Performance Analysis of MIMO Network Coding with SISO Physical-Layer Network Coding

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ABSTRACT

In this paper, a two-step communication protocol combined with MIMO (Multiple Input Multiple Output) and network coding is proposed. A three nodes network with two transceiver antennas on relay node is taken. In transmitting phase ZF (Zero-forcing) and MMSE (Minimum Mean Square) detecting techniques are used. In relay node forwarding network coding and Alamouti scheme are exploited together. Theoretical and simulative analyses prove that BER (Bit Error Rate) MMSE in transmitting phase is better than ZF and in relay forwarding MIMO alamouti outperforms the PNC (Physical-layer-network coding).

Keywords: MIMO (Multiple Input Multiple Output), ZF (Zero Forcing), MMSE (Minimum Mean Square) BER (Bit Error Rate) PNC (Physical-layer-network coding), SISO (Single Input Single Output).

1. INTRODUCTION

Network coding helps conveying more information by broadcasting mixed information from an intermediate network node to the receiver nodes, where interference can be canceled by appropriately altering the transmit information as in [1]. In MIMO transmission, multiple antennas are put at both the transmitter and the receiver to improve communication performance. MIMO technology has attracted attention in wireless communication, because it offer significant increase in data throughput and link range without additional bandwidth or transmit power. It achieves this by higher spectral efficiency (more bits per second per hertz of bandwidth) and link reliability or diversity (reduced fading). Because of these properties, MIMO is an important part of modern wireless communication standards. A technique known as Altamonte STC (Space Time Coding) is employed at the transmitter with two antennas. STC allows the transmitter to transmit signals (information) both in time and space, meaning the information is transmitted by two antennas at two different times consecutively. By combining network coding and MIMO technology into relay network, system can benefit throughput improvement from network doing as well as spatial multiplexing, and more reliable transmission from spatial diversity [2-5]. In this paper, a two-way relay network with combined MIMO and network coding is presented. The paper is organized as follows: Section II describes the system model of a three nodes relay network with two transceiver antenna at the relay. Section III is devoted to source node transmission. Section IV presented relay node forwarding. Finally, in sections V and VI simulation results and conclusion are presented.

2. SYSTEM MODEL

Our system model is based on the canonical two-way network shown in Fig.1. Source nodes, N₁ and N₂, out of eachother’s communication range, have messages to exchange. They communicate through a relay node R that is within the range of both N₁ and N₂. We assume that the transmission is organized in consecutive time slots enumerated by i. The source nodes use a single transceiver antenna each. The difference of our system compared to the classical model is that relay node R has two transceiver antennas instead of one. Since the signals originating at N₁ and N₂ are mutually independent from each other and are locally displaced, and since R has two transceiver antennas, the system can be seen as a 2x2 virtual MIMO system with spatial diversity scheme.

3. SOURCE NODE TRANSMISSION

Let us denote the signal transmitted by the source node N₁ in the iᵗʰ time slot by s₁⁽¹⁾(i) and the signal transmitted by the source node N₂ in the (i+1)ᵗʰ time slot by s₂⁽¹⁾(i+1). Similarly, the signals transmitted by the source node N₁ in the iᵗʰ and the (i +1)ᵗʰ time slots shall be denoted b s₁⁽²⁾(i) and s₁⁽²⁾(i+1), respectively. Let us assume that the transmit signals
$s_i^{(1)}(t), s_i^{(1)}(t), s_i^{(1)}(t), s_i^{(2)}(t)$ and $s_i^{(2)}(t)$ are Binary phase shift keying (BPSK) modulated signals, each comprising single bit. Let $E_b$ denote the bit energy and $T_b$ shall be the bit duration. Now, let $b_i^{(1)}$ be the data vector, representing the data, transmitted by the source node $N_1$ in the time slot $i$ using $b_i^{(1)} \in \{0,1\}$ and correspondingly, let $b_i^{(2)}$ denote the data vector at source node $N_2$ in the same time.[7-10]

