Statistical Analysis of Brain MRI Image segmentation for the Level Set Method

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ABSTRACT

Level set methods have been widely used in image processing and computer vision. Intensity inhomogeneity often occurs in real-world images, which presents a considerable challenge in image segmentation. MRI intensity inhomogeneities can be attributed to imperfections in the RF. The result is slowly-varying shading artifact over the image that can produce errors with conventional intensity-based classification. The most widely used image segmentation algorithms are region-based and typically rely on the homogeneity of the image intensities in the regions of interest, which often fail to provide accurate segmentation results due to the intensity inhomogeneity. In this paper statistical property of level set method for image segmentation is analyzed, which is able to deal with intensity inhomogeneities in the segmentation. In a level set formulation, the local intensity clustering property criterion is used to define energy that represent a partition of the image domain and a bias field that accounts for the intensity inhomogeneity of the image. Therefore, by minimizing this energy, the level set method is able to simultaneously segment the image and estimate the bias field, which can be used for bias correction. Statistical analysis is performed by calculating probability, variance and entropy for both the images i.e. Input image and Bias corrected image.

Keywords: Intensity inhomogeneity, Bias correction, level set, MRI, probability, variance and entropy.

1.INTRODUCTION

Accurate brain tissue segmentation from magnetic resonance (MR) images is an essential step in quantitative brain image analysis. However, due to the existence of noise and intensity inhomogeneity in brain MR images, many segmentation algorithms suffer from limited accuracy. Medical image segmentation has high impact on the digital image processing due to its spatial resolution enhancement and image sharpening. It has been used to derive useful information from the medical image data that provides the most accurate and robust method for diagnosis. This process is a compelling challenge due to the presence of inhomogeneities in the intensity of images. The removal of the spatial intensity inhomogeneities from MR images is difficult because the inhomogeneities could change with different MRI acquisition parameters from patient to patient and from slice to slice. For addressing this challenge, the region based level set method is used for segmenting the medical images with intensity inhomogeneity. Intensity inhomogeneity often occurs in real-world images due to various factors, such as spatial variations in illumination and imperfections of imaging devices, which complicates many problems in image processing and computer vision. In particular, image segmentation may be considerably difficult for images with intensity inhomogeneities due to the overlaps between the ranges of the intensities in the regions to which are to be segmented. This makes it impossible to identify these regions based on the pixel intensity. Those widely used image segmentation algorithms [2] [3] [4] [5] usually rely on intensity homogeneity, and therefore are not applicable to images with intensity inhomogeneities. In general, intensity inhomogeneity has been a challenging difficulty in image segmentation.

Existing level set methods for image segmentation can be categorized into two major classes: region-based models [3] [9] [10] [12] [14] [16] and edge-based models [2] [6] [7]. In the level set method, contours or surfaces are represented as the zero level set of a higher dimensional function, usually called a level set function. With the level set representation, the image segmentation problem can be formulated and solved in a principled way based on well-established mathematical theories, including calculus of variations and partial differential equations (PDE) [16]. An advantage of the level set method is that numerical computations involving curves and surfaces can be performed on a fixed Cartesian grid without having to parameterize these objects. Moreover, the level set method is able to represent contours/surfaces with complex topology and change their topology in a natural way.
Chunming et al. (2011) proposed a local clustering based variational level set framework for simultaneous intensity inhomogeneous image segmentation and bias correction [1]. First, the local intensity clustering property is derived based on the model of the images with the intensity inhomogeneity. For the image intensities in the neighbourhood of each point, the local intensity criterion function is defined which is then integrated with respect to the neighbourhood center to give a global criterion of image segmentation. In the level set formulation, to represent the partition of the image domain into distinct regions and a bias field, this criterion function defines energy in terms of the level set functions. Then the energy minimization is performed in the level set process to segment the images and also to estimate the bias field for the intensity inhomogeneity correction or bias correction.

The paper can be dismantled and explained as follows: In Section 2, the author’s Chunming Li et.al. Level Set method for image segmentation is presented. Statistical Analysis for the images by calculating the values of probability, Entropy and Variance of un-segmented input image and Bias- corrected image is done in Section 3. Section 4 concludes this paper.

2. Level Set Method
The flow of the level set method for image segmentation can be depicted by Figure 1. The local intensity clustering property is derived for the input image, based on the model of the images with the intensity inhomogeneity; a local criterion function is defined for the said image. Energy minimization function is performed by defining individual membership for each cluster’s of the image. A Bias- Field equivalent to the intensity inhomogeneity is generated. Thus energy minimization provides with a homogenous image which is termed as the bias corrected image. Then a statistical analysis of performed in which values of probability, entropy and variance is calculated which in turns provides us with the extent of success in image segmentation of Brain MR Image using the level set function methodology.

