

# MATHEMATIC MODEL OF DIRECTION CONTROL SYSTEM OF OPTICAL RADIATION

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## ABSTRACT

*It is pointed out in the article that the technology and equipment for optical transport networks is created and enhanced with rapid perfections today, however there are no scientifically substantiated and generally accepted methods of designing a mathematical model of control system of directing optical radiation. Therefore the development of the mathematical model of the optical system is of a great actuality. It has been shown that the quality characteristics of the displacement mechanism such as evenness step and moving force largely depends on the dynamic stability of the working body during operation in the oscillatory mode. Therefore, based on our analysis of known scientific works it has been developed a mathematical model comprising the scientific basis of modeling techniques of the operating mode of the actuator of the direction control system of the optical radiation. It is indicated that with the application of this control system, the loss of switching and the input of radiation into the fiber optic cable is 0.1...1.0 dB, which is 40-45% less than in the well-known systems. In the developed mathematical model of the system at the steps value of the angular rotation of the rotor from 4.0mm to 0.2mm, the rotors steering precision in the stepping mode constitutes 1.4...2.1 microns.*

**Keywords:** actuator, a piezoelectric element, a control system, moving, mathematical model, optical radiation, loss, input of radiation.

## 1. INTRODUCTION

In recent years there has been significant progress in the establishment of new promising telecommunications enhancing the quality and efficiency to transmit a variety of information, expanding services, reducing labor and material. Such assets include the optical system (OS), the introduction of which determines the development of not only communication, but also electronics, atomic power industry, space exploration, engineering, shipbuilding, and so on. Such attention to OS is due to broadbandness, high capacity, low attenuation in a wide frequency range, high immunity to external electromagnetic interference, small scale, lightness and availability of the layer on actual routes. Creating a highly reliable OS has become possible after the development of optical fibers with low losses. These qualities have stimulated the development of industrial fiber optic cables technologies (FOC), the development of specialized equipment and OS components base: emitters, modulators, photo detectors, optical connectors, couplers and splitters, optoelectronic switches (OS), optical gates and other optoelectronic components. The introduction of OS and FOC, where optical switches, optoelectronic switches, vents, splitters and modulators with electric and optical control are mainly used as the control devices of the direction of the optical radiation (OR) [1-3]. This poses challenges in the use of spatial controls device of the direction of the optical radiation (OR) in them and it is associated with high directivity of laser radiation. To realize such benefits it is required to ensure a high accuracy of mutual OR direction between NTC (OTKC) transceiver and transmission medium.

## 2. FORMULATION OF THE PROBLEM

Modern information technologies are increasingly using optical range of electromagnetic oscillations. Broadband OS, optical memory systems, optical switching and positioning devices require ever faster and more flexible management of ORs. At the same time one of the main tasks is to manage OR without an intermediate conversion of optical signals into electrical ones [4-7].

The design of the known controls with OR direction is much complicated and they do not provide a sufficiently high

precision switching of OR since in the process of spatial switching there must be automatic OR direction, relative to the surface of the end section of the optical fiber in the vertical and horizontal directions and the possibility of changes in the intensity of radiation to provide exposure to the beam at the boundary with the shell with all the energy of the radiation, to reflect inside of the fiber waveguide core [8-11]. Changing the direction of OR is due to the fact that when there is a need for design and laying of FOC, it needs to change the direction of FOC laying from a distance at an angle of twisting more than  $90^0$ . The high degree of localization of the OR field can significantly reduce the sizes, increase the speed and efficiency of the OI direction control. The expansion of the range of practical applications requires the creation of devices with new functional characteristics. Therefore, the development of the new models of three-point control systems of the optical radiation direction is an important task of research of theoretical and practical nature [12-16].