The relay node $R$ receives these signals via its two transceiver antennas. Let $h_{i}^{(1,1)}(t)$denote the channel impulse response between the first source node $N_1$ and the first antenna of the relay node $R$ in the time slot $i$. Furthermore, let $h_i^{(1,2)}(t)$ denote the channel impulse response between the first source node and the second antenna of the relay node $R$ in the time slot $i$. Similarly, $h_i^{(2,1)}(t)$ and $h_i^{(2,2)}(t)$ represent the channel impulse responses between the second source node $N_2$ and the first as well as the second antenna of the relay node $R$. In this case, the channel impulse responses approximately represent single path channels with negligible time variance; rather, inter-time slot time variations can occur. Hence, the general complex-valued numbers $h_i^{(1,1)}(t)$, $h_i^{(1,2)}(t)$, $h_i^{(2,1)}(t)$ and $h_i^{(2,2)}(t)$ can be used to represent the channel impulse responses. We will take into account that $h_i^{(1,1)}(t)$, $h_i^{(1,2)}(t)$, $h_i^{(2,1)}(t)$ and $h_i^{(2,2)}(t)$ represent Rayleigh-flat-fading channels with variance $\sigma^2$ equal to 1 with the additive white Gaussian noise signals $n_i^{(1)}(t)$ and $n_i^{(2)}(t)$ at the first and the second antenna of $R$ in the time slot $i$ and each having double-sided spectral noise power density $N_0/2$.

3.1 Numerical Analysis

Let us assume that $e_i^{(1)}_{R,i}(t)$ and $e_i^{(2)}_{R,i}(t)$ being the received signals at the first and the second antenna of $R$ in the time slot $i$, the communication system is given by

$$e_i^{(1)}_{R,i}(t) = h_i^{(1,1)}(t) s_i^{(1)}(t) + h_i^{(1,2)}(t) s_i^{(2)}(t) + n_i^{(1)}(t)$$

$$e_i^{(2)}_{R,i}(t) = h_i^{(2,1)}(t) s_i^{(1)}(t) + h_i^{(2,2)}(t) s_i^{(2)}(t) + n_i^{(2)}(t)$$

Equations (4) and (5) can be represented in matrix form as

$$\begin{bmatrix}
  e_i^{(1)}_{R,i}(t) \\
  e_i^{(2)}_{R,i}(t)
\end{bmatrix} =
\begin{bmatrix}
  h_i^{(1,1)}(t) & h_i^{(1,2)}(t) \\
  h_i^{(2,1)}(t) & h_i^{(2,2)}(t)
\end{bmatrix}
\begin{bmatrix}
  s_i^{(1)}(t) \\
  s_i^{(2)}(t)
\end{bmatrix} +
\begin{bmatrix}
  n_i^{(1)}(t) \\
  n_i^{(2)}(t)
\end{bmatrix}$$

Similarly, $e_i^{(1)}_{R,i+1}(t)$and $e_i^{(2)}_{R,i+1}(t)$ being the received signals at the first and the second antenna of $R$ in the time slot $i+1$. The relay node $R$ determines the information contained in the received signals, yielding the detected versions $b_i^{(1)}$ and $b_i^{(2)}$ of $b_1^{(1)}$ and $b_1^{(2)}$. Different detection techniques, for example Zero-forcing (ZF) based V-BLAST as in [4] or its Minimum mean square error (MMSE) as explained in [5] could be used.

3.2 MIMO ZF receiver

In this section, we will try to improve the bit error rate performance by trying out Successive Interference Cancellation (SIC). We will assume that channel is a flat fading Rayleigh multipath channel and the modulation is BPSK. Equations (4) and (5) can be represented in matrix form as shown in equation (6).

Equivalently,

$$e_i = h s + n$$

\[\text{Figure 1: Three Nodes Relay Network with Two-Step transmission}\]
where \( s = b \cdot p \) in this, \( b \in \{1,+1\} \) and \( p = 1 \) = BPSK impulse power. Now, equation (7) can also be written as:

\[
e = hb + n
\]

(8)

To solve for \( b \), the Zero-Forcing (ZF) in [4] linear detector for meeting the constraint is given by:

\[
W = (h^H h)^{-1} h^H
\]

(9)

Where \( H = \) Hermitian Transpose. To do the Successive Interference Cancellation (SIC), the receiver needs to perform the following:

Using the ZF equalization approach described above, the receiver can obtain an estimate of the two transmitted symbols \( b^{(1)} \) and \( b^{(2)} \):

\[
\begin{bmatrix}
\tilde{b}^{(1)} \\
\tilde{b}^{(2)}
\end{bmatrix} = (h^H h)^{-1} h^H
\begin{bmatrix}
e^{(1)} \\
e^{(2)}
\end{bmatrix}
\]