**Fig. 1**: Block diagram for the proposed method

2.1. Bias Field Formulation
The bias field in a brain MR image can be assumed to be formed of two components, i.e. one observed image and an unknown field, as follows:

\[ I = bJ + n \quad \text{…… (1)} \]

Where I: observed image, J: true image to be restored, b: an unknown bias field and n: additive Gaussian noise with zero-mean. The aim is to obtain the value of bias field component from the observed image and correct the intensity inhomogeneity. In general, the bias field b is assumed to be slowly varying in the entire image domain. Ideally, the intensity J in each tissue should take a specific value \( v_i \) reflecting the physical property being measured. This property, in conjunction with the spatially coherent nature of each tissue type, implies that the true signal J is approximately a piecewise constant map.
2.2. Local Intensity Clustering Property
Region-based image segmentation methods typically depend on a specific region descriptor of the intensities in each region to be segmented. For image corrupted due to intensity inhomogeneity, it is difficult to give a region descriptor. This also leads to one more problem of overlap between the distributions of the intensities in the $\Omega_1, \Omega_2, \ldots, \Omega_N$ regions. Hence the task of efficient segmentation based on the pixel intensities is incoherent. However, the property of local intensities is simple, which can be effectively used in the formulation of the level set method for image segmentation with simultaneous estimation of the bias field.

2.3. Energy Formulation
The local intensity clustering property explained above exemplifies that the intensities in the neighborhood can be classified into $N$ clusters, with centres $m_i \approx b(x) c_i$, $i = 1, 2, 3, 4 \ldots N$. Hence, standard K-means clustering to classify these local intensities can be applied. In particular, for the intensities $I(x)$ in the neighborhood $\Omega_i$, the K-means algorithm is an iterative process [15] to minimize the clustering which can be presented as follows

$$F_y = \sum_{i=1}^{N} \int_{Ox \cap \Omega_i} |I(x) - m_i|^2 dx.$$  

where $m_i$ : cluster center of the i-th cluster, $u_i$ : membership function of the region $\Omega_i$, i.e. to be determined, $u_i(x) = 1$ for $x \in \Omega_i$ and $u_i(x) = 0$, for $x \notin \Omega_i$.

In view of the clustering criterion in (2) and the approximation of the cluster center by $m_i = b(x) c_i$, a clustering criterion for classifying the intensities in $\Omega_i$ is defined as

$$E_y = \sum_{i=1}^{N} \int_{Ox \cap \Omega_i} K(y-x) |I(x) - b(y)c_i|^2 dx.$$  

Where $K(y-x)$ is introduced as a nonnegative window function, also called kernel function, such that for $K(y-x) = 0$ for $y \notin \Omega_i$, with the window function, the clustering criterion function $E_y$ can be rewritten as

$$E_y = \sum_{i=1}^{N} \int_{Ox \cap \Omega_i} K(y-x) |I(x) - b(y)c_i|^2 dx.$$  

2.4. Two-Phase Level Set Formulation
A local clustering criterion function for the intensities in a neighborhood of each point is established based on the local intensity clustering property, from a generally accepted model of images with intensity inhomogeneities. An energy function is defined by integrating the local clustering criterion over the neighborhood center, which is converted to a level set formulation. Using an interleaved process of level set evolution and estimation of the bias field minimization of this energy is achieved [8]. As an important application, the level set method can be used for segmentation and bias correction of magnetic resonance (MR) images.

A circular neighborhood with a relatively small radius $\rho$ centred at each point $x$ in the image domain $\Omega$, is considered which is defined by $Ox \_ \{y: |y-x| \leq \rho\}$. The partition $\{\Omega_i\}_{i=1}^{N}$ induces a partition of the neighborhood $\Omega_i$, i.e., $\{\Omega_i \cap \Omega_i\}_{i=1}^{N}$. For a smooth function $b$, the values $b(y)$ for all $y$ in the circular neighborhood $Ox$ can be well approximated by $b(x)$, which is at the center of $Ox$. Therefore, the intensities $b(y) J(y)$ in each sub region $Ox \cap \Omega_i$ is approximately the constant $b(x) c_i$. Thus the following approximation is arrived,

$$b(y)J(y) \approx b(x) c_i \text{ for } y \in \Omega_i \cap \Omega_i.$$  

The constants $b(x) c_i$ can be considered as the approximations of the cluster centres (or means) of the clusters $\{I(y): y \in \Omega_i \cap \Omega_i\}$ within the neighborhood $Ox$. Therefore, the intensities in the neighborhood $Ox$ are around $N$ distinct cluster centres $m_i \approx b(x) c_i$. The multiplicative components $b(x)$ and $c_i$ of the cluster centres $m_i \approx b(x) c_i$ can be estimated as the following.