### 3.DEVELOPMENT OF A MATHEMATICAL MODEL

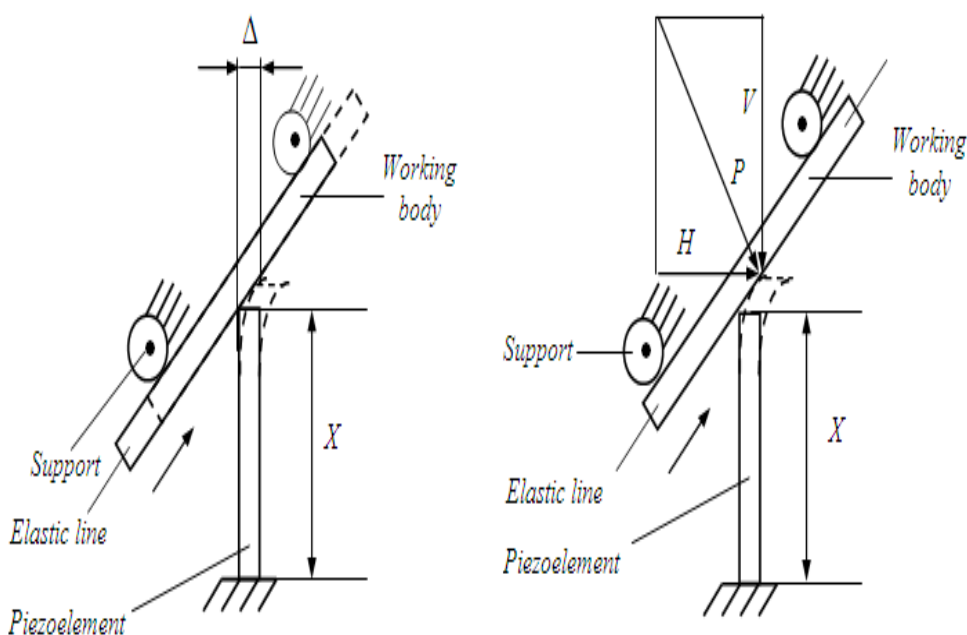
Despite the fact that the technologies and equipment designed for optical transport networks are established and improved with high rates, there are no scientifically substantiated and generally accepted methods of designing a mathematical model of control system of directing optical radiation. Therefore, the development of a mathematical model of such a system is of great actuality.

In devices [1,17-20], the piezoelement of rectangular section, receiving power from an external source, is subjected to longitudinal tensile strain, determined in accordance with [11]:

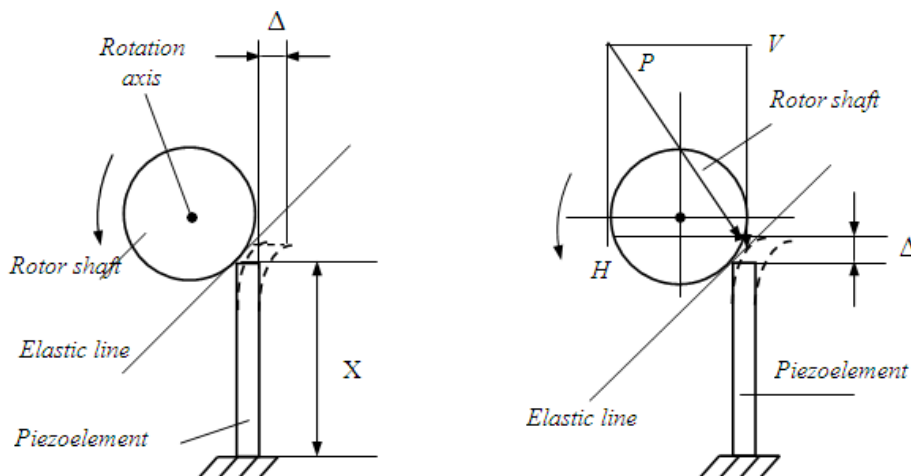
$$\Delta_{it} = \frac{2T_{mt} \cdot l_1}{\pi E_y} = \frac{2 \cdot 19,6 \cdot 10^6 \cdot 5 \cdot 10^{-2}}{3,14 \cdot 7 \cdot 10^{11}} = 6 \cdot 10^{-6} = 6 \text{ mkm}, \quad (1)$$

herein  $T_{mt}$  – is the mechanical tension in the center of the plate and equals to  $19,6 \cdot 10^6 \text{ (N / m}^2\text{)}$ ,  $l_1$  – is the length of piezoelectric element and is equal to  $5 \cdot 10^{-2} \text{ (m)}$ ,  $E_y$  – is modulus of elasticity (Young's modulus) and for the piezoceramic of mark ПТБС-3  $E_y = 0,8 \cdot 10^{11} \text{ (N / m}^2\text{)}$ .

Piezoelement interacts with the working member mounted on the two sliding supports (Figure 1, 2), and in Figure 3 and 4 it interacts with the rotor shaft installed on the rotation bearing.



**Figure 1, 2.** The scheme of interaction of the piezoelectric element with working body, mounted on two sliding bearings.



**Figure 3,4.** The scheme of the piezoelectric element interaction with of the rotor shaft, located on the rotation bearing

Thus, in the first and in the second case, as a result of longitudinal deformation of piezoelements, its bending occurs defined by formula:

$$\Delta_{bo} = \frac{F_m \cdot l_1^2}{3E_y \cdot J}, \quad (2)$$

here  $F_m$  – is the maximum force that acts from the part of the piezoelectric element ( $N$ ),  $J$  – is the moment of inertia for a rod with a rectangular cross-section ( $m^4$ ).