(10)

3.3 BER for ZF receiver

Till now we have seen that \( h \) is used to represent the channel impulse response and is called Rayleigh Random Variable. It is a identical distributed Gaussian random variable with mean 0 and variance \( \sigma^2 \). The magnitude \(|h|\) has a probability density function:

\[
p(h) = \frac{h}{\sigma^2} e^{-\frac{h^2}{2\sigma^2}}
\]

(11)

\[
e = hb + n
\]

(12)

where \( e \) is the received symbol, \( h \) is complex scaling factor corresponding to Rayleigh multipath channel \( b \) is the transmitted symbol (taking values +1’s and -1’s) and \( n \) is the Additive White Gaussian Noise (AWGN)

Assumptions:

1. The channel is flat fading; In simple terms, it means that the multipath channel has only one tap. So, the convolution operation reduces to a simple multiplication.
2. The channel is randomly varying in time; meaning each transmitted symbol gets multiplied by a randomly varying complex number. Since modeling is a Rayleigh channel, the real and imaginary parts are Gaussian distributed having mean 0 and variance 1/2.
3. The noise has the Gaussian probability density function with

\[
p(n) = \frac{1}{2\pi\sigma^2} e^{-\frac{(n-\mu)^2}{2\sigma^2}}
\]

with \( \mu = 0 \) and \( \sigma^2 = N_0/2 \).
4. The channel \( h \) is known at the receiver. Equalization is performed at the receiver by dividing the received symbol \( e \) by the apriori known as \( h \) i.e.

\[
\tilde{e} = \frac{e}{h} = \frac{hb + n}{h} = b + \tilde{n}
\]

(14)

Where, \( \tilde{n} = n/h \)

In the BER computation in AWGN, the probability of error for transmission of either +1 or -1 is computed by integrating the tail of the Gaussian probability density function for a given value of bit energy to noise ratio \( E_b/N_0 \) The bit error rate is,

\[
p_b = \frac{1}{2} erfc\left( \sqrt{\frac{E_b}{N_0}} \right)
\]

(15)

However, in the presence of channel \( h \), the effective bit energy to noise ratio is \(|h|^2 E_b/N_0 \). So, the bit error probability for a given value of \( h \) is,

\[
p_b = \frac{1}{2} erfc\left( \frac{|h|^2 E_b}{N_0} \right) = \frac{1}{2} erfc\left( \sqrt{\gamma} \right)
\]

(16)

Where, \( \gamma = |h|^2 E_b/N_0 \). To find the error probability over all random values of \( |h|^2 \), one must evaluate the conditional probability density function \( P_{(h|b)} \) over the probability density function of \( \gamma \).

Probability density functions of \( \gamma \):

From chi-square random variable, we know that if \( |h| \) is a Rayleigh distributed random variable, then \(|h|^2 \) is chi-square distributed with two degrees of freedom. Since \(|h|^2 \) is chi square distributed, \( \gamma \) is also chi square distributed. The probability density function of is,
\[ P_\gamma = \left( \frac{1}{E_b / N_o} \right) e^{-\frac{\gamma}{E_b / N_o}} \]  

(17)

Error probability: the error probability is,

\[ P_b = \int_0^1 \frac{1}{2} \text{erfc} \left( \sqrt{\gamma} \right) p(\gamma) d\gamma \]  

(18)

This equation reduces to

\[ P_b = 1 - \left( \frac{E_b / N_o}{1 + E_b / N_o} \right)^{1/4} \]  

(19)

or

\[ P_b = 1 - \left( \frac{\gamma_b}{1 + \gamma_b} \right)^{1/4} \]  

(20)

3.4 MIMO MMSE receiver

In this section, we extend the concept of successive interference cancellation to the MMSE equalization in [5] and simulate the performance. We will assume that the channel is a flat fading Rayleigh multipath channel and the modulation is BPSK. In the first time slot i.e. i, the received signal on the first receive antenna is shown in equation (4) and the received signal on the second receive antenna is given in equation (5). For convenience, the equations (4) and (5) can be represented in matrix notation as given by equation (6).