Here first consider the two-phase case: the image domain is $\Omega$ segmented into two disjoint regions $\Omega_1$ and $\Omega_2$. In this case, a level set function $\Phi$ is used to represent the two regions $\Omega_1$ and $\Omega_2$. The regions $\Omega_1$ and $\Omega_2$ can be represented
with their membership functions defined by $M_1 (\Phi) = H (\Phi)$ and $M_2 (\Phi) = 1 - H (\Phi)$, respectively, where $H$ is the Heaviside function [13]. Thus, for the case of $N=2$, the energy in (5) can be expressed as the following level set formulation:

$$
\mathcal{E} = \int \sum_{i=1}^{N} k|y-x||\nabla f| H_{\epsilon}(y) d\gamma.
$$

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The experimental results of the level set method are depicted in above figure. It can be seen the bias corrected image using level set function is more suitable for proper and correct diagnosis of Brain MR Images, due to reduction in the intensity inhomogeneities as compared to that of the original image. Now after the Bias correction the next step is to obtain values of statistical parameters of the Brain MR images like probability, entropy and variance.

3. Statistical Properties of Images

3.1. Probability

Calculate Probability for Each Pixel and Weight Value

Each pixel in the image needs to be scanned for intensity values. The total number of pixels must be calculated along with the number of occurrences of each pixel intensity value. The probability estimate of each pixel intensity value can then be calculated by dividing the occurrences over the number of pixels in the image. These values are then weighted by subtracting the resultant probability. The calculation for each pixel value for an image $M$ is as follows:

$$
P (n) = 1 - \frac{S \text{ occurrences of } n \text{ in } M}{S \text{ pixels in } M}
$$

The figure 3a. and 3b. shows that the probability of original image varies to that of corrected image.
3.2. Entropy

Entropy means to consider the neighbourhood of the pixel of an image. Entropy is a measure of disorder, or more precisely unpredictability. The probability of a image intensity occurring at particular pixel in 'k', where 'k' is the set of all pixels in a image, is defined as P {n} log (P {n}). The sum of all of these probability makes the Entropy of 'P', so,

\[ G (K) = - \sum P \{n\} \log (P \{n\}) \]  

Where \( n \in K \), \( P \{n\} \), is the probability mass function of particular pixel in 'k'. As its magnitude increases more uncertainty and thus more information is associated with the source. If the source symbols are equally probable, the entropy or uncertainty of Equation 8 is maximized and the source provides the greatest possible average information per source symbol. In this paper probability image is used as an input image to find entropy. Here it becomes necessary to select analyzing window size to find entropy for neighborhood of each pixel in the input image [11]. For this paper 3x3 window size is selected for to find entropy. By moving analyzing window on complete image, calculating entropy for each window, new entropy image was formed by replacing the central pixel of the particular window by entropy and displayed as entropy image.

The idea of maximization of mutual information comes out of communication theory in its attempt to determine which parts of a signal are used at both ends of a communication line. This idea was applied to the field of image registration by two independent groups, Viola and Wells in 1995 and Collignon et al. in 1997. The basic ideas of the entropy-based registration, as the theory is also called, uses the probability of occurrence of a pixel value at any point in the image to maximize the area of the images in which the pixel probabilities are shared. This can lead to error in images in which the background dominates or in which significant noise can alter pixel probability.

![Fig. 4a. Plot for Entropy of Original Image](image)

![Fig. 4b. Plot for Entropy of Corrected Image](image)

3.3. Variance

Variance is a measure of the dispersion of a set of data points around their mean value. It is a mathematical expectation of the average squared deviations from the mean. The variance of a real-valued random variable is its second central moment and it also happens to be its second cumulant. The variance of random variable is the square of its standard deviation.

\[ \text{Variance (X)} = \text{E}[(X - \mu)^2] \]  

if \( \mu = \text{E}(X) \), where \( \text{E}(X) \) is the expected value (mean) of the random variable \( X \). That is, it is the expected value of the square of the deviation of \( X \) from its own mean. It can be expressed as "The average of the square of the distance of each data point from the mean", thus it is the mean squared deviation. This same definition is followed here for images. Initially probability of image is calculated. Since the probability values are very small, equalized probability image is applied as an input image to find variance of probability image by using 3x3 window size, same as in case of entropy.

![Fig. 5a. Plot for Variance of Original Image](image)

![Fig. 5b. Plot for Variance of Corrected Image](image)
4. CONCLUSION

In this paper a variational level set framework for segmentation and bias correction of images with intensity inhomogeneities is presented. A local clustering criterion function for the intensities in a neighborhood of each point is established based on the local intensity clustering property, from a generally accepted model of images with intensity inhomogeneities. The energy of the level set functions represents a partition of the image domain and a bias field that accounts for the intensity inhomogeneity. Segmentation and bias field estimation are therefore jointly performed by minimizing the energy functional. This model efficiently utilizes the local image information and therefore able to simultaneously segment and bias correct the images with intensity inhomogeneity. Experimental results prove the probability of image (pixels) varies for both the images and is concentrated for certain pixel values, indicating refinement of image. Entropy diversity of bias corrected image as compared to original image is higher which indicates more information is available in the has changed entropy is maximized and the source provides the greatest possible average information, which in turn helps in obtaining better image segmentation results. Lastly reduction in value of variance for individual pixel indicates less deviation of intensity value to that of mean value for the cluster of pixel in a given neighbourhood.

References