In the described vibrational mode, the deflection " $\Delta$ " is a function of not only the coordinate " $x$ ", but also the time  $t$  that, that is

$$\Delta = f(x, t). \quad (3)$$

Studies show that the quality of characteristics of the transfer mechanism, that as a step uniformity and force movement in the first case of the working body, and the rotor rotational shaft in the second case, is largely dependent on the dynamic stability of the piezoelectric element in an oscillatory mode. Conducted experimental research of working characteristics by using sensors [17,20-22], measuring dynamic effects, show that in this specified mode, the piezoelectric element is exposed to the force of the working body with force  $P$  or in the second case, the engine rotor shaft aimed at its bending along a tangent to the elastic line (Figure 2, 4).

Expanding the force  $F$  into two components (Figure 2, 4), to vertical  $V$  and horizontal  $H$ , tangential force  $V$  can be considered constant for small deflections about  $1 \cdot 10^{-5}$  mm. Power  $H$  being specific, its value considerably depends on the deflection of the end sections, and the force is equal to the multiplication of the stiffness of piezoelement  $K_{sp}$  in the form of a rectangular bar with the deflection increment  $\Delta$ , i.e.,

$$F = K_{sp} \cdot \Delta. \quad (4)$$

The differential equation of the bent axis:

$$E_y J \frac{d^4 \Delta}{dx^4} + P \frac{d^2 \Delta}{dx^2} = q, \quad (5)$$

here  $q$  – is the intensity of the lateral load,  $J$  – is the moment of inertia for a rod with a rectangular cross-section:

$$J = \frac{l_2 \cdot l_3^3}{12}. \quad (6)$$

By using the principle of D'Alembert, we take the force of inertia of the piezoelectric element mass for intensity of load distribution in the form of a rod with a rectangular cross section, coming to its unit-length. Denoting piezoelement units of length by  $P$  weight we obtain:

$$E_y \cdot J \cdot \frac{d^4 \Delta}{dx^4} + P \cdot \frac{d^2 \Delta}{dx^2} = -\frac{P}{g} \cdot \frac{d^2 \Delta}{dt^2}, \quad (7)$$

here  $g$  – is the acceleration of gravity of the piezoelectric element ( $m/s^2$ ).

Introducing the notation  $K^2 = P/E \cdot J$  we arrive at the following equation:

$$\frac{d^4 \Delta}{dx^4} + K^2 \cdot \frac{d^2 \Delta}{dx^2} + \frac{P}{E_y J g} \cdot \frac{d^2 \Delta}{dt^2} = 0. \quad (8)$$

We shall seek a solution of equation (8) as a multiplication of two functions:

$$\Delta(x, t) = X(x) \cdot T(t). \quad (9)$$

Then, instead of the equation (8) we obtain:

$$\frac{1}{X} \cdot \frac{d^4 X}{dx^4} + K^2 \cdot \frac{1}{X} \cdot \frac{d^2 X}{dx^2} = - \frac{P}{E_y J g} \cdot \frac{1}{T} \cdot \frac{d^2 T}{dt^2}. \quad (10)$$

The left side of this equation depends only on  $X$ , and the right side depends on  $t$ .

The equation can only be satisfied in the case, if the left and right sides are constants:

$$\frac{1}{X} \frac{d^4 X}{dx^4} + K^2 \frac{1}{X} \frac{d^2 X}{dx^2} = \lambda, \quad (11)$$

$$- \frac{P}{E_y J g} \frac{1}{T} \frac{d^2 T}{dt^2} = \lambda. \quad (12)$$

From the expression (12) we find that the frequency of the piezoelectric element oscillation:

$$\omega^2 = \frac{E_y J g}{P} \cdot \lambda.$$

The equation can be rewritten in the following form:

$$\frac{1}{X} \frac{d^4 X}{dx^4} + K^2 \frac{1}{X} \frac{d^2 X}{dx^2} - \lambda X = 0. \quad (13)$$

The corresponding characteristic equation will have the form:

$$S^4 + K^2 S^2 - \lambda = 0. \quad (14)$$

This equation has two real roots and two imaginary. Introducing the notation

$$S_1^2 = \frac{\sqrt{K^4 + 4\lambda - K^2}}{2}, \quad (15)$$

$$S_2^2 = \frac{\sqrt{K^4 + 4\lambda + K^2}}{2}, \quad (16)$$

The solution of equation (13) can be written in the following form:

$$X(x) = A \cdot ch S_1 x + B \cdot sh S_1 x + C \cdot \cos S_2 x + D \cdot \sin S_2 x, \quad (17)$$

The boundary conditions are:

$$\left. \begin{aligned} X = 0, \quad \frac{dX}{dx} = 0 \quad \text{at } x = 0, \quad \frac{d^2 X}{dx^2} \\ \frac{d^3 X}{dx^3} = 0 \quad \text{at } x = l_1. \end{aligned} \right\} \quad (18)$$

