$u_t + \alpha u u_x + \beta u_{xxx} = 0, x \in \mathbb{R}, t > 0 \qquad (8)$$

where α and β are constants, (8) is a simple and useful model for describing the long-time evolution of dispersive wave phenomena in which the steepening effect of the nonlinear term is counterbalanced by the dispersion.

Choosing $\alpha, \beta = 1$, and choosing same the initial condition as

$u(x,0) = x \quad 0 < x < 1$, the equation (8) is become:

$$u_t + uu_{xx} = u_{xxx}, \qquad x \in \mathbb{R}, \qquad t > 0 \qquad (9)$$

Subject to initial condition:

$$u(x,0) = x \quad 0 < x < 1 \qquad (10)$$

$$\text{Let } L_t = \frac{\partial .}{\partial t} \text{ and } L_{xxx} = \frac{\partial^3 .}{\partial x^3} \text{ we have now}$$

$$L_t u(x,t) + u u_x = L_{xxx} u$$

$$L_t u(x,t) = L_{xxx} u - u u_x$$

Operating with $L_t^{-1} = \int_0^t (.)\, dt$

$$L_t^{-1} L_t u(x,t) = L_t^{-1} L_{xxx} u - L_t^{-1} u u_x$$

$$u(x,t) - u_0(x,o) = L_t^{-1} L_{xxx} u - L_t^{-1} u u_x$$

$$u = u_0 + L_t^{-1} L_{xxx} \sum_{n=0}^{\infty} u_n - L_t^{-1} \sum_{n=0}^{\infty} A_n$$

Since $= \sum_{n=0}^{\infty} u_n$, we can now write

$u_0 = x$

$$u_1(x,t) = L_t^{-1} L_{xxx} u_0 - L_t^{-1} A_0$$

$$u_2(x,t) = L_t^{-1} L_{xxx} u_1 - L_t^{-1} A_1$$

$$u_{n+1}(x,t) = L_t^{-1} L_{xxx} u_n - L_t^{-1} A_n$$

In the following, we outline the framework to generate these polynomials, where it was defined that

$$A_0 = u_0 \acute{u}_0,$$

$$A_1 = u_1 \acute{u}_0 + u_0 \acute{u}_1,$$

$$A_2 = u_2 \acute{u}_0 + u_1 \acute{u}_1 + u_0 \acute{u}_2$$

$$A_n = u_n \acute{u}_0 + u_{n-1} \acute{u}_1 + u_{n-2} \acute{u}_2 + \cdots + u_1 \acute{u}_{n-1} + u_0 \acute{u}_n,$$

$$u_0 = x$$

$$u_1 = -xt$$

$$u_2 = \frac{xt^2}{2}$$

Thus $u(x,t) = x\left(1 - t + \frac{t^2}{2} + \cdots\right), \qquad u(x,t) = \frac{x}{(1+t)}$

4. CONCLUSIONS

In this work, we used very efficient and powerfulAdomian decomposition methodology to solve korteweg-de vries (Kdv) equation. In this method, we are avoiding the difficulties and massive computational work.It is to be notedthat no linearization or perturbation was used and solutions can be said to be exact since anydesired accuracy is obtainable by the increasing n, normally a very small number. Each additionalterm depends simply on the preceding term. Randomness in the initial term is handled without restrictive assumptions or closure approximants.

References

[1] A domian G., "Solutions of nonlinear PDE", Appl. Math Lett., Vol.11, pp.121-123, 1998
[2] G. Adomain,"Anew approach partial differential equations", J.Math Appl. 102(1984),420_434.

[3] A. ElafJaafar, "A new Technique of Initialof Initial Boundary Value Problems Using Adomian Decomposition method", International Math. Forum, Vol. 7,2012,no. 17, 799_ 814
[4] A. M. Wazwaz, "A new algorithm for calculating Adomian polynomials for non-linear operators", Appl. Math. Comp. Vol.111, pp.53-69, 2000.
[5] E. Babolian, Sh. Javadi, “New method for calculating Adomianpolynomials”,Appl. Math. Comp., Vol. 153, pp. 253-259, 2004.
[6] J. Biazar, E. Babolian, A. Nouri, R. Islam, "An alternate algorithm for computing A domain polynomials in special cases", App. Math. Comp., Vol.138, pp. 523-529, 2003.
[7] J. Biazar and S. M. Shafiof, "A Simple Algorithm for Calculating Adomian polynomials", Int. J. Contemp. Math. Sciences, Vol.2, no. 20, pp. 975-982, 2007.