Equivalently,

\[ e = hs + n \]  

(21)

where,

\[ s = b \cdot p \]  

in this b Є {−1, +1} and p=1=BPSK impulse power. Now, equation (21) can also be written as

\[ e = hb + n \]  

(22)

The Minimum Mean Square Error (MMSE) approach tries to find a coefficient which minimizes the criterion,

\[ E = \{[We -b][We-b]^H\} \]  

(23)

Solving,

\[ W = [h^H h + N_o I]^{-1} \]  

(24)

Using the Minimum Mean Square Error (MMSE) equalization, the receiver can obtain an estimate of the two transmitted symbols \( b^{(1)} \) and \( b^{(2)} \) i.e.

\[ \begin{bmatrix} \hat{b}^{(1)} \\ \hat{b}^{(2)} \end{bmatrix} = (h^H h + N_o I)^{-1} h^H \begin{bmatrix} e^{(1)} \\ e^{(2)} \end{bmatrix} \]  

(25)

3.5 RELAY NODE FORWARDING

The decode-and-forward (DF) forwarding scheme is deployed in the second step i.e in relay node forwarding. The relay node R re-encodes and re-modulates the received signals \( \hat{b}_1^{(1)} \) and \( \hat{b}_1^{(2)} \), the detected version of \( b^{(1)} \) and \( b^{(2)} \) forwards them to source nodes. In this section, network coding and space-time block coding (STBC), like e.g. the Alamouti scheme [3] and [6] are exploited together to get coding multiplexing and spatial diversity gain. Network coding will improve network throughput, and STBC will help overcome channel fading [11-15]. In order to validate this paradigm, a simple network coding scheme will be applied, first. Let \( r_{is}^{(1)}(t) \) and \( r_{is}^{(2)}(t) \) represent the signals transmitted over the first and the second antenna of the relay node R, respectively, in the time slot (i+2) to both the source nodes \( N_1 \) and \( N_2 \). The network coding scheme is given by the bit-wise multiplication of the signals received in the i\(^{th}\) and the (i+1)\(^{th}\) time slots according to the following rule:

\[ r_{is}^{(1)}(t) = \hat{b}_1^{(1)} \cdot \hat{b}_1^{(2)} \cdot P(t) \]  

(26)

\[ r_{is}^{(2)}(t) = \hat{b}_{(i+1)}^{(1)} \cdot \hat{b}_{(i+1)}^{(2)} \cdot P(t) \]  

(27)

Similarly, Let \( r_{is}^{(1)}(t) \) and \( r_{is}^{(2)}(t) \) represent the signals transmitted over the first and the second antenna of the relay node R, respectively, in the time slot (i + 3) to both the source nodes \( N_1 \) and \( N_2 \) and are given by

\[ r_{is}^{(1)}(t) = \hat{b}_{(i+3)}^{(1)} \cdot \hat{b}_{(i+3)}^{(2)} \cdot P(t) \]  

(28)
After network coding, the Alamouti scheme is applied with the formats
\[ r_{i+2}^{(2)}(t) = \left( f_{i+3}^{(1)}(t) \right)^* \] (30)
\[ r_{i+2}^{(2)}(t) = \left( f_{i+3}^{(1)}(t) \right)^* \] (31)

The signal received at the source node \( N_i \) in the time slot \( (i+2) \) and \( (i+3) \) is given by
\[ e^{(1)}_{N,i+2}(t) = h_{i+2}^{(1,1)} f_{i+2}^{(1)}(t) + h_{i+2}^{(1,2)} r_{i+2}^{(2)}(t) + n_{i+2}^{(1)}(t) \] (32)
\[ e^{(1)}_{N,i+3}(t) = h_{i+3}^{(1,1)} f_{i+3}^{(1)}(t) + h_{i+3}^{(1,2)} r_{i+3}^{(2)}(t) + n_{i+3}^{(1)}(t) \] (33)

On applying Alamouti scheme in equation (33) from equations (30) and (31) we get
\[ e^{(1)}_{N,i+3}(t) = h_{i+3}^{(1,1)} f_{i+3}^{(1)}(t)^* + h_{i+3}^{(1,2)} f_{i+3}^{(2)}(t)^* + n_{i+3}^{(1)}(t) \] (34)

