Mathematical Modeling of Interacting and Non Interacting Tank System

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ABSTRACT

The main objective of this paper is to determine the mathematical model of a coupled tank system which will be useful for designing and tuning controllers consisting of a PID and a fuzzy logic for the system. In this paper, we adopt a two layer quadruple tank system whose transfer function was obtained via mathematical modeling. The simulation is being carried out using the Mat Lab/ Simulink Toolbox and the simulation results are compared with actual results performed during experiment. Based on procedure of Mathematical modeling for tanks in single, non-interacting and interacting mode this paper is an extensive comparative experimental study Liquid level controls. The system under investigation is a quadruple tank system apparatus. [8] The basic control principle of the quadruple tank system is to maintain a constant level of the liquid in the tank when there is an inflow and outflow of water in the tank.

Keywords: Interacting, Non-Interacting, Simulation, Quadruple Tank system.

1. INTRODUCTION

The first stage in the development of any control and monitoring system is the identification and modeling of the system. The present work is concerned with developing Mathematical model for interacting and non interacting tank process and comparing it with the real time experimental data. System identification from the experimental data plays a vital role for model based controller design. Derivation of process model from first principles is often difficult due to its complexity. [2] This paper presents a quadruple tank system is taken as a plant and will be modeled mathematically using Bernoulli’s law, simulate with Mat lab / Simulink and decentralized using control and estimation tool manager from Simulink model and mathematical model. This paper will help the method suitable for research findings concerning on the coupled-tanks liquid level control system.

2. MATHEMATICAL MODELING

A mathematical model is a description of a system using mathematical concepts and language. The process of developing a mathematical model is termed mathematical modeling. [2] As far as process control systems are concerned we need simple description of how process reacts to various inputs and this is what mathematical model can provide to control designer. In this report mathematical modeling of single and coupled tank system (interacting and non interacting mode) is developed using Quadruple Tank system. Multivariable level control trainer is designed for teaching the basic level control principles and advanced study in complex systems for control engineering. It is a combination of two double tank systems, usually also known as Quadruple Tank System. The setup consists of supply water tank with two variable speed positive displacement pumps for water circulation, four transparent process tanks fitted with level transmitters and flow dampers. The process signals from level transmitters are connected to Serial based duel loop PID controller. The controller is connected to computer through USB port communication. The process parameters are controlled through software and the output is fed to variable frequency drive used for the pumps. These units along with necessary piping are fitted on standalone support structure. The set up is connected to computer through USB port for monitoring and control by using PID logics. The product is supplied with 32 tag demo version software package along with licensed version of 64 Tag SCADA software. The experimental set up and process description is as given below.
3. MODELING OF SINGLE TANK SYSTEM

[1] The overall material balance on the cylindrical tank is:

\[ \frac{d(pAh)}{dt} = \rho Q_{\text{in}} - \rho Q_{\text{out}} \]  \hspace{1cm} (1)

Since there is no heating effects, density can be assumed constant for water, ODE becomes,

\[ \frac{A}{d} \frac{dh}{dt} = Q_{\text{in}} - Q_{\text{out}} \]  \hspace{1cm} (2)

\[ Q_{\text{out}} \] is determined by pressure exerted by liquid given by

\[ Q_{\text{out}} = K \]  \hspace{1cm} (3)

Therefore mass balance equation is given by [7]

\[ = (Q_{\text{in}} - K) / A \]  \hspace{1cm} (4)
4. MODELING OF NON INTERACTING TANK SYSTEM

In non-interacting tank system we have two tanks where the outflow from first tank feeds the second tank. We assume the tanks have uniform cross sectional area and the flow resistance is linear. We can write mass balance equations as

\[
\frac{dh_1}{dt} = \left( Q_m - K_1 \sqrt{h} \right) / A \tag{5}
\]

\[
\frac{dh_2}{dt} = \left( Q_m - K_2 \sqrt{h} \right) / A \tag{6}
\]
Figure 7 Simulink diagram for Non-Interacting Tank System

Figure 8 Simulation Response of Tank 1

Figure 9 Validation Graph for Tank 1 (Non-Interacting Tank System)

Figure 10 Simulation response for Tank 2
5. **MODELING OF INTERACTING TANK SYSTEM**

The coupled tank system consists of two tank coupled with a manually operated ball valve. The system is called interacting since flow from $Q_1$ depends on difference between $h_1$ and $h_2$. Mass balance equations for the system will be

\[
\frac{A_1}{dt} \frac{dh_1}{dt} = Q_{\text{in}} - Q_{1\text{out}} \tag{7}
\]

\[
\frac{A_2}{dt} \frac{dh_2}{dt} = Q_{1\text{out}} - Q_{2\text{out}} \tag{8}
\]

where

\[
Q_{2\text{out}} = K_2 \sqrt{h_2} \quad \text{and} \quad Q_{1\text{out}} = K_1 \sqrt{h_1 - h_2} \tag{9}
\]

\[
\frac{dh_1}{dt} = (Q_{\text{in}} - K_1 \sqrt{h_1 - h_2}) / A_1 \tag{10}
\]

\[
\frac{dh_2}{dt} = (K_1 \sqrt{h_1 - h_2} - K_2 \sqrt{h_2}) / A_2 \tag{11}
\]

the corresponding Simulink diagram is

**Figure 11** Validation Graph for Tank 1 (Non-Interacting Tank System)

**Figure 12** Interacting Tank System

**Figure 13** Simulink diagram for Interacting Tank System
6. CONCLUSION

The coupled tank system model development presented in the paper is nothing but the representation of the system in mathematical form. However, the complexity increases with increase in number of variables to be controlled. Interacting two tank system is considered as non-linear system which will be helpful in designing Manual PID and the simulation carried out using MATLAB will be helpful to ensure that controller perfectly regulates the desired output level as per the requirement. The Mathematical modeling helps in furnishing a convenient and flexible design that provides good performance in terms of disturbance rejection and set-point tracking. The values of parameters obtained from physical model are substituted in the theoretical model and are validated and the results obtained are compared with statistical method.

REFERENCES

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