The first two conditions give:

$$A + C = 0, \quad S_1 \cdot B + S_2 \cdot D = 0. \quad (19)$$

Other conditions lead to equations:

$$\left. \begin{aligned} (S_1^2 \cdot chS_1l_1 + S_2 \cdot \cos S_2l_1) \cdot A + S_1 \cdot (S_1 \cdot sh\lambda_1l_1 + S_2 \cdot \sin \lambda_2l_1) \cdot B &= 0, \\ (S_1^3 \cdot shS_1l_1 - S_2^3 \cdot \sin S_2l_1) \cdot A + S_1 \cdot (S_1^2 \cdot ch\lambda_1l_1 + S_2^2 \cdot \sin \lambda_2l_1) \cdot B &= 0. \end{aligned} \right\} \quad (20)$$

Equating to zero the determinant of this system of equations we obtain the following equation:

$$(S_1^4 + S_2^4 + 2S_1^2 \cdot S_2^2 \cdot chS_1l_1 \cdot \cos S_2l_1 - S_1 \cdot S_2 (S_1^2 - S_2^2) \cdot sh\lambda_1l_1 \cdot \sin S_2l_1) = 0. \quad (21)$$

Using expressions (15) and (16) we find:

$$S_1^4 + S_2^4 = K^4 + 2\lambda; \quad S_1^2 \cdot S_2^2 = \lambda; \quad S_1^2 - S_2^2 = -K^2. \quad (22)$$

Now the equation (18) takes the form:

$$K^4 + 2\lambda + 2\lambda \cdot chS_1l_1 \cdot \cos S_2l_1 + \sqrt{\lambda}K^2 \cdot shS_1l_1 \cdot \sin S_2l_1 = 0. \quad (23)$$

Turning to the dimensionless parameters:

$$K = (Kl_1)^2 = \frac{Pl_1^2}{E_{10}J}; \quad f = \lambda_1l_1^4 = \frac{P\omega^2l_1^4}{E_{10}Jg}; \quad \bar{S}_1 = S_1l_1; \quad \bar{S}_2 = S_2l_1. \quad (24)$$

Then we obtain the dependence:

$$F(Kf) = K^2 + 2f + 2f \cdot ch\bar{S}_1 \cdot \cos \bar{S}_2 + \sqrt{f}K^2 \cdot sh\bar{S}_1 \cdot \sin \bar{S}_2 = 0. \quad (25)$$

Parameter  $K$  determines the compressive force and  $f$  – oscillation frequency of the piezoelectric cell. The points lying on the x-axis correspond to the first two natural frequencies of flexural vibrations of the piezoelectric element in the form of a rod on the carrier compressive load. If we assume that  $P = 0$ , the equation (21) becomes the following:

$$ch\bar{S}_1 \cdot \cos \bar{S}_2 = -1. \quad (26)$$

As is well known, its roots determine the frequency of own oscillations of the piezoelectric element with one clamped and the other free end.

It should be noted that the change in direction of the force arising from the condition of task, must take place due to some external energy of sources for the system. If we seek the additional roots of the equation, we get new loops connecting the third and fourth own frequencies, fifth and sixth, etc. However, it should be noted that the limit point of the first loop is the lowest one, and namely this determines the critical load, which is approximately equal to:

$$P_{kr} \approx \frac{2\pi^2 E_y J}{l_1^2}, \quad J = \frac{l_2 \cdot l_3^3}{12}. \quad (27)$$

Taking into account [17,18] the geometric dimensions of the piezoelectric element, the tensile strength at longitudinal strain:

$$\sigma_{ls.} = P_{kr} / l_2 \cdot l_3. \quad (28)$$

Breaking load  $P$  can be measured if the piezoelectric element section is used as a sensor, and the value is calculated by the following formula with electric parameters:

$$\sigma_{din} = E_- d_{31} \cdot E_y \cdot Q_M, \quad (29)$$

here  $E_-$  – the electric field intensity at specimen ( $V/m$ ) destruction,  $d_{31}$  – piezoelectric modulus of longitudinal deformation of ( $m/V$ ),  $E_y$  – the Young's modulus of ( $N/m^2$ ),  $Q_M$  – the mechanical quality factor.

#### **4.CONCLUSION**

Based on the conducted analysis of the well-known scientific works, it has been developed a mathematical model constituting the scientific basis of modeling techniques of the working mode body of the control system of the optical radiation direction.

It is indicated that with the application of this system, the loss of switching and the input of radiation into the fiber optic cable is 0.1-1.0 dB, which is 40-45% less than in the well-known systems.

In the developed mathematical model of the system at the steps value of the angular rotation of the rotor from 4.0 mm to 0.2 mm, the rotors steering precision in the stepping mode constitutes 1.4-2.1 microns.

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