On rearranging the terms we get
\[ e^{(1)}_{N,i+3}(t) = h_{i+3}^{(1,2)} f_{i+3}^{(2)}(t)^* - h_{i+3}^{(1,1)} f_{i+3}^{(1)}(t)^* + n_{i+3}^{(1)}(t) \] (35)

Now, equations (32) and (35) can be represented in matrix form as
\[
\begin{bmatrix}
   e^{(1)}_{N,i+2}(t) \\
   e^{(1)}_{N,i+3}(t)
\end{bmatrix} =
\begin{bmatrix}
   h_{i+2}^{(1,1)} & h_{i+2}^{(1,2)} \\
   (h_{i+3}^{(1,2)})^* & -(h_{i+2}^{(1,2)})^*
\end{bmatrix}
\begin{bmatrix}
   f_{i+2}^{(1)}(t) \\
   f_{i+3}^{(2)}(t)^*
\end{bmatrix} +
\begin{bmatrix}
   r_{i+2}^{(1)}(t) \\
   r_{i+3}^{(2)}(t)
\end{bmatrix} +
\begin{bmatrix}
   n_{i+2}^{(1)}(t) \\
   n_{i+3}^{(2)}(t)
\end{bmatrix}
\] (36)

Similarly, the signals received at the source node \( N_2 \) in the time slots \( (i+2) \) and \( (i+3) \) can be represented in matrix form as
\[
\begin{bmatrix}
   e^{(2)}_{N,i+2}(t) \\
   e^{(2)}_{N,i+3}(t)
\end{bmatrix} =
\begin{bmatrix}
   h_{i+2}^{(2,1)} & h_{i+2}^{(2,2)} \\
   (h_{i+3}^{(2,2)})^* & -(h_{i+3}^{(2,1)})^*
\end{bmatrix}
\begin{bmatrix}
   f_{i+2}^{(2)}(t) \\
   f_{i+3}^{(1)}(t)^*
\end{bmatrix} +
\begin{bmatrix}
   r_{i+2}^{(2)}(t) \\
   r_{i+3}^{(1)}(t)
\end{bmatrix} +
\begin{bmatrix}
   n_{i+2}^{(2)}(t) \\
   n_{i+3}^{(1)}(t)
\end{bmatrix}
\] (37)

Now the source nodes \( N_1 \) and \( N_2 \) detect \( \hat{b}_{i+2}^{(1)} \) and \( \hat{b}_{i+3}^{(1)} \), then estimate the combined signal using Maximum likelihood (ML) decision rule. The corresponding bit error probability of this 2×1 MISO system [7], is given by
\[ P_{e,Alam} = \frac{1}{2} \left( 1 - \frac{1}{2} \sqrt{\frac{\gamma_e}{\gamma_b + \frac{3}{2} - \frac{\gamma_b}{2}}} \right) \] (38)

**Figure 2:** Overall BER Comparison of ZF and MMSE in transmitting phase.
5. SIMULATION RESULTS
In this paper, we compare the performance of the physical layer network coding (PNC) with the newly proposed Multiple Input Multiple Output Network Coding Scheme. Both schemes use BPSK, Rayleigh-flat-fading channel model is assumed between each antenna pair in the system. Fig.2 and Fig.3 provides a comparison of the overall bit error performance obtained in the three nodes relay network. In the source node transmission of the MIMO network coding scheme, two different spatial multiplexing receivers, ZF and MMSE are deployed at the relay node R. The BER comparison of both is shown in Fig.2. In the relay node forwarding of MIMO network coding scheme (BER comparison with PNC shown in Fig.3), the source node N1 or N2 applies Maximum Likelihood (ML) symbol detector followed by a binary exclusive or (XOR) operation to extract BPSK signals from the other. At last we found that MIMO network coding outperforms the PNC.

![Figure 3: BER for BPSK modulation with 2×1 MISO Alamouti and PNC (Rayleigh Channel)](image)

6. CONCLUSION
In this paper, a new two-way protocol with combination of MIMO and network coding has been proposed. The benefits of MIMO network coding come from two transreceiver antennas at the relay node R, where both code multiplexing and spatial diversity gains can be jointly exploited. Theoretical and simulative performance analyses show that MIMO network coding protocol with MMSE detection at the transmitting phase outperforms PNC schemes and provides more robust and efficient transmission.

REFERENCES

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